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PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON

From December 1921 to August 1922

VOL. XXXIV.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY OF LONDON.

SESSION 1921—1922.

October 28, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. **BRAGG**, K.B.E., M.A., F.R.S., President, in the Chair.

The following Papers were read :—

1. "On the Use of Anderson's Bridge for the Measurement of the Variations of the Capacity and Effective Resistance of a Condenser with Frequency," by S. **BUTTERWORTH**, M.Sc. (from the National Physical Laboratory).

2. "Notes on Earth Capacity Effects in Alternating Current Bridges," by S. **BUTTERWORTH**, M.Sc. (from the National Physical Laboratory).

3. "An Automatic Voltage Regulator," by Mr. F. G.H. **LEWIS** (with demonstration).

4. "The Flow of Viscous Liquids Through Slightly Conical Tubes," by Prof. A. S. **HEMMY**.

November 11, 1921.

Meeting held at the Imperial College of Science.

The Presidental Address was delivered by Prof. Sir W. H. **BRAGG**, K.B.E., M.A., F.R.S., who took as his subject "The Structure of Organic Crystals."

November 25, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. **BRAGG** in the Chair.

A Discussion on Hygrometry was held, the following Papers being first read :—

1. "Some Modified Forms of Hygrometers," by Dr. **EZER GRIFFITHS** (National Physical Laboratory).

2. "The Theory of the Hair Hygrometer," by F. J. W. WHIPPLE, M.A. (Meteorological Office).

3. "The Rationale of Glaisher's System of Hygrometry," by F. J. W. WHIPPLE, M.A.

4. "The Wet and Dry Bulb Hygrometer," by Principal SKINNER, M.A.

5. "Determination of the Proper Constant in Apjohn's Formula for Use in Aeroplanes," by WATSON WATTS.

6. "A New Form of Absorption Hygrometer," by H. G. MAYO, M.A., and Prof. A. M. TYNDALL.

7. "A Thermal Hygrometer," by Prof. A. M. TYNDALL and Prof. A. P. CHATTOCK.

The meeting was preceded by Demonstrations and an Exhibition of Apparatus.

December 9, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. BRAGG, and subsequently Dr. ALEXANDER RUSSELL, in the Chair.

The following Papers were read :—

1. "The Average Range of β-Particles in Different Metals," by G. A. SUTHERLAND, M.A., and L. H. CLARK, B.Sc.

2. "The Sensitivity of Ballistic Galvanometers," by Prof. ERNEST WILSON.

3. "The Estimation of the Radium Content of Radio-active Luminous Compounds," by E. A. OWEN, D.Sc., and WINIFRED E. FAGE, B.Sc. (National Physical Laboratory).

The following Paper was taken as read in the absence of the Author :—

4. "The Determination of the Damping Decrement of a Tuning Fork," by Prof. R. LL. Jones.

Dr. F. LL. HOPWOOD Exhibited a number of Optical Experiments :—

1. A New Effect with a Modified Strouhal Apparatus.
2. The Auto Stroboscope.
3. An Incandescent Colour Top.

January 4 and 5, 1922.

The Twelfth Annual Exhibition of Electrical, Optical and other Physical Apparatus was held by the Physical Society and the Optical Society at the Imperial College of Science.

The following Discourses were delivered :—

On January 4th and 5th :—"The Johnsen-Rahbek Electrostatic Telephone and its Predecessors," by ALAN A. CAMPBELL SWINTON, F.R.S.

On Janary 4th :—" Radium : its application in Peace and War," by Mr. F. HARRISON GLEW. Professor F. J. CHESHIRE, C.B.E., in the chair.

On January 5th :—" The Employment of Coarse Wire Gratings in Astronomy," by Sir FRANK WATSON DYSON, F.R.S., F.R.A.S., Astronomer Royal.

January 27, 1922.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. BRAGG in the Chair.

The following Papers were read :—

1. " On the Diffusion of Solutions," by T. H. LITTLEWOOD, M.A., B.Sc. (University College, Reading).

2. " On a Special Apparatus for the Measurement at Various Temperatures of the Thomson Effect in Wires," by H. R. NETTLETON, M.Sc. (Birkbeck College, London).

3. " A Defect in the Sprengel Pump : its Causes and a Remedy," by J. J. MANLEY, M.A. (Magdalen College, Oxford).

February 10, 1922.

Annual General Meeting.

Held at the Imperial College of Science.

Prof. Sir W. H. BRAGG in the Chair.

GENERAL BUSINESS.

The Annual Report was read by the Secretary, Mr. F. E. SMITH, O.B.E., F.R.S., and the Report of the Treasurer was read by the Treasurer, Mr. W. R. COOPER, M.A., B.Sc. Both Reports were unanimously adopted.

ANNUAL REPORT.

During the year 1921, 14 ordinary Science Meetings and one Special General Meeting of the Society were held. Of these the Special General Meeting and 13 ordinary meetings were held at the Imperial College of Science ; the remaining ordinary meeting was held, by invitation of the President, at University College.

Two of the ordinary meetings were devoted to discussions. The first of these was on " Absolute Measurements of Electrical Resistance and Instruments based on the

Temperature-Variation of Resistance," and the second was on "Hygrometry." Exhibits and demonstrations were arranged in connection with both discussions.

The Guthrie Lecture was delivered by Professor A. A. Michelson of the University of Chicago, who chose as his subject "Some Recent Applications of Interference Methods." The meeting was one of the most successful ever held by the Society.

On June 10th Sir Ernest Rutherford, F.R.S., delivered a special lecture on "The Stability of Atoms," and on November 11th Sir W. H. Bragg, F.R.S., gave his Presidential address, choosing as his subject "The Structure of Organic Crystals."

These discussions and lectures, five in all, were most successful; each was attended by over 130 Fellows and visitors and the average attendance was 155.

Excluding the Discussions, the Guthrie Lecture and the two special addresses, the average attendance at the ordinary meetings was 58.

In co-operation with the Optical Society the Eleventh Exhibition of Scientific Apparatus was held on January 5th and 6th. Fifty firms exhibited apparatus and many of them arranged very interesting demonstrations. About 2,500 Fellows and visitors attended the Exhibition. Dr. Morrison, on behalf of Emeritus Professor Archibald Barr, gave two lectures on "The Optophone, an instrument which enables the totally blind to read ordinary print," Professor Sir W. H. Bragg, F.R.S., delivered a lecture on "Sounds in Nature," and Professor C. R. Darling gave a discourse on "Some Unusual Surface Tension Phenomena." The lectures were experimentally illustrated and were attended by large audiences.

Early in the year the Council considered the size of page, the type and the quality of paper for the "Proceedings," and as a result of a Report by a special committee all of these were changed. The size of the new "Proceedings" is very similar to that of the "Proceedings of the Royal Society" and identical with the "Proceedings" of the Optical Society.

It is regretted that the arrangement has terminated by which Fellows obtained at a reduced rate the "Proceedings of the Royal Society." The Council is of opinion that the publications of any scientific society should be sold at a slightly reduced rate to Fellows of other Scientific Societies, and has asked the Conjoint Board to consider the practicability of such an arrangement.

Professor A. Fowler, F.R.S., has prepared for the Society a Report on "Series in Line Spectra." This will shortly be issued gratis to Fellows. A second Report on "Atomic Structure," by Professor Bohr, is in preparation.

An appeal for funds for a Duddell Memorial medal resulted in a sum of about £650 being raised. The Committee dealing with the memorial have asked Mrs. Mary G. Gillick to undertake the preparation of the medal, and it is hoped that the medal will be ready during the early part of 1922. The Society is greatly indebted to Mr. R. S. Whipple, who acted as Honorary Secretary to the Memorial Committee.

Mr. J. Guild, who acted as Assistant Secretary to the Society, for the purposes of its publications, resigned in June, and Capt. C. W. Hume was appointed Assistant Secretary in July. Mr. Guild had served the Society for nine years, and his resignation was received with much regret.

The number of Honorary Fellows on the Roll on December 31st, 1921, was nine, and the number of ordinary Fellows and Students was 534.

Forty-eight new Fellows and four Students were elected during the year and there were seven resignations.

The Society has to record with regret the deaths of Professor Lippmann, Mr. H. Durham, Mr. A. Howard, Mr. H. T. Gerrans, and Mr. H. E. Axford.

Professor Lippmann was an Honorary Fellow of the Society since 1907, Mr. Durham and Mr. Gerrans were Life Members of the Society, Mr. Howard was a Member for nearly 40 years, and Mr. H. E. Axford was a Student Member.

REPORT OF THE TREASURER.

Notwithstanding increased activities on the part of the Society, the year's revenue has been sufficient to meet the year's expenditure and to leave a small margin. The income has increased, being £1,660 5s. 4d., as compared with £1,513 2s. 8d. for the previous year. The increase has been due mostly to payments by exhibitors to meet the cost of printing the catalogue of the Exhibition, the Council having decided that this cost was becoming too high to impose on the Society as hitherto without any charge being made. The only other considerable increase in income is due to the steady rise in membership, which is a most satisfactory feature. The loss in revenue due to reductions in subscriptions under the arrangement made with the Institute of Physics amounted to £14 6s. 9d. The sum realised by sales of publications has fallen slightly, notwithstanding the increased prices that have been in force. This is probably due partly to the fact that the sales have depended less upon special reports than in the year before.

The expenditure on printing has been very heavy, not only on the "Proceedings" and usual publications (an increase of £88 5s. 9d.), but on special publications (£163 14s. 7d.). The latter include a reprint of Prof. Eddington's Report on Relativity, the discussion on Reflecting Surfaces, held with the Optical Society, and the discussion on Colloids, held with the Faraday Society. The total is thus £1,095 15s. 7d., as compared with £843 15s. 3d. in the previous year.

With regard to the Balance Sheet, it may be said that the subscriptions in arrears are becoming less serious. The investments have been valued at market prices through the courtesy of the Manager of the London County Westminster & Parr's Bank, and for the first time for many years they show a small appreciation. The deposit at the bank has increased by £150, and the total cash (including deposit), after adjustment, amounts to £427 5s. 7d., as compared with £408 17s. 1d. in the previous accounts.

Having regard to the high costs still prevailing the position may be said to be very satisfactory.

VOTES OF THANKS.

The following Votes of Thanks were passed :—

To the Hon. Auditors (Mr. R. S. WHIPPLE and Dr. E. A. OWEN), proposed by Dr. H. BORNS and seconded by Mr. B. W. CLACK.

To the Retiring Officers and Council, proposed by Col. O'MEARA and seconded by Mr. R. W. PAUL.

To the Governing Body of the Imperial College of Science, proposed by Prof. ECCLES and seconded by Dr. RANKINE.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1921.

INCOME.	£	s.	d.	£	s.	d.
Entrance Fees	49	7	0			
Subscriptions by Fellows direct	596	8	11			
" through Institute of Physics	79	19	3			
" Voluntary*	15	15	0			
" by Students	5	5	0			
" Arrears paid	36	17	6			
" Paid in Advance	39	7	6			
" for "Science Abstracts" and Advance Proofs	30	7	6			
				853	7	8
Composition Fees				73	10	0
Dividends:—						
Furness Debenture Stock	11	3	10			
Midland Railway	28	0	0			
Metropolitan Board of Works	7	0	0			
Lancaster Corporation Stock	8	8	0			
New South Wales Stock	8	17	10			
London, Brighton & South Coast Railway	17	10	0			
Great Eastern Railway	14	0	0			
India 3½% Stock	17	10	0			
Funding Loan	18	4	0			
				130	13	8
Income Tax refunded				44	18	4
Interest on deposit account				21	3	6
Sales of Publications (Fleetway Press, Ltd.)				426	18	11
Optical Society (for Exhibition)				11	0	0
Exhibitors' payments on account of catalogue				98	13	3
				£1,660	5	4

EXPENDITURE.	£	s.	d.	£	s.	d.
"Science Abstracts"				297	18	6
Fleetway Press, Ltd.:—						
"Proceedings"	605	17	5			
Bulletin	65	4	3			
Distribution (Postage)	82	14	7			
General, including printing for Exhibition	178	4	9			
				932	1	0
Sundry Publications:—						
Report on Relativity (reprint)	56	17	6			
Discussion on Reflecting Surfaces (report)	40	1	1			
Discussion on Colloids (report)—reserve	66	16	0			
				163	14	7
Reporting	71	14	5			
Refreshments and Attendance	96	3	2			
Petty Cash—						
Secretaries' Expenses	23	16	9			
Treasurer's Expenses	21	8	6			
Guthrie Lecture (Honorarium)	10	0	0			
Periodicals	3	17	8			
Insurance	2	3	0			
F. S. Spiers (registered address)	2	2	0			
Bank Charges		3	0			
Cheque Book		10	0			
				231	18	6
Conjoint Board of Scientific Societies				10	0	0
Total Expenditure for the Year				1,635	12	7
Balance, being excess of income over expenditure				24	12	9
				£1,660	5	4

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

ROBERT S. WHIPPLE } *Honorary Auditors.*
E. A. OWEN

February 7th, 1922.

* Voluntary subscriptions are those paid by Fellows who have compounded at the low rate of £10.

BALANCE SHEET AT DECEMBER 31ST, 1921.

ASSETS.	£	s.	d.	£	s.	d.
Subscriptions in arrears	59	6	6			
Less reserve for subscriptions probably unrealisable	25	0	0			
				34	6	6
Investments (valued at Dec. 31):—						
£533 Furness Railway 3 per cent. Debenture Stock	288	0	0			
£1,600 Midland Railway 2½ per cent. Perpetual Preference Stock	656	0	0			
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	174	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	208	0	0			
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	235	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock	240	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	350	0	0			
£500 India 3½ per cent. Stock	290	0	0			
£650 4% Funding Loan, 1960-90	494	0	0			
				2,935	0	0
Stock of Publications (Treasurer's valuation)				400	0	0
Cash at Bank, on Deposit	450	0	0			
Cash at Bank, Current Account	119	14	4			
	569	14	4			
Adjustment for outstanding cheques, &c.	142	11	4			
				427	3	0
Cash in hand (Treasurer's Petty Cash)					2	7
				£3,796	12	1

LIABILITIES.	£	s.	d.
Life Compositions	1,973	0	0
Balance, General Fund	1,823	12	1
	£3,796	12	1

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

ROBERT S. WHIPPLE } *Honorary Auditors.*
E. A. OWEN

February 7th, 1922.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1921

	£	s.	d.
133 Fellows paid £10	1,330	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20 10s.	20	10	0
11 Fellows paid £21	231	0	0
11 Fellows paid £31 10s.	346	10	0
159	£1,973	0	0

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*

February 7th, 1922.

ROBERT S. WHIPPLE } *Honorary Auditors.*
E. A. OWEN }

ELECTION OF OFFICERS AND COUNCIL.

The election of Officers and Council for the year 1922-1923 was carried out, with the following results :—

President.—Alexander Russell, M.A., D.Sc.

Vice-Presidents (who have filled the office of President).—Sir Oliver J. Lodge, D.Sc., F.R.S., Sir Richard Glazebrook, K.C.B., D.Sc., F.R.S., C. Chree, Sc.D., LL.D., F.R.S., Prof. H. L. Callendar, M.A., LL.D., F.R.S., Sir Arthur Schuster, Ph.D., Sc.D., F.R.S., Sir J. J. Thomson, O.M., D.Sc., F.R.S., Prof. C. Vernon Boys, F.R.S., Prof. C. H. Lees, D.Sc., F.R.S., Prof. Sir W. H. Bragg, K.B.E., M.A., F.R.S.

Vice-Presidents.—The Rt. Hon. Lord Rayleigh, F.R.S., Prof. T. Mather, F.R.S., T. Smith, B.A., Prof. G. W. O. Howe, D.Sc.

Secretaries.—F. E. Smith, O.B.E., F.R.S., "Redcot," St. James's Avenue, Hampton Hill; D. Owen, B.A., D.Sc., 62, Wellington Road, Enfield.

Foreign Secretary.—Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer.—W. R. Cooper, M.A., B.Sc., 82, Victoria Street, S.W.1.

Librarian.—A. O. Rankine, D.Sc., Imperial College of Science and Technology.

Other Members of Council.—J. H. Brinkworth, B.Sc., G. B. Bryan, D.Sc., C. R Darling, F.I.C., Prof. C. L. Fortescue, O.B.E., E. Griffiths, D.Sc., J. Guild, A.R.C.Sc., D.I.C., F. L. Hopwood, D.Sc., E. A. Owen, B.A., D.Sc., E. H. Rayner, M.A., D.Sc., J. H Vincent, D.Sc., M.A.

PAPERS.

The new President, Dr. ALEXANDER RUSSELL, M.A., D.Sc., being now in the Chair, the following Papers were read :—

1. "On the Measurement of the Radium Content of Sealed Metal Tubes," by E. A. OWEN, D.Sc., and BERTHA NAYLOR, B.Sc. (National Physical Laboratory).

2. "The Crystal Structure of Ice," by Sir WILLIAM BRAGG, F.R.S. (University College, London).

3. "A Method of Exciting Vibrations in Plates, Membranes, &c., Based on Bernouilli's Principle," by Dr. KERR GRANT (Adelaide University).

February 24, 1922.

Meeting held at the Imperial College of Science.

Dr. ALEXANDER RUSSELL in the Chair.

The following Papers were read :—

1. "The Number of Radio-active Transformations as Determined by Analysis of the Observations," by Dr. H. LEVY (Imperial College of Science).

2. "A Graphical Method of Treating Fresnel's Formulæ for Reflection in Transparent Media," by Prof. C. H. LEES, D.Sc., F.R.S. (East London College).

The following Demonstrations were given :—

3. Demonstrations of Apparatus by the Research Staff of the General Electric Co., London :—

(a) *Rapid Weighing Balance.*

(b) *Electrostatic Voltmeter.*

(c) *Apparatus for the Measurement of Density of Fine Wires.*

4. A Demonstration of the Physical Properties of Cellactite, by F. C. Dyche-Teague, B.Sc., F.I.C.

March 10, 1922.

Meeting held at the Imperial College of Science.

Dr. Alexander Russell in the Chair.

The following Papers were read, and illustrated by Exhibits and Demonstrations :—

1. "On the Electromagnetic Screening of a Triode Oscillator," by R. L. Smith-Rose, B.Sc., D.I.C. (The National Physical Laboratory).

2. "A New Form of High Vacuum Automatic Mercury Pump," by H. P. Waran, M.A., Ph.D. (University College, London).

3. "Viscosity Determination by Means of Orifices and Short Tubes," by W. N. Bond, M.Sc. (University College, Reading).

March 24, 1922.

Meeting held at the Imperial College of Science.

Dr. Alexander Russell in the Chair.

The Seventh Guthrie Lecture was delivered by Prof. N. Bohr, of the University of Copenhagen, the subject of his address being "The Effect of Electric and Magnetic Fields on Spectral Lines."

April 28, 1922.

Meeting held at the Imperial College of Science.

Dr. Aleaxnder Russell in the Chair.

The following Papers were read :—

1. "The Position of Best Focus in the Presence of Spherical Aberration," by T. Smith, B.A., F.Inst.P. (National Physical Laboratory).

2. "The Determination of the Absolute Stress-variation of Refractive Index," by F. Twyman, A.I.E.E., and J. Perry.

3. **"An Experimental Comparison of the Viscous Properties of (*a*) Carbon Dioxide and Nitrous Oxide, and (*b*) Nitrogen and Carbon Monoxide," by C. J. Smith, B.Sc., A.R.C.Sc., D.I.C. (Imperial College of Science and Technology).**

An optical sonometer was exhibited in action by **Mr. F. Twyman, A.I.E.E., F.Inst.P.** (A. Hilger, Ltd.).

May 12, 1922.

Meeting held at the Imperial College of Science.

Dr. **Alexander Russell** in the Chair.

1. A Demonstration of Some Electrical Properties of Neon-filled Lamps was given by S. O. **Pearson**, B.Sc., and H. St. G. **Anson**.

2. A Paper was read, entitled "A New Apparatus for the Measurement of the Polarisation Capacity of Platinum Plates in Sulphuric Acid," by A. **Griffiths**, D.Sc., and W. T. **Heys**, B.Sc.

3. Some remarks on "The Molecular Forces involved in Cohesion" were presented by Dr. **Herbert Chatley**.

May 26, 1922.

Meeting held at the Imperial College of Science.

Dr. **Alexander Russell** in the Chair.

A Lecture on "Atomic Weights and Isotopes" was delivered by Dr. F. W. **Aston**, F.R.S. (Trinity College, Cambridge).

June 9, 1922.

Visit to the National Physical Laboratory, Teddington.

June 23, 1922.

Meeting held at the Imperial College of Science.

Dr. **Alexander Russell** in the Chair.

The following Papers were read :—

1. "An Experiment on Molecular Gyrostatic Action," by J. W. **Fisher**, B.Sc. (King's College, London) (communicated by Prof. O. W. Richardson, F.R.S.).

2. "On the Viscous Properties and Molecular Dimensions of Silicane," by Prof. A. O. RANKINE, D.Sc., and C. J. SMITH, B.Sc. (Imperial College of Science).

3. "The Pressure-Gradient in Liquids Flowing through Cones," by W. N. BOND, M.Sc. (University College, Reading).

The following Demonstrations were given :—

4. "A Mercury-Drop Method of Producing Visual Effects by Means of Sound," by Dr. E. E. FOURNIER D'ALBE.

5. (*a*) "A Magnetic Pivot," and (*b*) "A Self-charging Electroscope," by MAJOR C. E. S. PHILLIPS, O.B.E.

I. *On The Use of Anderson's Bridge for the Measurement of the Variations of the Capacity and Effective Resistance of a Condenser with Frequency.*

By S. BUTTERWORTH, M.SC., *National Physical Laboratory.*

RECEIVED JULY 1, 1921.

ABSTRACT.

An analysis of the effect of residuals and earth capacities in Anderson's Inductance-capacity bridge is made, and it is shown that if balances are obtained

(*a*) by balancing the bridge with *direct* currents ;

(*b*) by making the alternating current adjustments by means of a small series resistance (s') and parallel condenser (C') in the condenser arm ;

then the changes required in s' and C' to hold the balance at different frequencies are equal and opposite to the variations of the effective (series) resistance and capacity of the condenser with frequency.

The assumptions made in obtaining the above conclusions are that the residual inductances and resistances of the " non-inductive " arms of the bridge are invariable with frequency and that the resistance of the inductive arm varies as the square of the frequency. No knowledge of the absolute values of the residuals, &c., is required for the method.

The method is illustrated by results obtained with a condenser of capacity $0{\cdot}5\mu F$, and details are given showing how the chief experimental trouble, viz., drift in D.C. balance owing to temperature variations, may be overcome.

I.—Introduction.

ALTHOUGH Anderson's bridge has been used frequently in precision measurements of inductance and capacity and the effects of the residual inductances of the arms have been discussed,* a valuable property of this bridge seems to have been overlooked. An examination of the equations of balance of this bridge in which the effects of residuals are included shows that if the balancing adjustments are carried out in a particular manner, the only factors which necessitate a readjustment of balance at various frequencies of current are :

(*a*) the variation of the capacity of the condenser with frequency,

(*b*) the variation of the effective resistance of the condenser with frequency,

(*c*) the variation of the inductance with frequency.

The assumptions upon which the above conclusions are based are that the resistances and residual inductances (or capacities) of the " non-inductive " arms are invariable with frequency and that the resistance of the inductive arm has a term varying as the square of the frequency.

The observed variations of adjustment with frequency may therefore be used to determine the frequency variations of capacity and effective resistance of a condenser, the frequency coefficient of the inductance being supposed given. This frequency coefficient may be determined by a measurement of the self capacity of the inductive coil if the coil is of stranded wire. The absolute magnitudes of the residuals need not be known if the only quantities required are the ***variations*** of capacity and effective resistance of the condenser.

II.

Anderson's bridge is shown in Fig. 1*a* in which X_1, X_2, X_3, X_4 are resistances having small inductances, X_5 is an inductive coil and X_6 is the condenser under test.

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* Rosa and Grover, Bull. Bureau of Standards, Vol. I., p. 291 (1905).

If X_3 is zero the bridge becomes Maxwell's inductance-capacity bridge (Fig. 1(*b*)). The latter bridge may also be derived from Anderson's bridge by converting the "star" combination X_1, X_2, X_3 to its equivalent "mesh,"[*] X'_1, X'_2, $X'_3=$

$$(X_3X_1+X_1X_2+X_2X_3)/X_2,\ X_1,\ X_3 \quad \ldots \ldots \quad (1)$$

the arm X'_3 acting as a shunt across the detector.

It will first be shown that small constant residuals in the arms X_1, X_2, X_3 convert to small constant residuals in the equivalent arms X'_1, X'_2, X'_3 and then the complete conditions of balance, including residuals, in Maxwell's bridge will be obtained.

Let $X_n=R_n+j\omega l_n \quad (n=1, 2, 3)$

Then

$$\begin{aligned} X'_1 &= X_1+X_3+X_1X_3/X_2 \\ &= R_1+R_3+j\omega(l_1+l_3)+(R_1+j\omega l_1)(R_3+j\omega l_3)/(R_2+j\omega l_2) \\ &= R_1+R_3+R_1R_3/R_2+j\omega\{l_1(1+R_3/R_2)+l_3(1+R_1/R_2)-l_2R_1R_3/R_2^2\} \end{aligned}$$

to the first order of small quantities

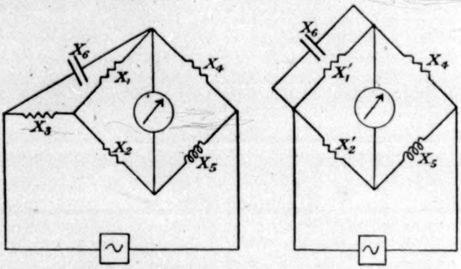

FIG. 1.—(*a*) ANDERSON'S BRIDGE. (*b*) MAXWELL'S BRIDGE.

Hence if l'_1, l'_2, l'_3 are the residual inductances of the equivalent arms X'_1, X'_2, X'_3

$$\left.\begin{aligned} l'_1 &= l_1(1+R_3/R_2)+l_3(1+R_1/R_2)-l_2R_1R_3/R_2^2 \\ l'_2 &= l_2(1+R_3/R_1)+l_3(1+R_2/R_1)-l_1R_2R_3/R_1^2 \\ l'_3 &= l_1(1+R_2/R_3)+l_2(1+R_1/R_3)-l_3R_1R_2/R_3^2 \end{aligned}\right\} \quad \ldots \ldots \quad (2)$$

If l_1, l_2, l_3 are invariable with frequency, then to the first order of small quantities l'_1, l'_2, l'_3 are also invariable.

For analytical simplicity it is convenient in certain cases to replace residual inductances by residual capacities. Thus if l is the residual inductance of a resistance R, the equivalent residual capacity acting as a shunt is

$$c=-l/R^2 \quad \ldots \ldots \quad (3)$$

III.—*Conditions of Balance in Maxwell's Bridge.*

Maxwell's bridge is redrawn in Fig. 2 with the residuals included.

R_1 is a resistance having residual inductance l; R_2 is an inductive coil of inductance L; G_2 is a conductance with residual capacity c; G_1 is a conductance shunting the condenser C, its residual capacity being included in C. The condenser C has leakage conductance g.

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[*] G. A. Campbell, Trans. Am.I.E.E., Vol. XXX., p. 891 (1911).

If the bridge is earthed at the point E the earth capacities (assumed to be sufficiently represented by condensers connecting the branch points to earth) are either included in c, C or are thrown across the whole bridge.

The vector equation of balance when using currents of frequency $\omega/2\pi$ is

$$(G_1+g+j\omega C)/(G_2+j\omega c)=(R_2+j\omega L)/(R_1+j\omega l) \quad . \quad . \quad . \quad (4)$$

from which the two conditions of balance are

$$(G_1+g)R_1-\omega^2 lC=G_2R_2-\omega^2 Lc \quad . \quad . \quad . \quad (5)$$

$$CR_1+l(G_1+g)=cR_2+LG_2 \quad . \quad . \quad . \quad (6)$$

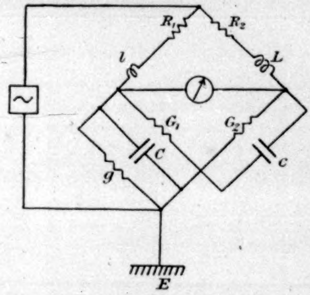

FIG. 2.—MAXWELL'S BRIDGE, INCLUDING RESIDUALS.

Now let the parallel combination of capacity C shunted by conductance g be replaced by its equivalent series combination of capacity K in series with resistance s, so that

$$g=s\omega^2K^2/(1+\omega^2s^2K^2),\ C=K/(1+\omega^2s^2K^2). \quad . \quad . \quad . \quad (7)$$

Since normally $s\omega K$ will seldom exceed 0·001, we may write to an accuracy 10^{-6}

$$g=s\omega^2C^2,\quad K=C \quad . \quad . \quad . \quad (8)$$

and the two conditions of balance are now

$$G_1R_1+\omega^2(C^2sR_1-lC+Lc)=G_2R_2 \quad . \quad . \quad . \quad (9)$$

$$CR_1=LG_2-lG_1+cR_2 \quad . \quad . \quad . \quad (10)$$

products of residuals being ignored.

IV.

Apart from the variations of C and s with frequency all the factors in the above equations are invariable with frequency except R_2 and L, and for a stranded wire coil we may write

$$L=L_0(1+a\omega^2),\ R_2=R_0(1+\beta\omega^2) \quad . \quad . \quad . \quad (11)$$

Suppose now that the condition

$$G_1R_1=G_2R_0 \quad . \quad . \quad . \quad (12)$$

has been satisfied by balancing with *direct* currents. Then, using (11) and (12) and ignoring products of small quantities, (9) and (10) become

$$C^2sR_1-lC+L_0c=G_2R_0\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$CR_1(1-a\omega^2)=L_0G_2-lG_1+cR_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If, therefore, the alternating current balance is secured by having a small adjustable air condenser C' in parallel with C and a small adjustable resistance s' in series, equations (13) and (14) show that *for all frequencies*, $s+s'$ *and* $C(1-a\omega^2)+C'$ *are constant.*

The variations required in s' and C' to restore balance at the various frequencies will therefore be equal and opposite to the variations of s and $C(1-a\omega^2)$.

V.—*Bridge Details for Effective Resistance Measurements.*

The bridge arrangements when used for the measurement of the variations in effective resistance of ½ microfarad condensers are shown in Fig. 3. The arms corresponding to X_1, X_2 in Fig. 1(*a*) were similar non-inductive coils each of 100 ohms. The inductive arm was made up of an inductance coil R_2, L and a non-inductive 10-ohm resistance. The coil L consisted of a single layer 7/36 stranded wire coil of inductance 780 microhenries and resistance approximately 2·6 ohms. Its self-capacity, including the connecting leads, was 20 micro-microfarads, so that its increase in effective inductance at a frequency of 2,000 cycles per second was $3/10^6$. The arm adjacent to the inductive arm was made up of a non-inductive 10-ohm coil and a non-inductive copper coil R_1 of resistance slightly greater than that of the coil R_2. The coil R_1 was shunted by a resistance box (*Sh.*). The inductive coil R_2 and the copper coil R_1 were enclosed in the same wool-lined box so that their temperature variations would be as nearly as possible identical, the function of R_1 being to balance any changes of resistance due to temperature variations of R_2. (*See* Section VII.)

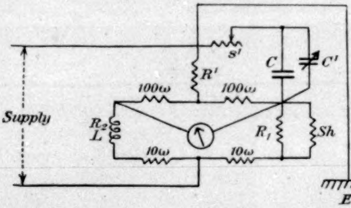

FIG. 3.—ANDERSON'S BRIDGE FOR EFFECTIVE RESISTANCE OF ½ μF CONDENSERS.

C was the condenser under test of nominal value ½ microfarad. It was shunted by an adjustable air condenser C' whose capacity ranged from 60 to 260 micro-microfarads, and the combination was in series with a constant inductance rheostat s' ranging from 0·1 to 1·1 ohm which could be read to 0·001 ohm. The resistance R' corresponding to X_3 in Fig. 1(*a*) was for purposes of preliminary balancing. It could be adjusted by steps of 0·01 ohm to 100 ohms.

VI.—*Modifications for the Measurement of Variations of Capacity with Frequency.*

The above arrangements required modification when the capacity variations were being studied. Any inductance in the condenser arm will introduce a frequency variation in the effective capacity of this arm, so that the presence of the constant inductance rheostat s' in the condenser arm is a disadvantage. However, the direct current balancing condition is no longer of importance, so that if s' is transferred to the inductive arm R_2 and the fine adjustments are made by s' in R_2 and C' in parallel with C, then, since in (14) we may replace R_0 by R_2 to the order of accuracy required, small variations of R_2[*] will only introduce second order errors in the capacity equation as R_2 is associated only with the residual capacity c. The capacity arm C may then be made to include only the condenser and its leads, and only a small correction is required for the inductance of the leads.

These modifications were accordingly used when capacity variations were being measured.

VII.—*Temperature Variations.*

The chief source of trouble in the effective resistance measurements was the difficulty experienced in holding the direct-current balance throughout the alternating-current adjustments, the balance tending to drift because of the large temperature coefficient of the resistance of the inductive coil in R_2. This difficulty was largely overcome by the use of the balancing coil R_1 of Fig. 3, both R_1 and R_2 being shielded from stray temperature variations by enclosing them together. It was found necessary also to shield the leads to these coils. For similar reasons the current supplied to the bridge, whether direct or alternating, was made to have the same R.M.S. value. Even with these precautions a small drift in balance often occurred so that observations were taken in a definite order, and a correction applied for want of D.C. balance. An examination of equation (9) shows that a departure from condition (12) introduces an error in s proportional to $1/\omega^2$, and is thus more serious at the lower frequencies. For this reason it was found advisable to take the lower frequency observations immediately after securing the D.C. balance. Also by deliberately throwing the D.C. balance out of adjustment and observing the change in s' required to re-balance with alternating currents of known frequency, the correction for a given want of balance could be determined. The method is illustrated by the following example.

VIII.—*Example Showing Correction for Drift in D.C. Balance.*

$Sh \doteqdot 400$ ohms. $R_1 \doteqdot 3$ ohms.

An increase of 10 ohms in Sh causes an increase in deflection of the D.C. galvanometer of 35 divisions, and to restore the A.C. balance at a frequency of 500 ∼ per second requires a diminution in s' of 0·129 ohm.

Hence for a want of balance represented by 1 division of the scale of the D.C. galvanometer the correction on $s' = 0{\cdot}0037$ ohm.

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[*] It is essential, however, that R_2 shall remain of constant *inductance* throughout the adjustments, so that s' must be of the type specified.

The correction is applied as in the following table :—

Time. Min.	D.C. observations.		A.C. observations.		Deduced.	
	Source.	Galvo. deflections.	Frequency.	s' ohms.	Galvo. zero.	Want of balance.
0	+	+2·0	...	...	...	...
1	—	+4·0	...	...	+2·8	...
2	+	+1·0	...	...	...	—1·8
4	...	...	500	0·520*	...	...
6	+	—5·0	...	...	...	—7·5
7	—	+10·0	...	...	+2·5	...
8	+	—5·0	...	...	...	...

Want of balance at moment of A.C. observation = —4·6.
Therefore corrected $s' = 0{\cdot}520 - 0{\cdot}0037 \times 4{\cdot}6$
$= 0{\cdot}503$ ohm

IX.—*Typical Results for the Change in Effective Resistance of a* 0·5 μF *Condenser.*

Preliminary adjustments are made by R' and Sh, and the remaining adjustments at *all* frequencies by s' and C' only, observations being taken as in the above table. The following results were obtained in a typical case :—

Condenser I (0·5 μF).
Bridge current = 30 milliamperes.

Frequency ∾ per second...	500	800	1,000	1,600	2,000
s' ohms	0·503	0·597	0·625	0·675	0·685
s ohms	0·287	0·193	0·165	0·115	0·105
Power-factor ωsC	$10^{-4} \times 4{\cdot}5$	4·8	5·2	5·8	6·6

The values of s have been obtained from the known value of s for this condenser at a frequency of 1,600 ∾ per second,† and the law $s + s' =$ constant for all frequencies.

X.—*Typical Results for the Variation in Capacity of a* 0·5 μF *Condenser.*

The rheostat s' is removed from the condenser arm, and placed in series in the arm R_2. After preliminary adjustments by R' and Sh the fine adjustments and adjustments for different frequencies are made by s' and C' only. The following results hold for condenser I. :—

Frequency. ∾ per sec.	C'* $\mu\mu F$.	$C - C_{1000}$ Uncorrected $\mu\mu F$.	Correction $\mu\mu F$.	$C - C_{1000}$ corrected $\mu\mu F$.
500	132	+46	+3	+49
675	154	+24	+2	+26
830	164	+14	+1	+15
1,000	178	0	0	0
1,170	185	—7	—1	—8
1,510	193	—15	—3	—18
2,000	193	—15	—10	—25

The correction in the fourth column is for the variation of the inductive coil L

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* Mean of source and telephone reversals.

† Butterworth, Proc. Phys. Soc., Vol. XXXIII, p. 346, 1921.

with frequency and for the residual inductance of the leads to the condenser. As regards the latter, if λ is the inductance of the leads, the effective capacity of the whole condenser arm is $C(1+\omega^2\lambda C)$. In the above observations $\lambda=0{\cdot}38$ microhenry, so that using $C=\frac{1}{2}\mu F$,

$$\omega^2\lambda C=30\times10^{-6}$$

at a frequency of 2,000 cycles per second. Now the correction due to L (section 5) is 3×10^{-6} and (equation 14) acts oppositely to the λ correction, so that the whole correction is 27×10^{-6} at 2,000 ∿ per second. For a $\frac{1}{2}\mu F$ condenser this leads to a negative correction of 13·5 $\mu\mu F$. The corrections for the lower frequencies follow from the square law. The corrected variations are plotted against frequency in Fig. 4. It is seen that the smoothed curve never differs by more than five parts in a million from the observed values.

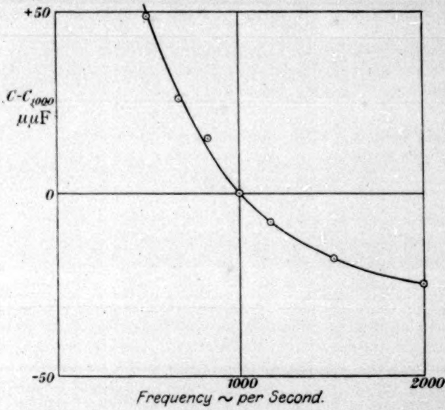

FIG. 4.—CHANGE OF CAPACITY OF $\frac{1}{2}$ μ F CONDENSER WITH FREQUENCY.

DISCUSSION.

Prof. FORTESCUE, referring to the typical results for the change in effective resistance of a condenser given in section IX. of the Paper, remarked that telephone engineers usually assume that the power factor of a condenser is constant, and would be specially interested to see that this assumption is incorrect. He enquired as to the type of condenser used.

Dr. D. OWEN congratulated the author on his valuable contribution, and expressed a belief that his methods would be adopted as the standard practice. It would be of interest to apply them to types of bridge other than that of Anderson. He asked if a sufficient number of condensers had been tested to determine whether the frequency coefficient of mica condensers is a definite quantity, or varies from one condenser to another like the temperature coefficient.

Mr. S. BUTTERWORTH, in reply to the discussion, stated that the results quoted in illustration of the method were obtained with a standard mica condenser. The power factors of four other mica condensers were also obtained, and the results varied between 0·0001 and 0·0007. Only one condenser was tested in regard to variation of capacity with frequency. However, a large number of mica condensers have been measured by H. L. Curtis (Bureau of Standards Circular, S.137), and his results are regarded as standard. The results of Curtis depend on the perfection of an air condenser of 0·1 μfd. capacity.

II. *Notes on Earth Capacity Effects in Alternating-Current Bridges.* By S. Butterworth, *M.Sc., National Physical Laboratory.*

Received March 23, 1921.

ABSTRACT.

It is shown that an earth capacity acting at any point in the arm of a bridge may be replaced by two earth-impedances acting at the ends of the arm together with an impedance in series with the arm. By integration the result is extended to small distributed capacities.

Two methods are given for the elimination of the error due to the end impedances. Complete elimination can only be obtained by the use of shields connected to the ends of the bridge arm.

I.

In precision measurements with alternating-current bridges it is always necessary to eliminate the effects due to the capacity of the various portions of the apparatus with regard to earth. The effect of these capacities is that the network with which we are dealing is not that composed of the visible arms, but includes other arms connecting various points to earth, and therefore different parts of the bridge to each other. Unless these earth paths are allowed for, different balances will be obtained as the bridge-earth conditions are changed. If it may be assumed that the earth capacities are sufficiently represented by condensers connecting the branch points of the bridge to earth, then there are two methods of eliminating the effect of these capacities in bridges which are reducible to one of the Wheatstone bridge type. These methods are described in Sections II. and III. below. The remainder of the Paper deals with the validity of the assumption in regard to earth capacities acting in the middle of an arm and in regard to distributed capacities. It is shown that the elimination is not complete by the two methods adopted. The error due to the incomplete elimination is not, however, serious if the parts of the apparatus are not brought too near to large masses of conductors connected to (or near to) earth, as the earth capacities tend rapidly to a constant value, and therefore behave as practically constant residuals in various bridge measurements. The effect is, therefore, that a different interpretation must be given to the residuals from that usually assumed. The residual inductance of a resistance coil, for example, is not merely that due to the magnetic field and its own intercapacity, but also includes a term which depends on the distributed capacity of the coil to earth. If in all measurements the coil is made to have the same position with regard to earth no discrepancy will appear when the coil is used in different bridges. This portion of the residual must, however, be taken into account in calculated standards of high resistance.

II.—*Elimination of Earth Capacities by the "Wagner" Earthing Device.*

The Wagner earthing arrangement as applied to a generalised Wheatstone bridge is shown in Fig. 1. a, b, c, d are the impedances of the arms of the bridge, while e and f are impedances so chosen that the relations $e/f=a/b=c/d$ may be approximately satisfied. The junction between e and f is connected to earth. Assuming that earth capacities or leakages may be represented by impedances connecting the branch points to earth, this earth point causes the paths of the earth currents to pass from the points 1, 2, 3, 4 to the point 5. The paths 5–1, 5–3 will merely modify the impedances e and f, but the paths 5–2, 5–4 will produce currents disturbing the balancing conditions if the points 2, 4, 5 are at different potentials. The procedure consists in bringing the points 5, 2, 4 to the same potential by successively adjusting the main bridge and the impedances *e* and *f* until the detector indicates balance whether connected to the point 2 or the point 5.*

III.—*Correction for Earth Capacities by Alteration of Earth Point.*

Let the symbols in Fig. 2 denote admittances, α, β being the admittances of the earth paths.

Let two balances be made (*a*) with the admittance *c* infinite, (*b*) with the admittance *c* finite. Suppose the balances secured by adjustment of A. Then, if A_1 and A_2 are the two adjustments of A, while $A_0=BX/Y$, we have

$$\frac{A_1+\beta}{B+\alpha}=\frac{X}{Y}=\frac{A_2+\mu\beta}{B+\mu\alpha}$$

in which

$$\mu=1+\frac{A_2+B}{c}$$

Hence, eliminating β/α

$$\frac{X}{Y}=\frac{A_2-\mu A_1}{B(1-\mu)} \text{ or } A_0=\frac{A_2-\mu A_1}{1-\mu}$$

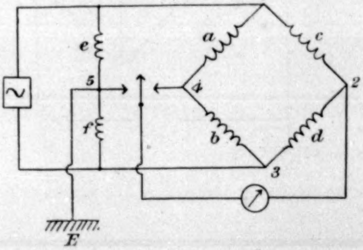

FIG. 1.—WAGNER EARTHING ARRANGEMENT.

If *c* is chosen to consist of an arm equivalent to A_2 and B in parallel, $\mu=2$ and

$$A_1=A_0+(A_2-A_1)$$

If a_0, a_1, a_2 are the corresponding impedances and a_1-a_2 is small,

$$a_1=a_0+a_2-a_1,$$

or, regarding a_0 as the unknown,

$$a_0=2a_1-a_2.$$

The change in adjustment from a_1 to a_2 thus doubles the error, and the method, therefore, also gives a measure of the earth capacity effect. It must be remembered in applications that a_1 and a_2 are vector impedances, and therefore each adjustment is a double adjustment.

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* The device was originally applied for the purpose of eliminating the "head" effect when using a telephone as detector in the comparison of small capacities by the Wien bridge. It has been extended to other bridges, particularly the Carey-Foster bridge, by Mr. D. W. Dye. The scheme given by G. E. Moore (Electrician, June 17, 1921) is defective.

This method suffers from the following disadvantages as compared with the Wagner method :—

(*a*) The body must be kept in the same position in both adjustments when a telephone is being used as detector.

(*b*) It is impossible in certain cases to secure a zero balance. This occurs sometimes in the measurement of small capacities by the Carey-Foster method.

The second difficulty may be overcome in the Carey-Foster bridge by using an auxiliary condenser for the zero balance and then placing the condenser to be measured in parallel with the auxiliary condenser.

It has the following advantages over the Wagner method :—

(*a*) Two adjustments only are necessary instead of a series of adjustments.

(*b*) An estimate may be made of the errors due to earth capacities.

The same auxiliary apparatus is required as in the Wagner method, the series arms *e* and *f* of the Wagner arrangement becoming paralleled to form *C* in the arrangement of Fig. 2.

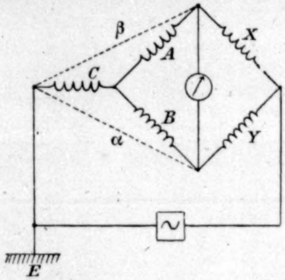

FIG. 2.—CORRECTION FOR EARTH CAPACITY BY ALTERATION OF THE EARTH POINT.

IV.—*Earth Capacities in Compounded Arms.*

The assumption that earth capacities may be represented by earth paths from the branch points of the bridge will now be examined.

If an arm is made up of several elements, appreciable earth capacities may be located at the junctions of these elements. This is particularly the case when there is a fairly large mass of metal at the junction, as in the case of condensers in series with a resistance or inductance. Even terminals may in certain cases produce an appreciable effect.*

Let an arm consist of two series elements of impedance α, β having an earth path of impedance γ at their junction. Transformation (A)† converts the arrangement to an arm of impedance $\alpha+\beta+\alpha\beta/\gamma$, with two end earth paths of impedances

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* A terminal approximating in size to a sphere of 1 cm. radius has an earth capacity of the order $1\mu\mu F$.

† Transformations (A) and (B) are described in the author's Paper on "Capacity and Eddy-current Effects in Inductometers." (Proc. Phys. Soc., Vol. XXXIII., p. 312, 1921.)

$(\alpha\beta+\beta\gamma+\gamma\alpha)/\beta$, $(\alpha\beta+\beta\gamma+\gamma\alpha)/\alpha$ respectively at the outer ends of α and β. If one of the methods described has been used to eliminate earth capacities, the effects of the end impedances are eliminated, but there is an uneliminated portion behaving as a residual $\alpha\beta/\gamma$ in series in the arm $\alpha+\beta$.

Thus, let α, β be two " non-inductive " resistances connected through a massive terminal.

If c is the earth capacity of the terminal, R_1, R_2 the values of the resistances, the uneliminated earth effect behaves as a residual inductance, cR_1R_2.

With resistances of 1,000 ohms, and with $c=1\mu\mu F$, this residual inductance is $1\mu H$.

If α is a coil of resistance R and inductance L, while β is a condenser of capacity C, then at a frequency $\omega/2\pi$

$$\alpha\beta/\gamma=\frac{c}{C}(R+j\omega L)$$

or the uneliminated portion of the earth capacity at the junction introduces an error in the power factor of the condenser C of an amount ωcR, and causes the inductance to appear too large by an amount cL/C.

In illustration, let $c=50\mu\mu F$, $R=100$ ohms, $\omega=10^4$. The error in power factor is then $5/10^5$. Large earth capacities at the junction should, therefore, be avoided by shielding the condenser and connecting the shield to that terminal of the branch point which is connected to the condenser.

V.—*Distributed Earth Capacities.*

Suppose the earth capacity to be small and let it be distributed uniformly throughout the arm. Suppose also that the impedance is distributed uniformly.

Then if Γ is the whole earth admittance and X the whole impedance of the arm of length l, the earth admittance $\Gamma\delta x/l$ of an element δx situated at a distance x along the arm contributes a series impedance $\Gamma X^2x(l-x)\delta x/l^3$, and end earth admittances $\Gamma(l-x)\delta x/l^2$, $\Gamma x\delta x/l^2$ to the arm, these results being obtained by transformation (A) retaining only first order quantities. With the same approximation the earth paths of all the elements may be treated as acting independently so that by integration the whole distributed earth admittance is represented by a series impedance of amount $\frac{1}{6}\Gamma X^2$, and two equal end admittances each of amount $\frac{1}{2}\Gamma$. The former is equivalent to a negative admittance $\frac{1}{6}\Gamma$, acting as a shunt across the arm.

We conclude that the uneliminated portion of the distributed earth capacity behaves as a negative residual capacity in parallel with the arm of an amount equal to $\frac{1}{6}$ the whole earth capacity.

VI.—*Calculation of Earth Capacity.*

Let a resistance be made of flat strip forming a loop of length $2a$, width $2b$ (Fig. 3). If b/a is small the system may be treated as an elliptic disc of semi-axes a and b. Now the capacity of an elliptic disc is $a/K(\sqrt{1-b^2/a^2})$ where $K(k)$ is the complete elliptic integral of the first kind to modulus k. Further if the elliptic disc is divided into narrow strips of equal width parallel to the minor axis, each strip will have the same charge when the potential of the whole disc is raised, so that in the limiting case the capacity of the strip is uniformly distributed throughout the length a. Further when b/a is small

$$K(\sqrt{1-b^2/a^2}) \doteqdot \log_e 4a/b$$

so that the whole earth capacity of the strip is $a/\log_e 4a/b$ electrostatic units.

In illustration let $a=10$ cm., $b=0{\cdot}5$ cm. Then the earth capacity is $10/4{\cdot}4$ cm. $=2{\cdot}3$ cm. $=2{\cdot}6\mu\mu F$.

The above result may be applied to give an approximate value of the earth capacity of a resistance wound in the manner shown in Fig. 4, a form often adopted to avoid large self-capacities in high resistance coils. The resistance consists of several separate loops wound on a mica cord.

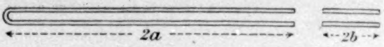


FIG. 3.

If each segment has a winding length of 3 cm. and winding width of 0·5 cm. the whole earth capacity of each segment would be about $1\mu\mu F$, so that if a 1,000 ohm coil is made up of 10 segments each of 100 ohms, the whole earth capacity would be about $10\mu\mu F$. When measured on a bridge in which end capacities are eliminated the uneliminated portion would contribute an inductance of the order $1{\cdot}7\mu H$.

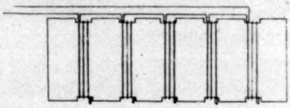


FIG. 4.

VII.—*Non-uniformly Distributed Capacity.*

The factor $\frac{1}{6}cR^2$ for the residual inductance introduced into a resistance R having earth capacity c has been obtained upon the assumption of uniform distribution. If the distribution is denser towards the outer ends of the coil the above estimate is too high, while if the capacity tends towards the middle, the estimate is too low. If the coil is of the form of a circular disc the effect may be calculated. Let a " non-inductive " coil be formed of two oppositely wound disc coils laid over each other.

If a is the radius of the coils, the whole earth capacity is $2a/\pi$ electrostatic units of which half is associated with one coil and half with the other. The density of distribution at distance r from the centre is proportional to $1/\sqrt{a^2-r^2}$. Now if l is the length of wire in one coil, and x the length of wire between radii r and a, $l/x=a^2/a^2-r^2$, so that the density of distribution at a wire distance x from the outer end is proportional to $x^{-\frac{1}{2}}$. Hence we write for the capacity per unit length $Ax^{-\frac{1}{2}}$, where since $A\int_0^l x^{-\frac{1}{2}}dx$ is the whole capacity of one coil (a/π), $A=a/2\pi\sqrt{l}$.

Again by the method of section 4, if ρ be resistance per unit length, the residual inductance due to the earth capacity of one coil is $\rho^2 A\int_0^l \sqrt{x}(l-x)dx$ $=\frac{4}{15}\rho^2 A l^{\frac{5}{2}}=\frac{2}{15}\frac{a}{\pi}R^2$, R being the total resistance of one coil.

The equivalent outer end capacity is

$$\frac{A}{l}\int_0^l (l-x)dx/\sqrt{x}=\frac{4}{3}A\sqrt{l}=\frac{2}{3}\frac{a}{\pi}$$

and the equivalent inner end capacity is $\frac{1}{3}\frac{a}{\pi}$

Now let the two oppositely wound coils be connected at their inner ends. The earth capacities of the combination are equivalent to three earth capacities each of magnitude $\frac{2a}{3\pi}$ joining the ends and centre to earth together with a residual inductance $\frac{a}{15\pi R^2}$ (R now referring to the resistance of the whole combination).

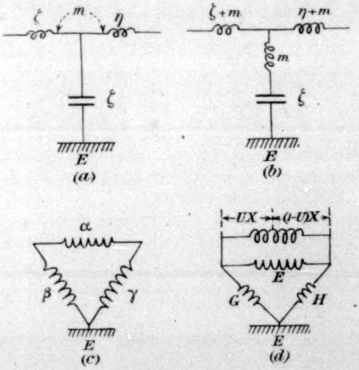


FIG. 5.

By transformation (A) the central earth capacity converts to a series inductance $\frac{a}{6\pi R^2}$, and two end capacities each of magnitude $\frac{1}{3}a/\pi$. Hence writing c for the whole earth capacity so that $c=2a/\pi$, the whole effect is equivalent to a residual inductance $7cR^2/60$, together with two equal end capacities $\frac{1}{2}c$.

If the outers had been joined, the final system would have been a residual inductance $7cR^2/30$, and equal end capacities $\frac{1}{2}c$.

The effect of interchanging the end connections is thus to double the residual inductance effect.

VIII.—*Distributed Earth Capacities in Inductive Coils.*

Let an earth path of impedance, ζ, be present between two coils having impedances ξ, η and mutual impedance m {(*a*) Fig. 5}. By transformation (*A*) the mutual impedance, m, may be removed {(*b*) Fig. 5}, and then by transformation (*B*), the mesh α, β, γ {(*c*) Fig. 5} is obtained in which

$$\alpha=\xi+\eta+2m+\frac{(\xi+m)(\eta+m)}{\zeta+m}$$

$$\beta=\xi+\zeta+2m+\frac{(\xi+m)(\zeta+m)}{\eta+m}$$

$$\gamma=\eta+\zeta+2m+\frac{(\eta+m)(\zeta+m)}{\xi+m}$$

If ζ is large compared with ξ, η, m, then to the first order

$$\alpha=\xi+\eta+2m+\frac{(\xi+m)(\eta+m)}{\zeta}$$

which (again to the first order) is equivalent to the original two coils paralleled by an admittance

$$-\frac{1}{\zeta}\,\frac{(\xi+m)(\eta+m)}{(\xi+\eta+2m)^2}$$

while

$$\frac{1}{\beta}=\frac{1}{\zeta}\,\frac{\eta+m}{\xi+\eta+2m},\quad \frac{1}{\gamma}=\frac{1}{\zeta}\,\frac{\xi+m}{\xi+\eta+2m}$$

In the case of distributed earth capacities there are an infinite number of paths such as ζ, and the above procedure may be applied to each, as to the first order of small quantities each path acts independently.

For an element of length $d\lambda$ of the coil, $1/\zeta$ will be of the form $Zd\lambda$ where Z is the earth admittance per unit length of the coil in the neighbourhood of the element. If X is the whole impedance of the coil, $X=\xi+\eta+2m$, so that, writing $\xi+m=UX$, the whole distributed earth admittances may be replaced by a shunt of admittance F, together with two end earth admittances G, H, F, G, H having the values

$$F=-\int ZU(1-U)d\lambda$$

$$G=\int X(1-U)d\lambda$$

$$H=\int ZUd\lambda$$

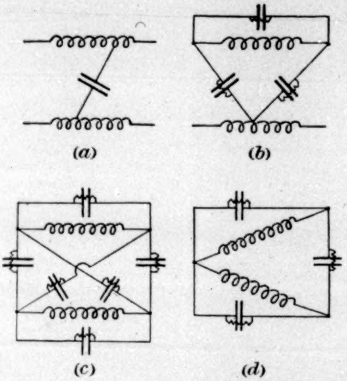
With pure earth capacities Z is of the form $j\omega C$ while U is a complex quantity, being the ratio of the potential difference up to the point λ to the whole potential difference between the ends of the coil. Hence except with special types of winding the equivalent end earth admittances and shunt admittance replacing a pure distributed capacity are not pure capacities, but rather behave as capacities having leakage.

IX.

A similar theory may be developed in regard to distributed intercapacities. The theory is sufficiently illustrated by Fig. 6.

A single pure capacity C {(*a*) Fig. 6} is replaced by three imperfect capacities in (*b*). The capacities C_2, C_3 are then transformed to the capacity system shown in (*c*) Fig. 6, so that to the first order this system of six leaky condensers is sufficient to represent the distributed capacity effects of two inductive coils.

For a single coil suppose the two coils to have a common point. The system then reduces to (*d*) Fig. 6. Finally the two condensers joining the ends to the common point are replaced by their equivalent end condensers, giving a single imperfect condenser across the whole coil, replacing the distributed pure capacity.


(*a*) (*b*)

(*c*) (*d*)

FIG. 6.—REPLACEMENT OF PURE INTERCAPACITY BY END SYSTEM OF LEAKY CONDENSERS.

X.

The general conclusions arrived at are that the ordinary methods of eliminating earth capacities do not completely eliminate the earth effect, either if the effect is distributed or if appreciable earth capacities are introduced in the middle of an arm. So long as the whole arm remains a single unit in various bridge measurements no discrepancies will arise unless the earth condition of the arm is considerably altered. If, however, the arm is split in transference from one bridge to another, discrepancies may appear with high resistances and small capacities. By providing shields in these cases, and arranging that the shields are connected to that terminal of the resistance or condenser which is connected to a branch point in the measurements it ought to be possible to make the observations agree within the limits of errors of observation.

DISCUSSION.

Dr. A. Russell remarked that the Paper contained much valuable matter and called for close study. He questioned whether it was justifiable at such high frequencies to lump the residuals or to leave mutual capacity out of consideration. In the examples given, *e.g.*, in section VI, of the Paper, nothing was said as to the distance from earth of the conductors whose earth-capacity was under consideration. Electrical engineers are much engaged upon the problem of protecting telephone circuits from disturbance by power circuits, and find that the mutual capacity of such circuits is far more important than their mutual inductance.

Mr. **Guild** referred to the method of measuring earth capacity by doubling the error, described in section III. of the Paper, and asked whether it was legitimate to assume that the capacity in question was unaffected by the operation of altering the connections.

Dr. D. **Owen** remarked that the Paper showed that standard resistances are subject to errors due to earth-capacity. Presumably standard inductances would be similarly affected.

Prof. **Fortescue** stated that in testing at frequencies of 20,000 ∿ or 100,000 ∿, errors of 10 per cent. were common. Would the proposed methods apply to such tests and make it possible to reduce this error ?

Dr. **Rankine** referred to the disturbances due to external agencies, such as motors, which are heard in thermionic valve amplifiers. Would the author's researches make it possible to determine points in the apparatus, such that by connecting them to earth the disturbances referred to could be eliminated ?

Mr. S. **Butterworth** replied to the discussion as follows : The calculations of distributed earth capacities are given in order to indicate the order of magnitude of these quantities, and the effect of proximity to earth has been ignored for simplicity. In regard to the method of doubling the error, it has been tested by adding artificial earth capacities and consistent results have been obtained. Errors due to earth capacities are only of importance in arms of high impedance, so that in calculated standards of inductance the earth capacity error only becomes appreciable when these are used at high frequencies. The errors in bridge measurements at radio frequencies should be considerably reduced by a proper choice of earth-point. In any case where it is desired to reduce the earth capacity effect in a particular arm the working principle is to bring the potential of that arm as near as possible to that of the earth.

III. *An Automatic Voltage Regulator.* *By* F. G. H. LEWIS.

RECEIVED JUNE 10, 1921.

ABSTRACT.

The Paper describes a method by which automatic voltage regulation to 0·15 per cent. may be obtained for such purposes as the operation of photometric standard lamps on an ordinary outside supply varying by as much as 10 per cent. The lamp is placed across an unbalanced Wheatstone bridge of which two opposite arms are composed of tungsten filament lamps. The increase of resistance of these lamps, due to the extra current passing through them when the outside voltage rises, causes a shift in the balance of the bridge such that the voltage across the photometer lamp remains unaltered if the values of the resistances in the arms be properly proportioned. The power taken is about 40 times that used in the regulated circuit.

THE object of the scheme described in what follows is to find a simple method by which the fluctuations of voltage of an ordinary electric current supply may be overcome, and a steady voltage obtained for the purpose of running a glow-lamp photometer standard or any other purpose for which a steady voltage is required.

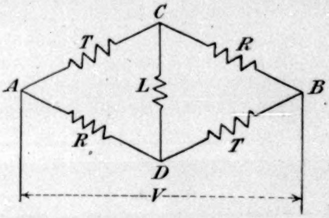


FIG. 1.—DIAGRAM OF CONNECTIONS.

The method depends on the fact that the resistance of a tungsten filament lamp rises with increase of temperature of the filament in such a way that the percentage increase in resistance is proportional to the percentage increase of current. This has been found experimentally by determining the change of current through a lamp due to change of voltage. The following experimental results show the values of current at various voltages for tungsten filament lamps of two different ratings :—

TABLE I.

100-volt Lamp.		200-volt Lamp.	
Voltage	Current	Voltage	Current
60	0·632	120	0·169
70	0·693	140	0·185
80	0·750	160	0·201
90	0·804	180	0·215
100	0·856	200	0·230
110	0·905	220	0·243
$dI/dV=0{\cdot}60(I/V)$ so that $dR/R=0{\cdot}67(dI/I)$		$dI/dV=0{\cdot}61(I/V)$ so that $dR/R=0{\cdot}64(dI/I)$	

Now suppose an unbalanced Wheatstone Bridge be set up as in Fig. 1, where

T,T are resistances composed of tungsten filament lamps in parallel, while *R,R* are constant resistances of any ordinary type. *L* is a standard lamp, the current through which it is desired to maintain constant, and *V* is a fluctuating supply voltage. It will be seen most clearly by means of a numerical example, how a constant voltage may be maintained at *L* when *V* varies.

As an example, suppose that the outside voltage, which may vary from 220 to 260 volts, be applied at *AB*, and that it is desired to have 40 volts constant across *CD* when the current flowing from *C* to *D* is 0·3 amperes. Then, when the voltage across *AB* is 220 volts, the voltage from *A* to *C* is 90 volts and that from *C* to *B* is 130 volts if the resistances in these two arms are respectively equal to the resistances in *BD* and *DA*. If, then, i_1 is the current in *CB*, the current in *AC* is $(i_1+0{\cdot}3)$.

Hence $90=T_1(i_1+0{\cdot}3)$ where T_1 is the resistance of *T* when the current through it is $(i_1+0{\cdot}3)$.

Also $130=R\,i_1$.

Similarly when the outside voltage is 260, the voltage between *A* and *C* must be 110 and that between *C* and *B* must be 150, in order to maintain 40 volts across *CD*.

Hence $110=T_2(i_2+0{\cdot}3)$ where T_2 is the resistance of *T* when the current through it is $(i_2+0{\cdot}3)$, for if the resistances of the arms *AC* and *DB* be equal at the lower value of current, they will also be equal at the higher current, since both are composed of lamps having the same current-resistance characteristic.

Also $$150=Ri_2$$

Hence $$T_2/T_1=11(i_1+0{\cdot}3)/9(i_2+0{\cdot}3)$$

and $$i_1/i_2=130/150$$

But since for a tungsten lamp $dR/R=a\ (dI/I)$ where a is about 2/3, it follows that

$$(T_2-T_1)/T_1=\frac{2}{3}\{(i_2+0{\cdot}3)-(i_1+0{\cdot}3)\}/(i_1+0{\cdot}3)$$

whence $$T_2/T_1=\frac{2}{3}(i_2+0{\cdot}3)/(i_1+0{\cdot}3)+\frac{1}{3}$$

Thus from the two expressions for T_2/T_1

$$\frac{2}{3}x^2+\frac{1}{3}x-\frac{11}{9}=0$$

where $$x\equiv(i_2+0{\cdot}3)/(i_1+0{\cdot}3)$$

This equation gives $$x=1{\cdot}127$$

and from this result, since $i_1/i_2=13/15$, we get $i_1=1{\cdot}43$, and hence $T=52$ and $R=91$ ohms.

Thus a regulated voltage of 40 volts with a current of 0·3 amps. may be obtained with a total current of approximately 3 amps. on a variable circuit of nominally 240 volts.

This result may be somewhat improved by using, instead of constant resistances in the arms *CB* and *AD* of the bridge, carbon filament lamps which have a small negative temperature coefficient of resistance.

The device has been tried in this form with the following values of resistances :—

Arm *AC*—2 120-volt, 60-watt tungsten filament lamps, and 2 105-volt, 25-watt t.f. lamps in parallel.

DB—3 120-volt, 60-watt and 1 110-volt, 30-watt t.f. lamps in parallel.

AD and *CB*, each 5 200-volt, 16-c.p. carbon-filament lamps in parallel.

The voltages measured across the lamp terminals in *CD* with various voltages across *AB* are given in Table II.

TABLE II.

Voltage across *AB*.	Voltage across *CD*.
260	68·0
250	68·2
240	68·2
230	68·1
220	67·9

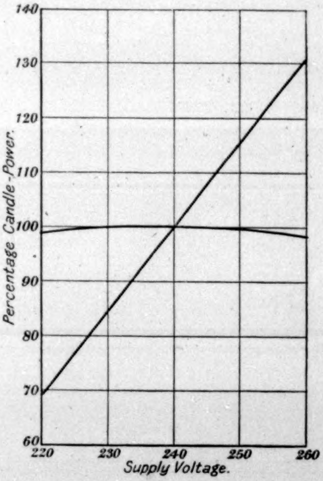


FIG. 2.—GRAPH OF RESULTS OBTAINED WITH REGULATOR.

Total current through circuit, 2·26 amps. ; current in *CD*, 0·20 amps.

It is suggested that this device should prove useful for a small photometric laboratory where the expense of upkeep of a storage battery is excessive for the amount of photometric work to be done. As will be seen from the table above, a voltage regulation to within 0·2 per cent. can be obtained on a supply fluctuating by as much as 5 per cent. from the mean, with a circuit in which the total power used is 40 times that needed for the photometer lamp.

The curves in Fig. 2 show the variation in the candle-power of a lamp operating (*a*) on the outside voltage as it varies from 220 to 260 volts, and (*b*) on the regulated voltage (over the same range of variation of supply voltage). In these curves the value 3·7 has been used for the percentage variation of candle-power due to 1 per cent. change of voltage.

The circuit is very easy and cheap to set up, and it can be readily adjusted by trial and error to give the required result. It is desirable to have a switch or series resistance in the regulated circuit, as the low resistance of the tungsten lamps when cold causes a rush of current in this arm when the main current is first switched on. The device might also find application in meter testing, or in other cases where a small current at a steady voltage is required.

DISCUSSION.

Dr. RAYNER suggested that the apparatus described might afford a new type of standard E.M.F.

Mr. F. E. SMITH dissented from the view of the previous speaker, and showed that the voltage which is to be kept constant is in fact a function of the resistance of the non-incandescent arms of the bridge. It can thus never be possible in such an arrangement to secure an absolutely constant voltage, such as would be required for the purposes of a standard.

Prof. FORTESCUE said that the method produced a distinct gain in steadiness of voltage and would undoubtedly be appreciated in testing work.

Mr. J. W. T. WALSH, who read the Paper on behalf of Mr. Lewis, referred, in reply, to Fig. 2 of the Paper, which shows candle-power plotted against supply-voltage. The curve is very flat-topped, thus permitting of steadiness within very close limits, even for variations of supply voltage which would be considered as very large.

COMMUNICATED REMARKS.

From **Mr. T. SMITH**: Mr. Lewis' problem can be treated theoretically by considering only algebraic forms. Taking the resistances of the outer arms of the bridge in cyclic order as P,Q,R,S, and the resistance on the regulated circuit to be G, the currents in these arms are evidently proportional to $S(Q+R)+G(R+S)$, $R(P+S)+G(R+S)$, $Q(P+S)+G(P+Q)$, $P(Q+R)+G(P+Q)$ and $PR-QS$ respectively. If then a constant current C is to be maintained in the regulated circuit the resistance in the first arm must be P when the current in this arm is

$$\frac{S(Q+R)+G(S+R)}{PR-QS}C$$

and similarly in the other cases. If P is the only variable resistance the regulation can only be exact if the law followed by the filament is of the form

$$\text{Voltage}=\alpha\times\text{current}+\beta\times\text{resistance},$$

where α and β are constants. When these constants are known the proper resistances for the bridge can be set up at once. The extension to the case when the resistance of more than one arm depends upon the current is obvious from the above expressions. The external voltage is

$$\frac{PQRS\left(\frac{1}{P}+\frac{1}{Q}+\frac{1}{R}+\frac{1}{S}\right)+G(P+Q)\ (R+S)}{PR-QS}C$$

and the battery current is

$$\frac{(P+S)(Q+R)+G(P+Q+R+S)}{PR-QS}C$$

From these expressions it is evident that the power efficiency of the regulation will be low unless the control can be effected by the use of a material having a large value for the coefficient β. If such a material were available it would evidently be desirable to employ it in the lamp of the regulated circuit, and with a suitable coefficient the need for the bridge would disappear. The condition would be exactly met if the resistance were proportional to the square of the voltage, or

$$\frac{dR}{R}=2\frac{dV}{V}=-2\frac{dC}{C}$$

which may be compared with the author's experimental results. It is difficult to imagine a material with the property that the current decreases continuously over a certain range in consequence of a continuous increase in the applied voltage, but the author's scheme may enable the construction of standard lamps with a self-contained automatic voltage steadier.

From Mr. A. CAMPBELL: The thermal system of voltage regulation used by Mr. Lewis is not novel, except in so far as tungsten is employed as the heated material. The general system was described by the writer many years ago (Proc. Inst. El. Engs., Vol. XXX., p. 889, 1901), and the mathematical conditions were investigated for one case. The hot wires actually used were of nickel, platinum or copper, and very exact automatic regulation was obtained. A 40-volt lamp was exhibited regulated on a variable circuit of about 150 volts. In the discussion on the Paper referred to, Professor Callendar described his use of the system for potentiometer purposes; he was probably the first to employ the method. Mr. Lewis has made the method more easily available by putting ordinary tungsten filament lamps in place of specially constructed resistances.

From the AUTHOR: I feel that Mr. T. Smith's mathematical treatment of the problem adds much to the value of the Paper, but I would emphasise the fact that in any particular case the final adjustments are always more conveniently made by trial and error, since the resistances of the opposite arms of the bridge need not necessarily be exactly equal. As Mr. Smith says, the efficiency of the device depends upon the quantity he calls β, and if β be so large compared with α that the latter may be neglected, the need for the device would disappear. Unfortunately, as he remarks, the substance in which the current decreases as the applied voltage increases is one somewhat difficult to imagine.

I regret very much that I was not aware of Mr. Campbell's previous description of the device before the Institution of Electrical Engineers, or of Professor Callendar's use of it. It clearly appears that the principle of the method was first employed by Professor Callendar, but I would point out that the use of tungsten lamps instead of nickel resistances not only simplifies the setting up of the device, as Mr. Campbell remarks, but it also has another more important result, viz., that of making the regulation independent of external temperature. Mr. Campbell achieved this by the use of compensating resistances placed in the same enclosures as the nickel arms of his bridge, but any such arrangement is unnecessary in the case of tungsten lamps where the conductor operates at a temperature of something like 2000 deg. in a vacuum. It can be shown both theoretically and practically that the effect of even a 100-deg. change in the temperature of the surroundings is quite inappreciable in its effect on the temperature, *i.e.*, the resistance of the filament.

It thus seems that Dr. Rayner's suggestion is one well worth investigating further, and a small form of the bridge, to be used on a circuit regulated to the nearest volt (as it might easily be), could be made very handy and suitable for use with a potentiometer or for similar standardising purposes, and should give a voltage which remains constant and unaltered by temperature to at least 0·01 per cent., and it is possible that a very much greater constancy than this might prove to be obtainable.

IV. *The Flow of Viscous Liquids through slightly Conical Tubes.* *By* PROF. A. S. HEMMY, *Government College, Lahore.*

RECEIVED MAY 5, 1921.

ABSTRACT.

A formula is obtained for the flow of a viscous liquid through a slightly conical tube by neglecting terms containing the square of the obliquity.

The approximation is shown to be justified by the agreement in the values of the viscosity found experimentally with tubes of differing degrees of conicality.

THE corrections to be applied to Poiseuille's formula for the flow of a viscous liquid through a cylindrical tube have been considered in a number of Papers; in this communication the case where the tube differs slightly from the true cylinder is discussed.

I. MATHEMATICAL.

The radius a_2 of the tube at a distance z from one end is taken to be $a(1+bz)$, and all terms containing the square of b are assumed to be negligible.

The stream lines are generators of circular cones having the same apex, so that the velocity v at any point on a stream line at distance r from the axis can be represented by

$$v=\frac{v_0 r_0^2}{r^2}=v_0(1-2bz) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v_0 and r_0 are the values of v and r when $z=0$.

Likewise, if p is the pressure and G the value of dp/dz when $z=0$

$$\frac{dp}{dz}=G(1-2bz) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Integrating along the whole length l of the tube

$$P=\int_0^l G(1-2bz)dz=G(l-bl^2)$$

or

$$G=\frac{P}{l}(1+bl) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

P being the pressure difference between the ends of the tube. The tangential stress per unit length of the sides of the cylinder of radius r equals

$$2\pi r\eta\frac{dv}{dr}$$

where η is the coefficient of viscosity. The difference of tangential forces on the two surfaces of the cylindrical shell of thickness dr is therefore

$$2\pi\eta\frac{d}{dr}\left(r\frac{dv}{dr}\right)dr$$

This differs from the axial component by a quantity depending on b^2 and may therefore be taken as axial. We have, therefore,

$$2\pi\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)dr=-2\pi r dr\,\frac{dp}{dz}-2\pi r dr\sigma\,\frac{d\sigma}{dt}$$

where σ is the density and t the time, or

$$\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)=-rG(1-2bz)-\sigma r\,\frac{dv}{dz}\cdot\frac{dz}{dt}\quad\ldots\ldots\quad(4)$$

From (1) $\frac{dv}{dz}=-2bv_0$ and (4) becomes

$$\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)=-rG(1-2bz)+2\sigma rbv_0\sigma$$

Substitute for v from (1), and, neglecting b^2,

$$\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)=-rG(1-2bz)+2b\sigma rv_0^2\quad\ldots\ldots\quad(5)$$

In the third term, as b is small, we may substitute the value of v_0 as obtained by Poiseuille on the assumption that the tube is a true cylinder, viz.:

$$v_0=\frac{A}{4\eta_0}(a^2-r^2)$$

where $A=P/l$ and η_0 is the first approximation to the value of the viscosity on this assumption, *i.e.*, introducing the usual correction for the kinetic energy of the entering liquid,

$$\eta_0=\frac{\sigma}{8l}\left(\frac{gH\pi m^4t}{v}-\frac{V}{\pi t}\right)\quad\ldots\ldots\quad(6)$$

where m is the mean radius, H the head, and V the volume flowing through the tube in time t.

(5) now becomes

$$\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)=-rA(1+bl)(1-2bz)+2b\sigma r\,\frac{A^2}{16\eta_0^2}(a^2-r^2)^2$$

or

$$\eta\,\frac{d}{dr}\left(r\frac{dv}{dr}\right)=-rA(1+bl-2bz)+r\,\frac{b\sigma A^2}{8\eta_0^2}(a^2-r^2)^2\quad\ldots\quad(7)$$

Integrating, we get

$$\eta\, r\,\frac{dv}{dr}=-\frac{r^2}{2}A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{2}-\frac{2a^2r^4}{4}+\frac{r^6}{6}\right)+B$$

$B=0$, as stress$=0$ when $r=0$.

$$\therefore\quad\eta\,\frac{dv}{dr}=-\frac{r}{2}A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r}{2}-\frac{a^2r^3}{2}+\frac{r^5}{6}\right)\quad\ldots\quad(8)$$

Integrating again, we get

$$\eta v=-\frac{r^2}{4}A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{4}-\frac{a^2r^4}{8}+\frac{r^6}{36}\right)+C$$

To find C, put $z=0$; then when $r=a$, $v=0$.

$$\therefore \qquad C=\frac{a^2A}{4}(1+bl)-\frac{b\sigma A^2}{8\eta_0^2}\;\frac{11}{72}a^6$$

$$\therefore \qquad \eta v_0=\frac{a^2-r^2}{4}A(1+bl)-\frac{b\sigma A^2}{8\eta_0^2}\;\frac{11}{72}a^6+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{4}-\frac{a^2r^4}{8}+\frac{r^6}{36}\right)\,..\,(9)$$

The volume V of liquid which passes across the mouth of the tube in time t is given by

$$V=2\pi t\int_0^a rv_0dr$$

or

$$\frac{\eta V}{2\pi t}=\frac{a^4}{16}A(1+bl)-\frac{b\sigma A^2a^8}{8\eta_0^2}\cdot\frac{1}{32}$$

whence

$$\eta=\frac{\pi ta^4P}{8Vl}\left(1+bl-\frac{b\sigma Pa^4}{16\eta_0^2l}\right)\quad .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (10)$$

Put $P=g\sigma H$, where H is the head, and, including the correction for the kinetic energy of the liquid on entering the tube, we have, finally,

$$\eta=\left(\frac{\pi ta^4g\sigma H}{8lV}-\frac{\sigma V}{8\pi lt}\right)\left(1+bl-\frac{b\sigma^2gHa^4}{16\eta_0^2l}\right)\quad .\;.\;.\;.\;.\;.\quad (11)$$

II. EXPERIMENTAL.

By Prof. A. S. Hemmy and L. Sant Ram, M.Sc.

The need for the formula arose from the difficulty of obtaining perfectly cylindrical tubes for the usual laboratory method of determining coefficients of viscosity. So-called thermometer tubing is rarely perfectly cylindrical, and it is easier to obtain evenly conical lengths. It was desirable to determine how much a variation of radius affected the result. It was found that variations such as commonly occur exercise a marked influence. The correction introduced by formula (11) for a piece of tubing selected as being more cylindrical than the average was found to be nearly 2 per cent., *vide* exp. 9, Table II.

TABLE I.—*Dimensions of Tubes Used.*

Tube.	A	B	C	D	F
Length (l)	10·20	10·40	9·80	6·90	11·405
Mean radius (m)	0·10750	0·10720	0·07661	0·06500	0·10620
Radius at narrow end (a)	0·10632	0·10495	0·07560	0·06359	0·10593
Slope (b)	0·002154	0·004042	0·002767	0·006195	0·0003409

The usual method of correcting for variations in the radius of a tube is to replace l/m^4 in equation (6) by the integral of dz/a_z^4, but for an evenly conical tube this makes no change in the value of η to the first order of bl.

The formula was verified experimentally by one of us using tubes drawn to various degrees of conicality. A tubulated vessel was provided with a simple overflow arrangement to maintain the head constant at any desired value. Care was taken to keep the tubes horizontal. The head was adjusted in most cases to be just sufficient to maintain a steady jet. Tap water was used throughout. The tubes were calibrated and b determined by means of a mercury thread and a micrometer microscope. The value of m was determined by finding the weight of mercury filling the tube, and a calculated from the values of m and b.

TABLE II.—*Values of the Viscosity.*

Col. 1	2	3	4	5	6	7	8	9	10
No. of Exp.	Tube used.	H in cm.	V in c.c.	t in secs.	Temp. Cent.	b	η_0	η	η_{15}
1	A	6·040	250·0	154·8	15·0	0·002154	0·01252	0·01124	0·01124
2	B	6·040	250·3	164·2	15·0	0·004042	0·01351	0·01129	0·01129
3	B	6·830	250·3	150·4	15·0	0·004042	0·01366	0·01128	0·01128
4	D	13·215	70·24	93·0	15·0	0·006195	0·01304	0·01127	0·01127
5	C	10·248	89·00	101·0	15·4	0·002767	0·01213	0·01118	0·01129
6	B	7·493	88·11	49·7	14·8	0·004042	0·01385	0·01131	0·01125
7	C	11·050	62·50	67·5	14·9	0·002767	0·01236	0·01136	0·01133
8	C	11·131	104·55	109·8	16·0	0·002767	0·01193	0·01093	0·01121
9	F	5·852	93·57	61·2	16·5	0·000341	0·01108	0·01087	0·01128

In column 10, Table II., for the sake of comparison all results are reduced to the temperature 15°C. by interpolation with the aid of Kaye and Laby's tables. Whilst the values of η_0 in column 8 show wide variations, those in columns 9 and 10 justify the approximations used in obtaining formula (11) and show that a good value of the viscosity can be obtained with a conical tube.

DISCUSSION.

(Communicated after the Meeting.)

W. N. BOND, M.Sc. This communication consists of a criticism of certain parts of the theory of the Paper; and secondly, of an analysis of the author's experimental results.

The variation in the pressure gradient along the tube, given by equation (2) of the above Paper, was put forward without explanation, and does not appear to follow from the equations for either purely turbulent or purely viscous flow through tubes.

For purely viscous flow we have $dp/dz \propto \eta V/r^4 t$, and for purely turbulent flow $dp/dz \propto \sigma V^2/r^5 t$, where the z axis is along the length of the tube, and r, V/t, σ and η denote the radius, volume passing per second, and density and viscosity of the liquid respectively. Then, adopting the rotation of the Paper, $r=a(1+bz)$, where a is the radius of the tube at the narrow end. Hence, if G denotes the pressure gradient at the narrow end, we have $dp/dz=G(1-4bz)$ and $dp/dz=G(1-5bz)$ in the cases of purely viscous and purely turbulent flow respectively. In the case considered we may probably assume the first of these equations to be true.

According to equation (2) of the Paper $dp/dz=G(1-2bz)$.

In obtaining equation (11) of the Paper, pressure differences occurring in equation (10) are replaced by the corresponding heads, or effective differences in level of the liquid. The effective difference of head between the ends of the tube is deduced from the total head of liquid employed, after subtracting a term due to the gain in kinetic energy of the liquid on entering the tube.

This kinetic energy correction (which amounts in the case of some of the above experiments to 40 per cent. of the total head employed) is, however, apparently not applied in obtaining the last term in equation (11). Hence this last term (which itself in some cases affects the estimated value of η by 10 per cent.) is considerably in error.

It is true that the equation (11) is in very good agreement with the experimental results; but the agreement is not so close when the above changes are made. In any case, it will be noticed that the values obtained for the viscosity are uniformly some 1 per cent. lower than those usually accepted.

If the usual correction due to the kinetic energy at entrance be assumed, we may suppose the effective pressure difference between the ends of the tube to be partly opposed by viscous resistance, and partly by the resistance due to the negative acceleration of the liquid caused by the tube being divergent. Hence we may write

$$H-\frac{V^2}{\pi^2a^4t^2g}=\frac{8l\eta V}{\pi m^4g\sigma t}+kbl\frac{V^2}{\pi^2a^4t^2g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

where $m=a(1+bl/2)$ = mean radius of tube ;
l = length of tube ;
H = total head of liquid employed.

It would be expected that k should have the value 4. And if that were so, the above equation would correspond to Prof. Hemmy's theory, except for the two changes mentioned above.

If we assume the usual values for the viscosity of water, and suppose that $g=978{\cdot}5$ (Lahore), we can determine k from the experimental results, employing equation (I).

The following table gives the values of k thus obtained, together with the values of η calculated from the experiments, using equation (I) and assuming firstly $k=4$, and secondly $k=6$.

TABLE I. (*Analysis of Prof. Hemmy's Results.*)

l	m	a	b	H	V/t	°C.	η calculated $k=4$	η calculated $k=6$	η assumed	k calculated
10·20	0·10750	0·10632	0·002154	6·040	1·614	15·0	0·01163	0·01134	0·01142	5·5
10·40	0·10720	0·10495	0·004042	6·040	1·523	15·0	0·01193	0·01140	0·01142	6·0
10·40	0·10720	0·10495	0·004042	6·830	1·663	15·0	0·01191	0·01136	0·01142	5·8
10·40	0·10720	0·10495	0·004042	7·493	1·772	14·8	0·01199	0·01140	0·01149	5·8
9·80	0·07661	0·07560	0·002767	10·248	0·881	15·4	0·01147	0·01129	0·01130	5·8
9·80	0·07661	0·07560	0·002767	11·050	0·926	14·9	0·01166	0·01146	0·01146	6·0
9·80	0·07661	0·07560	0·002767	11·131	0·951	16·0	0·01124	0·01102	0·01110	5·3
6·90	0·06500	0·06359	0·006195	13·215	0·756	15·0	0·01180	0·01142	0·01142	6·0
11·405	0·10620	0·10593	0·0003409	5·852	1·528	16·5	0·01096	0·01089	0·01096	4·0

The fair agreement between the different values of k obtained might be taken to indicate that its true value was of the order 5·7 : but the writer has been unable to find any theoretical reason why k should have such a value. It is just possible that the flow was on the verge of turbulence in the case of these experiments, and that this might cause a higher value of k than 4.

V. *The Estimation of the Radium Content of Radio-active Luminous Compounds.*

By E. A. OWEN, *M.A., D.Sc., and* WINIFRED E. FAGE, *B.Sc.* (*The National Physical Laboratory, Teddington.*)

RECEIVED AUGUST 15, 1921.

ABSTRACT.

(1) The absorption of γ-radiation in zinc sulphide has been measured by different methods, and in each case its value was found to be approximately the same.

(2) No indication was observed of the occurrence of an abnormal absorption when the radioactive salt was mixed with the sulphide.

(3) A table has been compiled giving the values of the ratio of the true to the apparent radium content for tubes of different diameters filled with radioactive luminous compound. This table applies to the case of a compound whose apparent density is 2·03 grammes per cubic centimetre.

(4) The values of the absorption of γ-radiation from radium have been measured in different salts, particular attention being paid to the barium salts with which radium is often found in combination.

THE apparent radium content of a sample of luminous compound may be directly measured by means of any one of the γ-ray methods. To arrive at the true content, it is necessary to know how much of the radiation is absorbed in the salt. This absorption has been measured by Paterson, Walsh and Higgins,* who found that different values were obtained according to whether it was measured before or after mixing the radium with the responsive base—the base employed being hexagonal zinc sulphide. The value of the absorption coefficient of γ-radiation in the sulphide under the conditions of experiment was found to be about three times as much after mixing as it was before mixing with the radium. It was suspected that this difference was due to a selective effect which occurred in the one case but not in the other. This, however, was not definitely proved. The object of the present investigation was to obtain further information on this point. The absorption coefficient of γ-radiation in zinc sulphide has been re-determined by a number of different methods, and, in addition, data regarding the absorption of other salts have been obtained.

APPARATUS.

The most convenient method of measuring the gamma ray activity of a radium salt is the Direct Method. This is one of the methods in constant use at the Laboratory, and has always been employed to measure the radium content of luminous compounds. The apparatus consists of a combined ionisation chamber and gold-leaf electroscope as shown in Fig. 1. The sample whose radium content is desired is placed some distance in front of the ionisation chamber. The distance between the sample and the chamber is decided by the amount of radium which the sample contains—about 30 divisions per minute is a convenient rate of fall of the leaf. The face of the ionisation chamber is covered with lead 0·5 cm. thick, so that only γ-radiation enters the chamber. To diminish the effect of stray radiation the windows of the electroscope are shielded with lead, and the walls of the chamber and the electroscope are made of lead 0·6 cm. thick. The amount of radium in a sample is measured by comparing it with a standard tube of known content, the experimental conditions during the observations being in each case exactly similar. The time taken by the leaf to fall over a constant number of divisions of the scale is measured both for the standard and for the sample. After correcting these times for the natural leak of the leaf system, which should be determined at the beginning and at the end of the observations, the corrected times are inversely proportional to the radium contents of the standard and sample, and thus the radium content of the sample is directly determined.

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* Proc. Phys. Soc., Vol. XXIX., p. 215 (1917).

METHODS OF MEASURING THE ABSORPTION OF THE RAYS IN THE SULPHIDE.

The absorption of the radiation in zinc sulphide has been measured in the following ways: (1) The sealed tube containing the radium which is eventually to be mixed with the sulphide is first measured. The zinc sulphide is placed in a glass cell of known thickness, and the radium placed very close to the face of it, remote from the chamber. Observations are taken with the cell empty and with it full, and from the knowledge of the apparent density of the material in the cell the mass absorption coefficient under the conditions of experiment is calculated. (2) The sulphide is put into a glass tube of known diameter. The radium tube is then placed on the axis of this tube and its radium content again measured. (3) The contents of the radium tube are intimately mixed with a known quantity of the sulphide and placed in a glass tube of known diameter. The values of the radium content before and after mixing give sufficient data to calculate the absorption coefficient of the radiation in the sulphide. (4) The luminous compound made for the third of the above methods is placed in a glass cell of known thickness and the absorption of the radiation from another tube of radium measured in it by the first method.

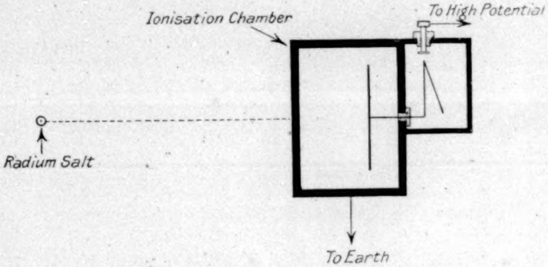

FIG. 1,

ABSORPTION OF γ-RADIATION IN SODA GLASS.

A preliminary investigation was carried out on the absorption of different kinds of soda glass. It was necessary to know this figure in order to correct for the absorption of the radiation in the walls of the glass vessels containing the compounds. A number of determinations of the absorption coefficient (μ) in soda glass, under the conditions of experiment, gave the value 0·105 cm.$^{-1}$.

OBLIQUITY CORRECTION.

In the first of the above methods of measurement the cone of the rays passing into the ionisation chamber, which was 14 cms. in diameter internally, has an appreciable angle at the vertex. The semi-angle in the present case varied from about 4 deg. to about 17 deg., and for the larger angles a correction had to be made for the obliquity of the radiation. This was calculated by means of the following formula, due to Soddy and Russell*:—

$$\frac{I}{I_0} = \frac{e^{-\mu t} - \cos\theta\, e^{-\mu t/\cos\theta} + \mu t[Ei(-\mu t)] - \mu t[Ei(-\mu t/\cos\theta)]}{(1-\cos\theta)}$$

where I_0 and I are the intensities of the radiation before and after traversing the salt.

A full curve was drawn, showing the relation between $\frac{I}{I_0}$ and μt, for given values of θ, from which the value of μ could be obtained for any intensity ratio measured.

RESULTS.

Two radium standards were employed in the work, the one (Standard A) containing about 15 mgms. and the other (Standard B) 2·5 mgms. of radium element. The following table shows the variations that were obtained in the values of the absorption coefficient with the two standards when placed at different distances in front of the ionisation chamber:—

TABLE I.

Standard A.		Standard B.	
Semi-angle θ of cone of rays.	μ(cm.$^{-1}$).	Semi-angle θ of cone of rays.	μ(cm.$^{-1}$).
8° 19′	0·100	16° 31′	0·095
7° 27′	0·099	13° 45′	0·101
5° 50′	0·099	11° 46′	0·100
5° 6′	0·101	10° 17′	0·101
4° 31′	0·103	9° 7′	0·100
4° 1′	0·097	8° 12′	0·102

The values of the absorption coefficient given in the table have been corrected for the obliquity of the rays. The mean value of μ is 0·100 cm.$^{-1}$.

Another determination of μ by the same method for a different sample of sulphide gave the value 0·103 cm.$^{-1}$. The average value of the apparent density of the zinc sulphide employed for these determinations was 2·03 grammes per cubic centimetre, so that the mean value of the mass absorption coefficient is 0·050.

A determination of the absorption coefficient, when the radium tube was placed axially inside a wider tube filled with zinc sulphide, gave the value μ=0·098 cm.$^{-1}$.

The radium was now carefully mixed with a known quantity of the sulphide. This was carried out by the dry method and it may be of interest to indicate the procedure adopted. The contents of the radium tube were transferred into an agate mortar; it was difficult to remove all the salt from the tube as some of it adhered to the walls, even when the tube was gently tapped. The final cleaning was carried out by putting a little zinc sulphide into the tube and rubbing it on the inside walls by means of a small camel-hair brush. The operation was repeated five or six times, using only a small quantity of sulphide in each operation, until all the material clinging to the walls was removed. Having now transferred the radium with the small quantity of zinc sulphide used in the cleaning process into the mortar, it was ground for about 20 minutes into a fine powder. A small quantity of zinc sulphide was afterwards added, say, about 1 gramme, and mixed with the radium by means of a spatula, taking care not to apply much pressure to the sulphide so as to avoid breaking up the crystals, which should be kept as large as possible for the best luminous effect. More sulphide was added and again thoroughly mixed with the salt already in the mortar. If the quantity of zinc sulphide is not too large it may all be added little by little to the radium, stirring constantly and keeping about 2 grammes of sulphide in hand for final washings of camel-hair brush, spatula, and funnel used to transfer the compound from mortar to the tube in which it is sealed. It is advisable not to fill the tube, so that the salt may be well shaken for final mixing. In this way a very homogeneous mixture was obtained.*

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* Soddy and Russell. Phil. Mag., XIX., 725 (1910).

The compound was finally sealed in a glass tube whose internal diameter was 14·1 mm. and whose thickness of wall was 0·85 mm. The growth of activity of the compound was kept under observation for about five weeks, after which period it had reached a state of radioactive equilibrium. The maximum value of its activity was found to be equivalent to 2·314 milligrammes radium element, whilst the activity of the radium tube before mixing with the sulphide was equivalent to 2·519 milligrammes. Calculating the value of μ to a first approximation from the simple formula $I=I_0e^{-\mu}$, where t is 7·05 mm., we have, after correcting for the absorption in the glass, $\mu=0{\cdot}107$ cm.$^{-1}$. There will undoubtedly be a slight loss of radium during the process of mixing the compound which will tend to increase the value of μ. A loss of 0·5 per cent. of the salt during mixing would in the present case alter the value of μ from 0·100 cm.$^{-1}$ to 0·107cm.$^{-1}$, so that we may say that the above figure is in fair agreement—allowing for experimental error—with the value obtained by the other methods.

The fourth method of measurement, in which the luminous compound was removed from the sealed tube, placed in a glass cell and the absorption of the radiation from another radium tube in it measured, gave 0·100 cm.$^{-1}$ for the value of μ.

There is, therefore, no appreciable difference between the values of the absorption coefficient of the radiation in zinc sulphide when measured by any one of the four methods described. The two values of μ obtained by Paterson, Walsh and Higgins may be explained if it be assumed that the compound when finally measured had not reached a state of radioactive equilibrium. It is possible for a radium salt to reach a steady maximum value of activity and yet not be in a state of true radioactive equilibrium. This may occur if the containing tube is imperfectly sealed, so that there will be a constant small leakage of emanation. The rate of production of emanation by the radium and the rate of leakage of emanation from the tube will, after a time, reach a steady state giving a fictitious state of equilibrium. The maximum value of the activity will in this case be smaller than the value attained when the salt is in a state of true radioactive equilibrium, so that if the luminous compound were not properly sealed in its enclosure, the value of the absorption coefficient of the radiation measured by the third of the above methods would be too great.

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* **The authors are indebted to Mr. F. H. Glew for the details of this method of preparing luminous compound.**

THE RATIO OF THE TRUE TO THE APPARENT RADIUM CONTENT OF TUBES OF LUMINOUS COMPOUND OF DIFFERENT DIAMETER.

The most reliable value of the absorption coefficient of γ-radiation in zinc sulphide is that measured by the first of the above methods, which gave the mean value $0{\cdot}101$ cm.$^{-1}$. Knowing this figure and assuming that the radium is distributed uniformly throughout the body of the sulphide, we can calculate the ratio of the true to the apparent radium content of tubes of different diameter filled with luminous compound. Suppose the tube containing the compound is cylindrical and of internal radius a cm. If I is the measured intensity of the γ-radiation emitted by the tube and I_0 the intensity if no absorption had taken place, it can readily be shown,* if the correction due to the obliquity of the rays be neglected, that

$$\frac{I}{I_0}=\frac{1}{\mu a\sqrt{\pi}}\sum_{r=1}^{r=\infty}(-1)^{r+1}\frac{(2\mu a)^r}{\underline{\left| r+1\right.}}\,.\,r\,.\,\frac{\Gamma\frac{r}{2}}{\Gamma\frac{r+1}{2}}$$

$$=C_0+C_1(\mu a)+C_2(\mu a)^2+\;.\;.\;.\;.\;.\;.$$

where μ is the coefficient of absorption of the rays per centimetre thickness of compound. The values of the constants $C_0, C_1, C_2, \ldots$ are given in the following table :—

TABLE II.

Coefficient.	Value.	Coefficient.	Value.
C_0	$+1{\cdot}000000$	C_7	$-0{\cdot}001642$
C_1	$-0{\cdot}848826$	C_8	$+0{\cdot}000347$
C_2	$+0{\cdot}500000$	C_9	$-0{\cdot}000067$
C_3	$-0{\cdot}226354$	C_{10}	$+0{\cdot}000012$
C_4	$+0{\cdot}083333$	C_{11}	$-0{\cdot}000002$
C_5	$-0{\cdot}025869$	C_{12}	$0{\cdot}000000$
C_6	$+0{\cdot}006944$		

Since the value of the absorption coefficient for soda glass is approximately the same as that for zinc sulphide we may, without serious error, calculate by means of the above series the ratio of the true to the apparent (or measured) radium content for tubes of different *external* diameters on the assumption that $\mu=0{\cdot}101$ cm.$^{-1}$. The values calculated in this manner are given in the following table :—

TABLE III.

External Diameter of containing tube in cm.	$\frac{\text{True Content}}{\text{Apparent Content}}$
0·5	1·02
1·0	1·04
1·5	1·07
2·0	1·09
2·5	1·11
3·0	1·13

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* See Paterson, Walsh and Higgins (loc. cit.). There is a misprint in the general term of the series as given in their Paper, but the values of the constants are the same as given here.

ABSORPTION OF γ-RADIATION IN OTHER SALTS.

The absorption of γ-radiation was also measured in a number of other salts, special attention being paid to barium salts with which radium is closely associated. The measurements were made by the first method outlined above. Each salt was powdered in a mortar and its apparent density determined from the weight of the salt required to fill the glass cell in which the absorption was measured. The thickness of material traversed by the rays was 1·12 cm. The following table contains a summary of the results obtained :—

TABLE IV.

Salt.	Apparent Density in gms. per c.c.	Absorption Coefficient cm.$^{-1}$.	Mass Absorption Coefficient.
ZnS	2·03	0·101	0·050
$BaSO_4$	2·41	0·079	0·050
$BaCl_2$	1·58	0·100	0·053
$BaCO_3$	1·88	0·046	0·055
PbO	4·21	0·294	0·070
Bi_2O_3	2·89	0·206	0·071
U_2O_5	3·93	0·315	0·080

The mean value of the mass absorption coefficient in the barium salts may be taken as 0·053. By the aid of this value and a knowledge of the apparent density of a barium salt containing a small percentage of radium, a table similar to Table III. above may be compiled, giving the ratio of the true to the apparent radium content of the salt for tubes of different diameters.

It should be noted that the above values of the absorption coefficients hold when the radiation passes through 0·5 cm. of lead on its way to the chamber. Since γ-radiation from radium is heterogeneous the values of the absorption coefficients would be different for other thicknesses of lead in front of the ionisation chamber.

DISCUSSION.

Mr. C. C. PATERSON: I have been much interested by the explanation which the author has offered of the divergent results obtained in the previous work to which he has so kindly alluded. We were greatly puzzled by this divergence at the time, but I think the suggestion that there was a false equilibrium due to leakage of emanation provides a very probable and satisfactory solution of the question.

VI. *The Structure of Organic Crystals.* *By.* PROF. SIR W. H. BRAGG, *K.B.E., M.A., F.R.S.*

(PRESIDENTIAL ADDRESS, DELIVERED NOVEMBER 11, 1921).

SUMMARY.

For many reasons the structure of crystals of organic substances invites examination by the methods of X-ray analysis; but their molecular complexity would seem to throw great difficulties in the way. It is possible, however, that the difficulties in the case of aromatic compounds may be surmounted by adopting a certain hypothesis, viz., that the benzene or naphthalene ring is an actual structure, having definite size and form, and that it is built as a whole into the organic substances in which it occurs. Reasons can be given why this is *a priori* probable.

The examination of certain organic crystals has been made. The results are in general agreement with the hypothesis, and lead to various deductions of interest.

THE structure of organic crystals offers a very inviting field of research by the methods of X-ray analysis. To the organic chemist the relative positions of the atoms in the molecule, as also of the molecules in the crystal, are of fundamental importance; and it is with these relations that the X-rays deal in a manner which is new and unique. Moreover, the multiplicity of crystalline forms—and this is true of both organic and inorganic substances—each so precise and invariable, and so obviously related to the atomic and molecular forces, is a sign that if the forces were better understood it would be possible to account for the forms that are known, and possibly to build others that are unknown. But in order to acquire such a power we must learn the crystalline structure, so that the physical characteristics of the whole may in the end be referred to the characteristics of the individual atom. Progress has been made with the examination of the structure of some of the simpler inorganic crystals; but organic crystals have been neglected. Their molecular complexity has been somewhat of a deterrent. Yet, if a way could be found of making determinations of structure, in spite of the complexity, it seems likely that they would quickly be fruitful. The substitutions and additions which are so characteristic of organic chemistry take place in such an immense variety of combinations and grades that the slightest knowledge of the underlying mechanism might lead to useful comparisons and rules.

I have made a careful study of a few crystals, principally naphthalene and some of the naphthalene derivatives, in order to discover, if possible, some way of handling the complex molecules. The numerical results will be set out later, and may, I think, be taken as sufficiently accurate to make foundations for a theory.

I shall endeavour to show that the results can be explained, so far as can be seen at present, by supposing the benzene ring or naphthalene double ring to have definite form and size, preserved with little or perhaps no alteration from crystal to crystal, and that there are good *a priori* reasons for the supposition. If this principle be accepted the problem is simplified at once. Naphthalene itself is then to be regarded as a structure in which there is but one element, the naphthalene double ring, and no longer as an aggregate of 10 carbon atoms and eight hydrogen atoms of unknown mutual arrangement. A more complex molecule such as either of the naphthols is not to be regarded as an addition of one oxygen atom to these 18, an idea on which nothing can be built, but as a naphthalene double ring of the same size and form as before, except that one particular hydrogen has been replaced by a hydroxyl group. It is then possible to think what changes in the disposition of the molecules might be caused by such a substitution and to compare conceivable solutions with observations on the dimensions of the new crystal. Such a method of procedure is obviously in good agreement with the ideas of organic chemistry.

It is convenient to distinguish the facts regarding crystalline structure which can be obtained by the goniometer and various other means, from the new facts which can be obtained by the use of X-rays. The former are recorded in crystallographic tables such as are given by von Groth in his " Chemische Krystallographie." Naphthalene may be taken as an example. In the fifth volume of von Groth's work, on p. 363, a description of naphthalene is given from which the following data are taken :—

MONOCLINIC PRISMATIC.

$$a : b : c = 1{\cdot}3777 : 1 : 1{\cdot}4364 ;$$

$$\beta = 122^\circ\ 49'$$

The monoclinic prismatic class has the highest symmetry of which the monoclinic system is capable, having a diagonal axis and a plane of symmetry perpendicular to it. The figures give the angular, but not the linear, dimensions of the unit cell of the structure (Fig. 1) ; the unit cell being the smallest volume, of which, by continual repetition without any change in contents or disposition, the whole crystal can be formed. In any crystal it is possible to choose the unit cell in many ways, but they must all be capable of derivation from one another and must all have the same volume. The angular dimensions are to be considered as including the angular relations to one another of any pair of planes in the crystal, not merely of the planes bounding the cell.

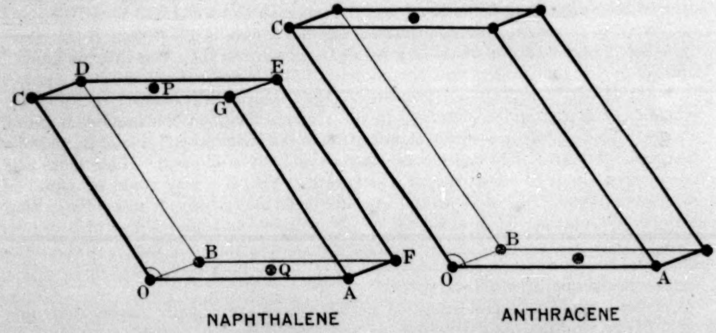

Fig. 1.—Unit Cells of Naphthalene and Anthracene Drawn to the Same Scale.

	$OA = a$	$OB = b$	$OC = c$
Naphthalene	8·34	6·05	8·69
Anthracene	8·7	6·1	11·6

Naphthalene $\alpha = BOC = 90^\circ$, $\beta = COA = 122^\circ\ 49'$, $\gamma = AOB = 90^\circ$

Anthracene $\alpha = BOC = 90^\circ$, $\beta = COA = 124^\circ\ 24'$, $\gamma = AOB = 90^\circ$

It is also stated that the (001) face is the cleavage plane; and that, in addition, the faces (110), $(20\bar{1})$, $(11\bar{1})$ are found as bounding planes of the crystal. The angles between various pairs of these faces are also given as observed. The specific gravity is stated to be 1·152. Other information is given by von Groth concerning the optical properties of the crystal; also concerning the methods that have been adopted in growing the crystals from various solutions and the consequent effect on the development of different faces. These facts do not concern us for the moment, but they must be taken into account eventually.

The examination by X-ray analysis gives us the spacings between the planes and, therefore, the linear as well as the angular dimensions of the unit cell. The specific gravity being known, and the actual weight of the molecule, it is possible to find how many molecules are contained in each cell; generally, two or four. In the case of naphthalene, it is found that, assuming the angular dimensions to be correctly given by the crystallographers, the linear dimensions are:—

$$a=8{\cdot}34,\ b=6{\cdot}05,\ c=8{\cdot}69.$$

These figures are obtained in the following way:—

The actual length of the b axis being represented by b, the mass contained in the cell is

$$b^3 \times 1{\cdot}3777 \times 1{\cdot}4364 \times \sin 122^\circ\ 49' \times 1{\cdot}152 \text{ A.U.}$$

(It is convenient in this work to extend the Angstrom system of units so that an A.U. of area is 10^{-16} cm.^{2}, of volume 10^{-24} cm.^{3}, and of mass 10^{-24} gr.)

The mass of the hydrogen atom being 1·662 A.U., the mass of the molecule $C_{10}H_8$ is $128 \times 1{\cdot}662 = 213$ A.U. Now, from the full results of the X-ray measurements which will be given presently, it is perfectly clear that there are two molecules in each unit cell. Hence, the value b is readily calculated, and the values of a and c also.

Besides these determinations of length, the X-ray method gives also the angle between any pair of planes, whether they form faces or not, provided that a measurable reflection can be found. Also, the relative intensities of the reflection by different faces, as well as the relative intensities of the spectra of different orders given by any one set of planes, yield information as to the distribution of the scattering centres and of the atoms which contain them.

There are two distinct methods of using the X-rays. In the one, which has been used from the beginning, a single crystal is employed. It need not usually weigh more than 2 or 3 milligrammes, and, in fact, it is more convenient that it should be small, since the pencil of reflected rays is then conveniently limited by the size of the crystal without the need for slits, which are still used, however, but as stops only. This method is at present the more precise of the two. Also it permits the measurement of the angles between reflected planes, a knowledge of which is often very useful for identifying the planes.

The second method, first used for crystal analysis independently by Debye, and by Hull, can be used when the crystal is in powder, and can, therefore, be employed when no single crystal can be obtained of sufficient size to measure by the first method. All the spectra of the different planes are thrown together on the same diagram or photograph, and must be disentangled. This is not so difficult as it might seem, because there are not more than one or two lines in each spectrum, and there is generally independent evidence as to the angular dimensions of the crystal. The spectra of copper, and of some organic substances, shown in Figs. 2, 3, 4 and 5, were obtained in this way. The X-ray bulb employed was of the form described by Müller.* The way of using the powder in conjunction with an ionisation spectrometer has been described in the Proceedings of this Society.† Müller's

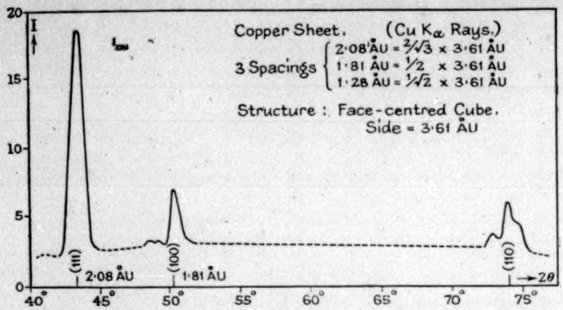

FIG. 2.—COPPER SPECTRUM, OBTAINED BY POWDER-IONISATION METHOD.

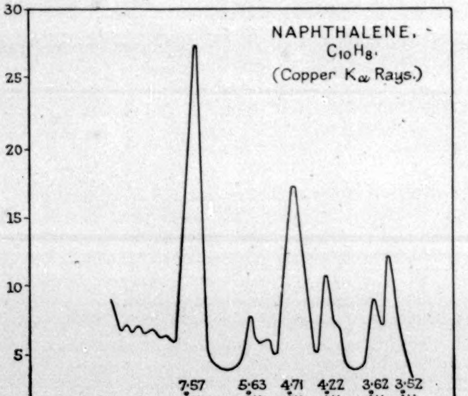

FIG. 3.—SPECTRUM OF NAPHTHALENE—POWDER METHOD. THE RESULTS AGREE WELL WITH THOSE OBTAINED BY THE SINGLE CRYSTAL METHOD, BUT ARE SLIGHTLY LARGER.

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* Phil. Mag., p. 419, September (1921).

† Proc. Phys. Soc., Vol. XXXIII., Part 4, p. 222, June (1921).

bulb is very well suited to this method. The spectrometer slit can be brought very close to the radiator; so that the slit becomes the source of a strong beam of rays which is sufficiently divergent to cover a large surface of the plate on which the

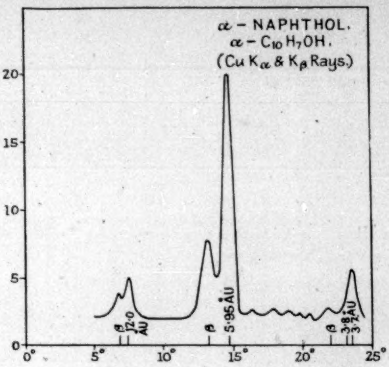

FIG. 4.—SPECTRUM OF α-NAPHTHOL—POWDER METHOD.

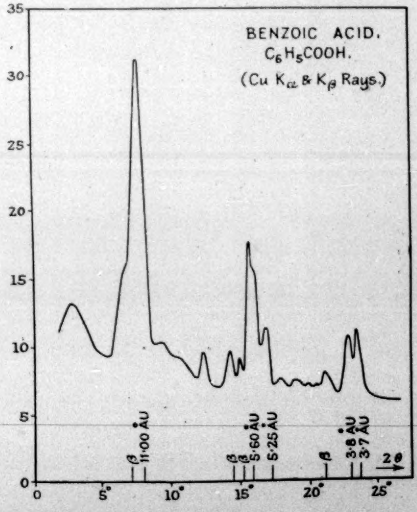

FIG. 5.—SPECTRUM OF BENZOIC ACID—POWDER METHOD.

powder is spread. By a known focussing action (X-rays and Crystal Structure, W. H. and W. L. Bragg, p. 31) the rays are converged upon the slit of the ionisation chamber. The anticathode used in these experiments was made of copper ; the long wave lengths of the *K* series of copper give suitable angles of deflection even for the wide spacings that are found in organic crystals. The curve in Fig. 2 shows the spectrum of copper obtained by the use of copper-rays, and gives at once all the necessary information from which the structure of the copper crystal is derived. The piece of copper employed was of ordinary sheet, not prepared in any way. The spectra of the organic substances show how very diversified they are, and illustrate the power of a method of analysis which promises to be of great use, since every crystal has its own characteristic spectrum. The bulb was driven by a transformer capable of developing half a kilowatt ; but not working to half of its full capacity. Each determination of a point on the curve took about 10 to 30 seconds.

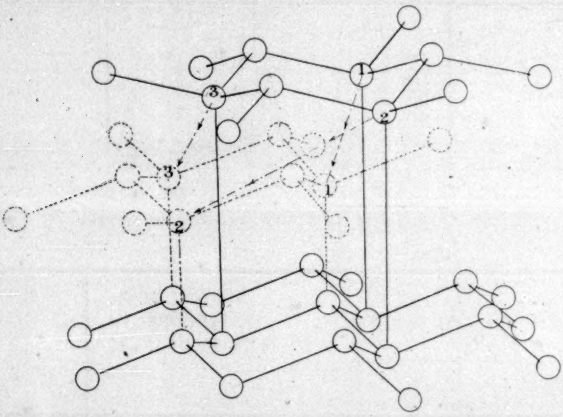

FIG. 6.—THE FINE LINES OF THE DIAGRAM SHOW THE STRUCTURE OF GRAPHITE. BY MOVING THE TOP LAYER TO THE POSITION SHOWN BY THE BROKEN LINES THE DIAMOND STRUCTURE IS OBTAINED.

We may now consider what reasons can be put forward for assuming the concrete existence of the benzene and naphthalene rings. If we examine the structure of the diamond we find that the atoms of carbon are tied together so that each is at the centre of gravity of four others. The distance from centre to centre is 1·54 A.U. As I have already pointed out* the rigidity of the diamond and the open character of its structure, imply that great force is required to alter the orientation of any coupling with respect to the other three belonging to the same atom. Were it otherwise, all atoms would seek to be surrounded by as many neighbours as possible ; the substance would be close packed, and its density would be more than double what it is. The structure of the diamond may also be looked on as consisting of a series of puckered

* Proc. Phys. Soc., Vol. XXXIII., Part 5, August 15 (1921).

layers parallel to a given tetrahedral plane (*see* Fig. 6). A sharp blow may cleave the diamond, along one of these layers. If we take a model showing two layers as in graphite, lay hold of the upper layer, and move it to the new position shown in the figure, the structure is now that of diamond. I am following Hull's determination;* Debye and Scherrer† would flatten out the puckers in the planes of graphite. The point is perhaps not important for our present purpose, but it is necessary, for descriptive purposes at least, to choose one form. According to Hull's measurements, the shortest distance between each pair of atoms lying in the same layer is shortened from 1·54 A.U.—the diamond spacing—to 1·50 A.U. The distance between two successive layers has been increased by 1·35 A.U. A carbon atom in one layer is now at equal distances from its three nearest neighbours in the next layer, the distance being 3·25 A.U.

The bonds between one layer and the next are now greatly weakened; the substance cleaves readily in thin flakes. One layer slides with great ease over the other, though the bonds between the atoms in any layer are at least as strong as before. When all the bonds were of the strong kind, the substance, as diamond, was the hardest thing known. When the one set of bonds has been weakened, the substance, as graphite, is used as a lubricant. Probably its efficiency as such depends both on the weakness of the one set of bonds and the strength of the other. Yet these new bonds are perfectly definite, and the distance between two layers and, therefore, the 3·25 distance between atomic centres is a perfectly constant and determinate quantity. It should be of great interest to compare the physical constants of graphite along and perpendicular to the axis, since in the two cases the two kinds of bonds are separately involved. Some of the comparisons would be difficult, but the thermal expansions can probably be compared by X-ray methods.

If the strong bonds between the atoms in a layer remain, and are even drawn a little tighter when the graphite form replaces that of diamond, it seems very reasonable to suppose that the single ring or multiple rings which are so clearly to be distinguished in the network may be separated out as such without loosening the bonds between their component atoms. In fact, these latter bonds might be expected to tighten even a little more. Let us assume that a single ring is a benzene ring, a double ring a naphthalene ring, and so on. Taking the spacings as given by Hull for graphite, the dimensions of the benzene and naphthalene frameworks are as shown in Fig. 7. The figure is constructed to show the arrangement of atoms in the naphthalene crystal, but it will also serve to illustrate the point under discussion. The carbon atoms *A* to *F* form a benzene ring, those from *A* to *J* the double ring of naphthalene.

The atom centres *A*, and *G*, are 0·71 A.U. above, and the centres *D* and *J* the same distance below the plane of the diagram which is supposed to contain all the remaining centres. It should be observed that circles are used to represent the atoms as a convenient method of designation, not as implying that the radius (1·50) may *always* be used in calculating the distance between the centre of any one atom, and the centre of any other atom.

We may now go on to consider individual crystals; and we take naphthalene first. It might have seemed more natural to attack the benzene crystal before the naphthalene; but the latter was chosen because it is a very well-shaped crystal, and is solid at ordinary temperature. Benzene can only be examined under special temperature conditions and then only, with convenience, as a mass of small crystals. The study of benzene and some of its derivatives has been begun, but the greater attention has been given to the naphthalene crystals, and I will describe now the results that have been obtained in their case:

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* Physical Review, X., p. 692, December (1917).

† Phys. Zeit., 13, p. 297 (1917).

In the second column of Table I. are given the angles between certain pairs of planes as calculated from the data already given, viz. :—

$$a=8{\cdot}34,\ b=6{\cdot}05,\ c=8{\cdot}69,\ \beta=122^\circ 49',$$

and in the third the corresponding values as observed.

TABLE I.

Angles between.	Calculated.	Found.
100 : 001	57°11′	57°16′
110 : $11\bar{1}$	36°32′	36°34′
$20\bar{1}$: 001	85°45′	85°48′
021 : 001	67°47′	67°35′
$20\bar{1}$: $10\bar{1}$	30°21′	30°23′
210 : $21\bar{1}$	24°12′	24°24′

These figures are quoted in order to show that the X-ray method gives angular measurements with sufficient accuracy. Indeed the agreement between the values in the two columns is as good as is usually reported by different observers who have used the goniometer (*see* the descriptions recorded by von Groth); and the X-ray method could really be relied on to a minute of arc if the necessary care and time were justified. In this case no special pains were taken.

In the next table are set out the results of the linear measurements. They are compared with calculations based on the assignment of *one* molecule to each cell, which is equivalent to supposing each corner of the cell to represent a molecule. The object of making this provisional assumption is to show how the position of the second molecule can be found by comparing the observed with the calculated results.

TABLE II.

Plane.	Calculated spacing.	Observed spacing.	Nature of reflection.
100	7·00	3·46	Strong: indication of spacing 6·92
010	6·05	2·95	Very weak.
001 (Cleavage)	7·30	7·30	Very strong: also higher orders.
110 (Natural)	4·59	4·55	Strong.
$11\bar{1}$ (Natural)	4·70	4·63	Moderate.
$20\bar{1}$ (Natural)	4·17	4·12	Strong.
021	2·79	2·76	Very weak.
$10\bar{1}$	7·51	3·71	Very weak.
210	3·04	2·99	Strong.
$21\bar{1}$	3·44	3·39	Strong.

No reflection obtained from following planes :—

011, 012, $10\bar{2}$, 101, 111, $22\bar{1}$, 112, $11\bar{2}$, 221, $21\bar{2}$, 211, 212.

The table shows that the 100 and 010 spacings are only half what they should be if there were molecules at cell corners only. (N.B.—Only one-eighth of a corner molecule is within the cell, and the whole eight count for one whole molecule within the cell.) But the 001 spacing is right. We conclude that there is a molecule at each of the points *P* and *Q* (*see* Fig. 1), each contributing half a molecule to the cell: and that these are in all respects similar to the corner molecules. Molecules placed at *P* and *Q* interleave the planes 100, 010, and also $10\bar{1}$ by other planes of equal density which halve the corresponding spacings. The planes 110, $20\bar{1}$, 021, are unaffected because they already contain *P* and *Q*.

It should be mentioned, however, that the 100 plane seemed to give a small spectrum at half the angle which gave the principal reflection; this would indicate that the second molecule was not quite similar, in orientation or some other particular, to the first, or was not exactly at *P* and *Q*. This is also suggested by the fact that 210 and $2\bar{1}\bar{1}$ give the calculated spacings, whereas half values might be expected.

Although so much information is given in these tables, some of which we have used and some of which we cannot fully use for lack of knowledge, yet it would be hopeless to try to arrange the eighteen atoms of naphthalene on the basis of what has been learnt, without some helpful hypothesis. But we now take the naphthalene double ring as described. Its dimensions are such that it seems quite possible to fit two of them into the cell, if we had some indication as to their orientation thereto.

As to this we get a strong hint from a comparison of naphthalene with anthracene ($C_{14}H_{10}$), whose construction shows three rings in a line, as against the two of naphthalene. The crystallographic data of the latter are:—

$$a : b : c = 1{\cdot}4220 : 1 : 1{\cdot}8781, \quad \beta = 124^{\circ}\ 24'.$$

Specific gravity = 1·15.

The crystals themselves are very small flakes, and it was not possible to find one which could be conveniently treated by the single crystal method. However, by pressing a number of them together against a flat disc, so that all the 001 planes were parallel thereto, however oriented they might be otherwise, it was easy to get a sufficiently accurate determination of the 001 spacing, and therefore the linear dimensions of the unit cell. It appeared that there were two molecules to the cell, as for naphthalene. The dimensions were:—

$$a = 8{\cdot}7, \quad b = 6{\cdot}1, \quad c = 11{\cdot}6.$$

If these dimensions are compared with those of naphthalene (*see* Fig. 1), it will be seen that while *a*, *b*, and β remain nearly the same, the *c* axis has lengthened considerably, the difference amounting to 2·9 A.U. nearly. Now the extra ring, if of the benzene dimensions, should be responsible for an addition of 2·5 A.U. nearly to the molecule.

It is reasonable to conclude that the molecules in both crystals lie end to end along the *c* axis, and that the structures are similar.

The over-all lengths of the two molecules, without allowance for the hydrogen atoms at their ends, that is to say, in the β-positions, are 6·41 and 8·86 respectively. There is, therefore, a vacant space between the ends of two molecules of rather more than 2 A.U., into which two hydrogen atoms have to be fitted. This agrees very well with what might be expected; only it must be remembered that we have no definite indication from studies of crystal structure as to the actual distance between the centres of a carbon and a hydrogen when united by a valency bond, nor between two hydrogen atoms not so united.

We have still to decide in what plane, passing through the *c* axis, the molecule is to be placed, and we have less clear indications with respect to this point than those that have guided us hitherto. On making up a model, however, it is seen that it is much more likely that the plane of the molecule lies nearer to the *ac* plane than the *bc* plane. The molecules lock together much better if that is so. Moreover, if the molecules lie in the *bc* plane, they would be close neighbours in that plane, and at the same time there would be wide gaps between consecutive planes. The plane 100 should, therefore, be prominent, most probably a natural face, perhaps even a cleavage plane: whereas it is neither of these things. But if the molecules lie in the 010 plane the form of the crystal seems much easier to understand, as we shall see later. The β-hydrogens of each molecule lie up against the corresponding hydrogens of the next and the 001 plane passes through them all. It would appear to be the weakest junction in the crystal, and therefore the 001 plane is the cleavage plane.

It must be observed that in the junctions between molecule and molecule there are forces far weaker than the valency forces, which latter unite the atoms of the same molecule. It is the former which bind the molecules into the crystal, nevertheless.

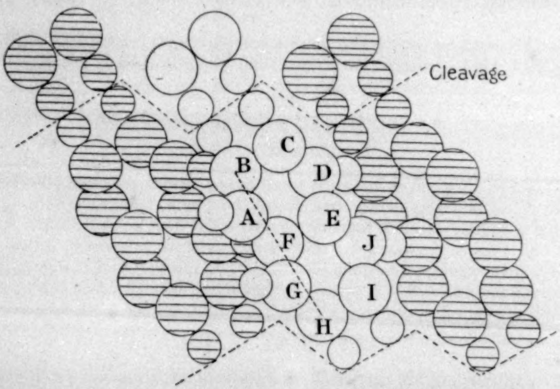


FIG. 7.—SHOWING MUTUAL RELATIONS OF THREE NAPHTHALENE MOLECULES AND PARTS OF OTHERS.

The unshaded circles between the two cleavage planes represent a molecule as at *Q* (Fig. 1). The shaded represent molecules *B* and *F* in the same figure. The small circles represent hydrogen atoms, but their size is uncertain.

Diameter of carbon atom = 1·50. *BH* = 4·92. Projection of *AD* on the plane of the diagram = 2·50.

When the model is put together in the way now indicated it is found that all the α-hydrogens, those that are atached to the sides of the molecule, lie up against carbon atoms of the next molecule, and that there is an appropriate space waiting for each, of magnitude about 1 A.U., the actual value depending on the orientation of the molecule. The forces exerted at these junctions, though far weaker than valency forces, are stronger than the forces between two β-hydrogens; and therefore if the crystal is ruptured it is the latter which give way first. The forces exerted by the α-hydrogens do so across the 110 and $1\bar{1}0$ planes, and it is not surprising to find that the latter make natural faces of the crystal and give a strong reflection. The structure now found is a very empty one: it is like lace-work in space. That must be expected, since the specific gravity is so low. The structure is shown in Figs. 7 and 7A.

We may now attempt to find the structure of a naphthalene molecule in which some complexity has been introduced by a substitution. We take acenaphthene. Dr. J. B. Cohen was kind enough to give me some good crystals weighing each a few milligrams.

In this case the molecule has been made lop-sided by the substitution of a group of two carbon and four hydrogen atoms for the two hydrogens on one side. It is a striking fact that the crystal has now gained in symmetry; the unit cell is rectangular. The crystallographic data are:—

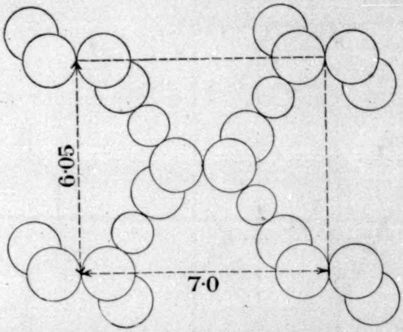


FIG. 7A.—SECTION OF NAPHTHALENE CELL PERPENDICULAR TO THE AXIS OF *c*, SHOWING α-HYDROGENS CONNECTING THE MOLECULES SIDE TO SIDE.

Orthorhombic (with the fullest symmetry of that class)—

$$a : b : c = 0{\cdot}5903 : 1 : 0{\cdot}5161.$$

It appears that there are four molecules to the cell, and that

$$a = 8{\cdot}32, \quad b = 14{\cdot}15, \quad c = 7{\cdot}26 \quad (\textit{see}\ \text{Fig. 8.})$$

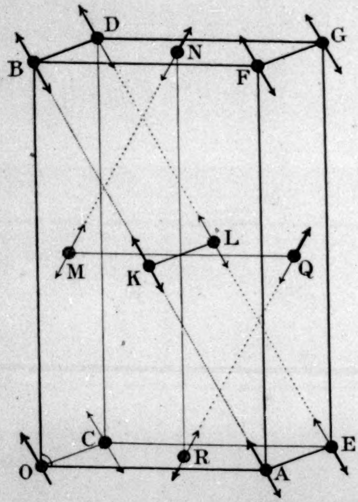
The doubling of the number of molecules, as compared with naphthalene, suggests that symmetry has been obtained by an arrangement in which two of the unsymmetric molecules are the images of the other two across one of the principal planes, *ab*, *ac*, or *bc*.

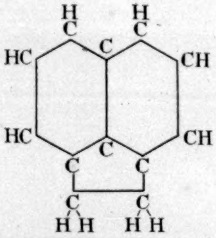
As the symmetry is so high, it is natural to test first the hypothesis that there is a molecule at each corner of the unit cell, and one in the middle of each face, which arrangement will account for four molecules. The calculated spacings in the following table are in accordance with this supposition.

TABLE III.

Plane.	Calculated spacing.	Observed spacing.	Remarks.
100 (Natural)	4·16	4·16	Moderate strength
010 (Cleavage)	7·07	7·1*	—
001	3·63	3·63	Moderate strength
110 (Natural)	3·57	7·18	Moderate strength
101	2·74	2·74†	Very weak
011	3·23	3·19	Very weak
111 (Natural)	5·10	5·1	Weak

There is complete agreement, except for the 110 plane. Assuming that there will be an explanation of this as we proceed further, we now look where a molecule of length about 8·69 (as in naphthalene) may be placed in position, and observe that $KB=KF=\sqrt{(4\cdot16^2+7\cdot07^2)}=8\cdot23$, and these two are images of each other in the 100 plane. This is not quite long enough. Let us, however, suppose provisionally that the molecules at the corners of the cell and at *K* and *L* lie parallel to *AB*, and with their planes coincident with (001). Let those at *MNQR*, which between them contribute to the cell the other two molecules that belong to it, lie in the parallel





ACENAPHTHENE

FIG. 8.—UNIT CELL OF ACENAPHTHENE, CONTAINING FOUR MOLECULES.
$OA=8\cdot32$, $OB=14\cdot15$, $OC=7\cdot26$.

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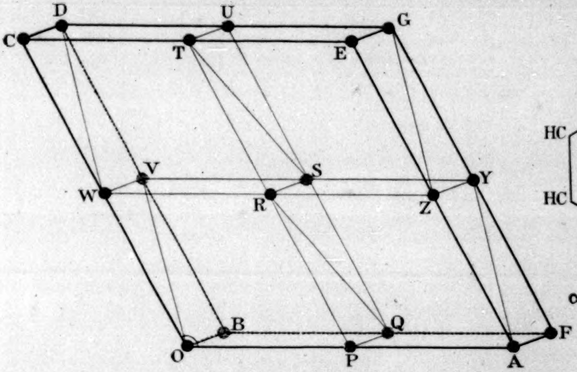
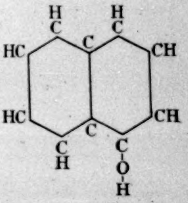
* Not observed directly, but inferred from other observations.

† There is an indication that alternate planes differ somewhat.

plane, but slope the other way, parallel to *MN* or *QR*. If this is so, we see why the 110 plane *ABDE* in the figure seems to have a double spacing; it is because the two sets of molecules—viz., such as *A*, *B*, *D*, *E* of the one set, and *M*, *N*, *Q*, *R* of the other, are very differently disposed towards this plane. One set lies parallel to it, the other not. Consequently the spacing is twice what it would have been had the molecules been all alike with respect to this plane, and is equal to the perpendicular from *F* or *G* upon the plane *ABDE*. This is true of the 110 plane alone, out of those set out in the table above.

Once more the cleavage plane (*e.g.*, *BDGF*) passes through the junctions of β-hydrogens. On constructing a model it is found that the projecting addition on the side of a molecule seems to engage with a neighbouring molecule on that side of the latter on which its own addition does not lie. The crystal is thus laced together; and in lines which for alternate layers parallel to the cleavage plane (010) are parallel to *OE* and *AC* respectively.





α-NAPHTHOL

FIG. 9.—UNIT CELL OF α-NAPHTHOL CONTAINING FOUR MOLECULES.
$OA=13{\cdot}1$, $OB=4{\cdot}9$, $OC=13{\cdot}4$, $\beta=117^\circ\ 10'$.

The two points left doubtful are the small difference between the length of the molecule in this crystal and in naphthalene, and the fact noted with respect to the plane 101 (*see* the table).

We now take α-naphthol. Here one of the hydrogens at the side of the molecule has been replaced by *OH*. It was difficult to obtain without special preparation a crystal large enough for the single crystal method; but on sorting over a considerable amount of material a few flakes were found, each two or three millimetres across and a fraction of a millimetre thick. With one of these some very good observations were made, as recorded in the table below. The crystallographic data are:—

Monoclinic prismatic.

$$a:b:c=2{\cdot}7483:1:2{\cdot}7715,\ \beta=117^\circ\ 10'.$$

Specific gravity$=1{\cdot}224$.

It is natural to suppose that the similarity of these figures to naphthalene, when the *a* and *c* have been halved, means that the naphthalene cell has been increased four times in order to allow all the four *a* positions to be represented in due order. But it turns out that the crystal is of a totally different structure. There are four molecules in the cell (*see* below), as defined in accordance with the above data. If the naphthalene cell had been repeated four times, there would have been eight molecules in the larger cell.

The linear dimensions are :—

$$a=13{\cdot}1,\quad b=4{\cdot}9,\quad c=13{\cdot}4 \text{ (see Fig. 9)}$$

and it is clear that the molecules are placed as in the figure, one at each point indicated by a letter. This is, perhaps, seen most readily if the calculated figures of the following table are reckoned as if there were one molecule at each corner of the large cell only—viz., *OAFBCEGD*. We then have the following comparison :—

TABLE IV.

Plane.	Calculated spacing.	Observed spacing.	Remarks.
Cleavage 001	11·92	12·00 *	Strong
002	5·96	5·95*	Very strong
210	3·75	3·76	Weak
$21\bar{2}$	3·69	3·68	Rather weak
100	11·82	...	...
200	5·91	5·84	...
$10\bar{1}$	11·38	...	...
$20\bar{2}$	5·69	5·62	†

Also angles between 210, $21\bar{2}$ 36° 24′ (calc.) 36° 16′ (obs.)
and between $10\bar{1}$, 100 57° 48′ „ 58° 5′ „

In the cases where a bracket occurs in this table, it is meant that a reflection shows the existence of a certain spacing between planes, while a very strong reflection at twice this glancing angle shows that this spacing is halved by other planes differing considerably from the first. See, for example, Fig. 4, where a small peak shows the 001 spacing 12·00, and a much larger peak at twice the angle corresponds to half the spacing. This agrees with a supposition that there are molecules at every point indicated by a letter, but those at *VSYZRW* are very different in some way from the others.

The reflection of 100 in the powder photograph would be indistinguishable from 001, and perhaps also the $10\bar{1}$. The powder method does not give very clear indications, therefore, of the existence of the spacings 11·82 and 11·38, because all that is observed may be due to the 11·92, which must be strong, as 001 is the cleavage plane. The single crystal method gave the proper reflection of the 200 and $20\bar{2}$ planes, as shown in the table ; but the reflections belonging to the double spacings 100 and $10\bar{1}$ were not looked for, and probably would have been very hard to find with so small a crystal. However, the combined data are amply sufficient for the conclusion that the four cells in the figure really imply four different orientations of the molecule, and that if we place molecules exactly at the corners of the large fourfold cell the other molecules are very nearly at all the other corners.

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* Powder method only.

† The symbol 002 is used to denote the second order reflection of 001. When it is abnormally strong, it follows that the 001 planes are interleaved.

We have this further indication that, though the half of the c axis, $OW=6{\cdot}7$, is too short to take in the naphthalene molecule, yet the distance OV is equal to 8·3, exactly the distance into which the molecule seemed to fit in acenaphthene. Once again, then, it would seem that these lopsided molecules lie criss-cross, as represented by the lines parallel to OV in the figure ; but they now lie edgeways on top of one another, not flatways, as in the other crystal. It is the "a" axis that now runs along the line of crossings, not the "c" axis. The cleavage plane again passes through the β-junctions.

When a model is made, it is found that the hydroxyl groups fit into their places very naturally, and that if they link together the tops of the molecules in any one layer parallel to the 001 plane, they link in the same way the bottom ends in the next layer. This brings pairs of hydroxyl groups rather close together, as one might expect ; and it looks as if the attraction existed specially between the two oxygens. The attraction is exerted across every alternate 001 plane. The other 001 planes are the cleavage planes. All the other linkings appear to be done by α-hydrogens.

The crystallographic data of β-naphthol are :—

$$a : b : c = 1{\cdot}3662 : 1 : 2{\cdot}0300 \; ; \; \beta = 119°48'$$

Specific gravity$=$1·217.

This has been examined by the powder method only, as a single crystal was not readily obtainable. There is a very strong reflection for the spacing 7·90. This is sufficient to fix the linear dimensions of the cell. If we put

$$a = 5{\cdot}85,\; b = 4{\cdot}28,\; c = 8{\cdot}7 \quad (\textit{see}\ \text{Fig. } 10)$$

the weight of material in the cell is equal to that of one molecule, and the 001 spacing is 7·76 in close agreement with what is found.

Comparing these figures with those for the smaller cell of α-naphthol such as *OBQPWVSR* (Fig. 9) which are 6·55, 4·9 and 6·7 respectively, it is clear that the removal of the OH group from the side to the end of the molecule has caused the cell to shrink sideways in both a and b directions and to grow along the c axis ; this confirms the hypothesis that the molecules in these crystals lie mainly along the c axis. The cleavage plane still cuts across the β-junctions.

The question naturally arises as to whether this is really the unit cell. Ought it not to be multiplied by four, as for α-naphthol, because there are four β-positions, as well as four α-positions ? The evidence in favour of the extension seems very strong. In the first place, it would be the only case, if the present crystallographic data were correct, in which a cell contained one molecule only. Moreover, if all the molecules were oriented alike, their dissymmetry would lower the symmetry of the crystal. The symmetry which is assigned to it can only be obtained by making the unit cell large enough to contain four molecules so that all the β-positions may be represented. Also the hydroxyl groups will surely tend to draw together and cause pairs of molecules to point opposite ways. For these reasons alone it seems likely that the structure is really the same as that of α-naphthol. The length OV which was 8·3 for α naphthol becomes 9·7, a growth of 1·4 which seems consistent with the value of the diameter of the oxygen atom, viz., 1·30.*

No other naphthalene derivatives have yet been examined by the X-ray methods, but there are, of course, large numbers that may be expected to give useful results.

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* W. L. Bragg, Phil. Mag., p. 169, Aug., (1920).

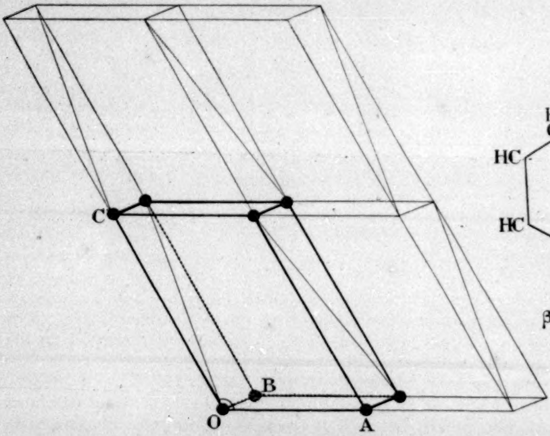
It is worth remarking that the unit cell of α-naphthylamine has almost exactly the same dimensions as that of acenaphthene, if it be supposed to contain four molecules. Both are rectangular ; the acenaphthene dimensions are :—

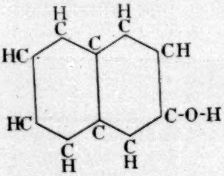
$$a=8{\cdot}32,\ b=14{\cdot}15,\ c=7{\cdot}26,$$

those of α-naphthylamine are :—

$$a=8{\cdot}62,\ b=14{\cdot}08,\ c=7{\cdot}04,$$

provided that the a and b axes of the crystallographers are interchanged in the latter crystal. Their habits also correspond closely, and although the latter crystal has no clear cleavage plane it has a very prominent (010) face which may represent the 010 cleavage of the former. Assuming that the arrangement is therefore the same in the two crystals, the length of the molecule in the new case is $\sqrt{(4{\cdot}31^2+7{\cdot}04^2)}=8{\cdot}25$; as against 8·23 for acenaphthene and 8·31 for α-naphthol. In all these three cases the lopsided lines of molecules are laid criss-cross upon one another, whereas in naphthalene the lines are parallel to each other. The somewhat greater length of the molecule in naphthalene, viz., 8·7, may be connected with the difference in arrangement.





β-NAPHTHOL

FIG. 10.—UNIT CELL OF β-NAPHTHOL, IN THICK LINE, CONTAINING ONE MOLECULE. $OA=5{\cdot}85$, $OB=4{\cdot}28$, $OC=8{\cdot}7$, $\beta=119°\ 48'$.

I have made some measurements of benzene compounds, but the analysis of the results is incomplete ; and until benzene itself is examined data are wanting that must be important. It is worth mentioning, however, that benzoic acid has a very wide spacing between the (001) planes. The dimensions of the unit cell containing four molecules, are :—

$$a=5{\cdot}44,\ b=5{\cdot}18,\ c=21{\cdot}8,\ \beta=97°05'.$$

Experiment by the powder method shows that there is a very strong 10·9 spacing implying that the sheets of molecules lie at this distance apart parallel to the 001 plane. If the crystallographic data are correct these sheets differ alternately. Even a 10·9 spacing is very wide, and this suggests that extended $COOH$ groups are required to bridge across the intervening space. Such a bridge may be made, of course, by a CO extension from one ring joining on to an OH extension from another ring. These bridges part very easily, it would appear; the crystal flakes at the least touch.

The work that I have now described may conveniently be divided into two parts. The first is experimental, and shows, I think, that a large mass of new and valuable information respecting the linear dimensions of organic crystals is to be gained by the methods of X-ray analysis. In the second, I have ventured to suggest and apply a certain principle, viz., that the benzene and naphthalene rings as well as other ring combinations have actual form and dimensions which are nearly, if not quite, the same when they are built into different compounds. Such a principle requires much more illustration before it can be finally established; for the present, however, it shows promise of providing an entry into a very wide field which otherwise seems difficult to break into.

Moreover, the idea fits in extremely well with the work of Langmuir on surface films, and with such results as those described by Adam,* and with the ideas and experimental results of several other workers.† We see clearly that the forces that bind atoms together are of more than one kind. The very strong valency bonds, whether explained as due to electron sharing, or in any other way, are exemplified by all the linkings in diamond and by the linkings in the planes of the graphite flakes. But, besides these are other bonds of much weaker character, such as those extending between an atom in one graphite sheet and its three nearest neighbours in the next. Such bonds as these unite the molecules of the organic compound so as to form the crystal. They are of varying degrees of strength; the cleavage plane shows where they are weakest. They are definitely associated with special points on the molecule as we see from the facts of crystallisation. When a crystal forms in a liquid, or by sublimation, the molecule that attaches itself correctly, and with proper orientation to others already in position, is the one that stays there and resists the tendencies of other drifting and thermally-agitated molecules to remove it. It is fixed by the attachment of certain points on its own structure to certain points on the structure of the other molecules. The beautiful exactness of crystal structure is evidence of the precision with which this adjustment is made; and at the same time of the definite molecular form without which precision would be impossible.

The molecules of naphthalene arrange themselves side by side, the *a*-hydrogens of each molecule seeking to attach themselves to the carbon atoms of its neighbours; valency bonds do not enter into this. Whether there is some special tendency to make an attachment with a carbon atom in the centre of the molecule as seems to be the case, or whether the general slant of the molecules is due to some other cause, is a matter for further inquiry. But it is the side-to-side combination that is preferred in comparison with the end-to-end; the former gives up more energy than the latter. Consequently the crystal grows out quickly into thin sheets. We may further imagine that sometimes, and more often in certain crystals than in others, as well as more often with respect to one plane in a given crystal than to other planes, the geometry may permit a molecule to attach its active points to those of other molecules, already settled, in more than one way which does not carry on the regularity of previous attachments. The crystal starts a new growth, and twinning results.

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* Proc. Roy. Soc., XCIX., July (1921.)

† *See* a list given by Adam, loc. cit. p. 351.

The surface films studied by Langmuir and by Adam and others were not formed of aromatic compounds such as the few that have been investigated as crystals. But the strength of the side-to-side attachment of their molecules as compared with the end-on attachments seems to be exactly the same in effect and to lead to similar results. In a mass of oleic acid the long molecules are lying in all ways, and their heat motions do not give them a chance of attaching themselves at sufficient points at the same instant. But when the hydroxyl ends of some of them root themselves in the water, these molecules are held quiet for a moment and the others quickly slip into their places and link on side to side. So the film spreads out quickly over the water. It is a kind of catalytic action ; the water does not itself draw the film flat, but helps the molecules of the film to arrange themselves. In a sense, the film has negative surface energy.

The arrangement of molecules in crystals or in surface films or interfaces between liquids and solids cannot be fully explained as due to forces which are merely functions of the distances between their centres. Confining ourselves to the cases where there is no obvious separation of electron charges, as there is none in the crystals described above, it is clear that we must think of the molecules as bodies of very definite form. These attachments to one another are made at definite points and the forces there exerted may have very short ranges. The molecules are locked into crystalline structure, when attachments are made at sufficient points, and the whole has the stability of an engineering structure. It may well be that in a liquid there are always some completed attachments, but insufficient in number to give rigidity to the whole. In a gas there are no attachments at all.

I have to thank Mr. Shearer for his assistance in examining crystals by the powder method, which method he has greatly improved ; and Mr. J. Reid, for the labour and skill which he has devoted to making models.

Part of the apparatus employed was obtained through a grant from the Department of Scientific and Industrial Research.

I should also like to thank the General Electric Company of America for their great kindness in presenting me with Coolidge X-ray bulbs of special design.

Dr. C. CHREE, in moving a vote of thanks, said that the President's researches had opened up fascinating vistas of investigation, which should lead to momentous discoveries.

Sir R. ROBERTSON, expressing the hope that these accurate measurements and brilliant speculations would in time be applied to the compounds of nitrogen, seconded the vote of thanks, which was carried by acclamation.

VII. *On the Average Range of β-Rays in Different Metals.* *By* G. A. SUTHERLAND, *M.A.*, *and* L. H. CLARK, *B.Sc.*

ABSTRACT.

Measurements of the range of β-particles in various metals were made in 1910 by Prof. W. H. BRAGG. The present Paper relates to a repetition of these measurements, employing 3 grams of radium bromide. The method consists in determining the ionisation in a lead chamber exposed to very penetrating γ-radiation, when various linings are inserted in it. The quantitative results obtained earlier are substantially confirmed by the present investigation.

THE present Paper owes its origin to a suggestion of Professor Sir W. H. Bragg that it would be worth while repeating some of his earlier γ-ray determinations with the larger quantities of radium now available. In particular the redetermination of the average range of β-rays in different substances seemed of special value. By the courtesy of the authorities at the Middlesex Hospital access to a considerable quantity of radium was arranged for the purpose of this investigation.

The theory of the method followed has already been explained by W. H. Bragg,* and it seems unnecessary to repeat it here. The experiments consisted in measuring the ionisation in a lead chamber, exposed to very penetrating gamma radiation, when different linings were inserted into it.

The apparatus consisted of a cylindrical ionisation chamber placed in the beam of gamma radiation from a brass box containing 3 grams of radium bromide. The source *S* consisted of a number of small glass tubes containing the radium salt, which were distributed uniformly over a disc of 11·7 cm. diameter and were screened by a lead disc 2 cm. thick. The tubes with their lead shield were contained in a flat, cylindrical brass box 13 cm. in diameter and 4 cm. deep. The ionisation within the chamber *C* was measured by means of the electroscope *E*, which was very efficiently shielded from direct radiation by placing the radium container in a lead cubical block of 52 cm. side.

The diagram shows the apparatus with the ionisation chamber in the end-on position and gives the dimensions of the latter, which was made of lead 0·49 cm. thick. It was arranged that the ionisation chamber could be disconnected from the electroscope and by making one of its ends removable the insertion of various metal linings into the chamber was easily effected.

It will be observed that the ionisation chamber is arranged unsymmetrically with respect to the radio-active source *S* in order to prevent considerable ionisation taking place within the stem connecting the chamber to the electroscope. As it was, 5 per cent. of the total ionisation as given by the electroscope was due to that occurring in the stem.

The experiments of Bragg for the end-on position of the lead chamber have been repeated for linings of tin, zinc, iron and aluminium. In addition a lining of copper was used.

Table I. shows the results obtained for the different linings when the ionisation chamber was in the one case in the end-on and in the other in the broadside-on positions with respect to the radium container. The second column gives the mean thickness of the lining and the figures in the third and fourth columns are corrected for differences in volume and for differences in the strengths of the γ-rays due to the differing absorptions by the various linings. For comparison, the figures in the fifth column are taken from the Paper already mentioned,* and were obtained by using a source of only 5 milligrams of radium bromide. The agreement between the earlier and later figures is striking.

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* Phil. Mag., Sept. (1910).

TABLE I.

Metal.	Mean Thickness of lining in cm.	Ionisation in *C* when broadside-on.	Ionisation in *C* when end-on.	The Earlier Values.
Lead	0·49	100	100	100
Tin...............	0·16	67·2	68·2	68
Zinc	0·21 0·35 0·42	55·3, 54·4	52·5, 53·6 54 54·9	55
Copper	0·19 0·25 0·31	55: 55·5	57·3, 57·9 57·1 56·5	...
Iron	0·25 0·155	55·3	56·6	54
Aluminium ...	0·26 0·21	49·6	48·2	49

The theory of the experiment requires that the relative figures for the different linings should be independent of the thickness of the lining, provided that the initial thickness is sufficient to prevent the passage of β-rays originating elsewhere. The figures obtained for different thicknesses of the copper and of the zinc linings show that for these two metals, at least, this is approximately the case. The other metals were not subjected to a similar test.

TABLE II.

Metal.	Broadside-on. Ionisation, I.	End-on. Ionisation, II.	I./II.
Lead	268	202	1·33
Tin..............	140	107	1·31
Zinc	118	84·6	1·39
Copper...........	115	90·2	1·28
Iron	105	81	1·30
Aluminium	103	75	1·37

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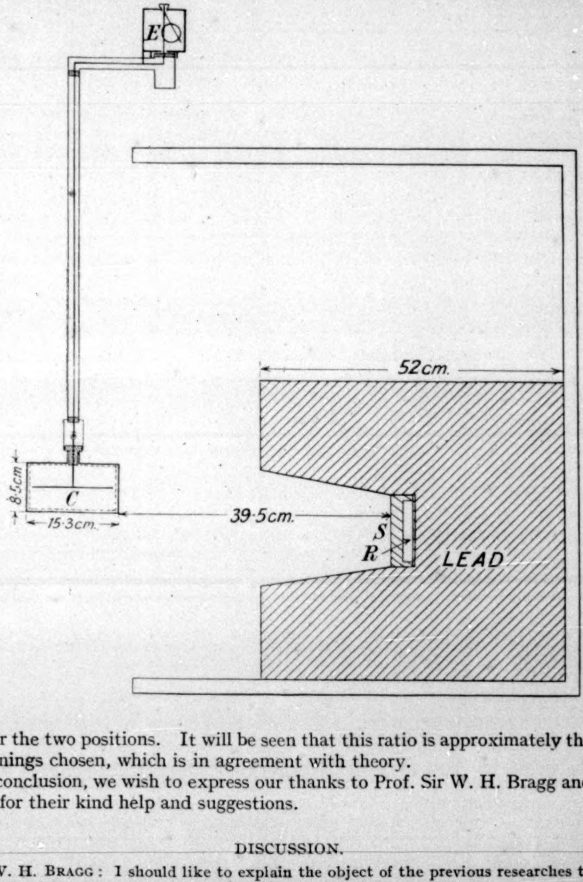
* Loc. cit.

Table II. gives the values of the ionisation obtained for the various linings when the ionisation chamber was (*a*) in the end-on position and (*b*) in the broadside-on position. The last column gives the ratios of the ionisations for one and the same lining for the two positions. It will be seen that this ratio is approximately the same for all linings chosen, which is in agreement with theory.



In conclusion, we wish to express our thanks to Prof. Sir W. H. Bragg and Prof. S. Russ for their kind help and suggestions.

DISCUSSION.

Sir W. H. Bragg: I should like to explain the object of the previous researches to which Mr. Sutherland has referred in his extremely interesting Paper. When an electron meets an atom there are two changes which may take place—changes, namely, in its speed and in its direction—and it was my object to measure their effects separately. In dealing with α-particles the separation of these two factors is comparatively easy, because in this case changes of direction are rare—although it is by investigating those changes of direction, when they do occur, that Rutherford has been able to find out so much about the atom. But suppose, for instance, that you put screens of various thicknesses in the path of a pencil of β-rays, and measure the so-called " absorption coefficient." The effects of the two factors—change of speed and change of direction—are compounded, and it is very difficult to interpret your result, or to say what it really is that has been measured. However, by adopting the conception of the *total range* of a β-particle—the length, that is, which its total path would have if it were straightened out—we get the result that given a certain number of β-particles per gramme of metal, the density of their tracks is independent of the amount of twisting and turning which they exhibit. The idea may be illustrated by taking a square of paper and drawing on it a zigzag line of given length. Whatever shape you may give to the line, the total quantity of ink you use will be the same. I personally am much indebted to Messrs. Sutherland and Clark for having seized the opportunity of repeating these measurements with such a large quantity of radium.

VIII. *On the Sensitivity of Ballistic Galvanometers.* By PROFESSOR ERNEST WILSON.

RECEIVED AUGUST 20, 1921.

(COMMUNICATED BY D. OWEN, D.SC.)

ABSTRACT.

(1) The Paper deals with a method whereby the effects due to excessive *damping in moving coil galvanometers* are largely diminished. The circuit is opened after an interval of time which is small compared with the periodic time of the instrument. This gives rise to an increased first deflection, and consequently, greater sensitivity.

(2) *Analytical Treatment.*—The theory indicates the greater sensitiveness of the galvanometer when the method described in the Paper is employed. For the measure of *Quantity* it is equally as satisfactory and more simple in its application than the ordinary method.

(i.) *Constant Time Interval of Closing the Secondary Cricuit and Constant Total Resistance.*—The values of the Quantity of Electricity are directly proportional to the first deflection. The experiments demonstrate that this is true within the limits of error in observation.

(ii.) *Variable Resistance of Secondary Circuit.*—It is shown that the Quantity of Electricity, Q, is proportional to $\frac{\text{(deflection)}}{R}\varepsilon^{\alpha t_1/R}$ where R is the total resistance, t_1 the time interval of closure of the secondary circuit, and α is a constant.

(iii.) *Constant Quantity and Variable Total Resistance.*—The observed first deflection is equal to

$$\beta/R\varepsilon^{\alpha t_1/R},$$

where α and β are instrumental constants.

(3) *Experimental Confirmation of Theory.*—Experiments were made *to test the accuracy of the theory*, the time interval t_1 being determined (*a*) by a weight falling with uniform velocity, which strikes two levers in turn, the first lever interrupting the primary coil of a mutual inductance, and the second interrupting the secondary circuit; and (*b*) by a freely falling weight released by the operation of the primary switch.

(4) *The Testing of Iron.*—The method has been applied to the testing of iron when the magnetic force, H, was of the order 7×10^{-6} C.G.S., and the corresponding magnetic induction, B, was of the order $1{,}000 \times 10^{-6}$ C.G.S. The gain in sensitivity has been fruitful of greater accuracy in the results.

INTRODUCTORY.

WHEN a galvanometer of the moving-coil type is employed ballistically for the measurement of a small quantity of electricity in a secondary coil connected to the instrument, and when in an extreme case the resistance of the circuit is small, the damping effect in the instrument may be considerable.

Suppose that the secondary circuit is opened after an interval of time at least equal to and coincident with the time of duration of the transient current, but small compared with the periodic time of the galvanometer, the first deflection will be increased and greater sensitivity will be secured. The necessary analysis underlying the action of the galvanometer under the two different conditions of damping is now given; it is, of course, in its main essentials similar to that worked out in general terms by Lord Kelvin in the theory of the siphon recorder. But it does not appear that this method of securing greater sensitivity in the galvanometer has been adopted hitherto.* The analytical treatment is accompanied by the results of experiment.

THEORETICAL.

Let R be the resistance of the swinging coil of a ballistic galvanometer, and A its area of cross section. When swinging with deflection θ in a magnetic field of intensity H, an E.M.F. $HA\dot{\theta}$ is induced in the coil. If we take no account of its self-induction—as is evidently justifiable in the majority of cases, and especially so in the present instrument—the induced current is $i=HA\dot{\theta}/R$, where R is the resistance of the coil and the remainder of its circuit. Energy is dissipated by induction at a rate given by

$$\tfrac{1}{2}Ri^2=\tfrac{1}{2}\,.\,\frac{H^2A^2\dot{\theta}^2}{R}$$

If mk^2 is the moment of inertia of the coil, and κ a decrement factor, the equation of motion of the coil becomes

$$mk^2\ddot{\theta}+\left(\kappa+\frac{H^2A^2}{R}\right)\dot{\theta}+\mu\theta=0$$

where μ is a constant of the instrument, related to its *free* period T by the equation

$$\frac{\mu}{mk^2}=\frac{4\pi^2}{T^2}$$

Writing $\kappa/mk^2=p,\quad \kappa/mk^2+H^2A^2/Rmk^2=q$
for brevity, we have

$$\ddot{\theta}+q\dot{\theta}+\frac{4\pi^2}{T^2}\theta=0$$

This equation is valid while the key is kept closed, so that the induced current continues to flow through the coil of the instrument. During the period in which the key is kept open, q is replaced by p, the inductive agencies not being then in operation.

While the key is down, if we write

$$D=-\frac{q}{2}\pm\tfrac{1}{2}\sqrt{q^2-\frac{16\pi^2}{T^2}}=-\frac{q}{2}\pm i\,\rho \text{ (say)}$$

$$\theta=A\varepsilon^{-qt/2}\sin\,\rho t$$

making θ vanish with t. Thus

$$(\dot{\theta})_{t=0}=A\,\rho.$$

For $(\dot{\theta})_{t=0}$, the initial angular velocity of the needle, we may use the usual relation connecting it with the quantity Q of electricity to be measured, and if G is the usual constant of the ballistic galvanometer,

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* Since the reading of this Paper my attention has been called to a communication by B. Wwedensky in the current number of the Annalen der Physik (Nov., 1921), who, in addition to using the method, refers to a Paper by M. Gildemeister (Ann. d. Physik, Vol. XXIII., p. 401, 1907), in which it is also described. I wish to take this opportunity of acknowledging these publications. I should like to add that the authors have not made a special study of the instrument itself as in the present communication.

$$(\dot{\theta})_{t=0}=\frac{MGQ}{mk^2}=A\,\rho$$

Thus

$$\theta=\frac{MGQ}{mk^2\,\rho}\,\varepsilon^{-qt/2}\sin\rho t \quad \dots\dots\dots\dots \quad (1)$$

during the initial period when the key is kept down. Suppose that the key is kept down for an interval t_1—afterwards made small in comparison with the period T of the instrument. Then at the end of this interval t_1, the values of θ and $\dot{\theta}$, say θ_1, $\dot{\theta}_1$, at $t=t_1$ are derived from (1), and are the initial values for the second interval with the key open, and q equal to p. In this second interval, the general solution of the differential equation is

$$\theta=\varepsilon^{-pt/2}\left\{A\cos\frac{2\pi t}{T}+B\sin\frac{2\pi t}{T}\right\}$$

Moreover, p is very small, being the *natural* decrement of the instrument, not involving the inductive agencies.

Introducing now the consideration that t_1 is very small in comparison with T, the natural period of the instrument, we may write with a sufficient approximation

$$(\theta)_{t=t_1}=A+\frac{2\pi t_1}{T}B$$

$$(\dot{\theta})_{t=t_1}=\frac{2\pi B}{T}\,\varepsilon^{-pt_1/2}$$

$$\sin\rho t_1=\rho t_1$$

Thus

$$A+\frac{2\pi t_1}{T}B=\frac{MGQ}{mk^2}\,t_1\varepsilon^{-qt_1/2}$$

$$\frac{2\pi B}{T}=\frac{MGQ}{mk^2}\,\varepsilon^{-qt_1/2}$$

using the values of $(\theta,\dot{\theta})_{t=t_1}$ derived from (1) directly, A being small in comparison with B.

Accordingly, at time t on the second interval—the zero of time being the beginning of the first interval—we have to a sufficient order of accuracy

$$\theta=\frac{T}{2\pi}\cdot\frac{MGQ}{mk^2}\cos\frac{2\pi t}{T}\,\varepsilon^{-pt/2-qt_1/2}$$

showing the natural decrement. The effect of the first interval is, to this order, effectively confined to the exponential factor $\varepsilon^{-qt_1/2}$. The maximum of θ occurs when $t=\frac{1}{4}T$ as usual, and is given by

$$(\theta)_{\text{max.}}=\frac{T}{2\pi}\cdot\frac{MGQ}{mk^2}\,\varepsilon^{-pT/8-qt_1/2} \quad \dots\dots\dots \quad (2)$$

If the key had been depressed throughout, we should have obtained from (1)

$$\theta=\frac{MGQ}{mk^2}\left\{\frac{4\pi^2}{T^2}-\frac{q^2}{4}\right\}^{-\frac{1}{2}}\varepsilon^{-qT/8}\,. \quad \dots\dots\dots \quad (3)$$

The wide difference in the values of p and q in (2) and (3) indicates the much greater degree of sensitiveness of the galvanometer in the former case. In other respects the formula (2) indicates that, as a measure of Q the method is equally satisfactory, and if t_1 is very small and is maintained constant, together with the resistance R, while measuring different values of Q, such values are directly proportional to the first deflections. The formula is also of great simplicity in its application, as compared with (3). Effectively, the natural decrement operates during the greater part of the first swing.

The effect of a varying resistance in the circuit of the coil is of some interest. For fairly small resistance, we may write

$$q=\frac{H^2A^2}{Rmk^2}=a/R \text{ (say)}$$

or Q varies inversely as the resistance, a being a constant. Then

$$Q \propto \frac{\text{(deflection)}}{R}\,\varepsilon^{at_1/R}$$

In measuring the same value of Q—or the same magnetic induction in an actual experiment—with varying resistances by this method, we should find that the law connecting the deflection with the interval t_1 of closing the circuit, and the resistance R of coil and circuit, is of the form

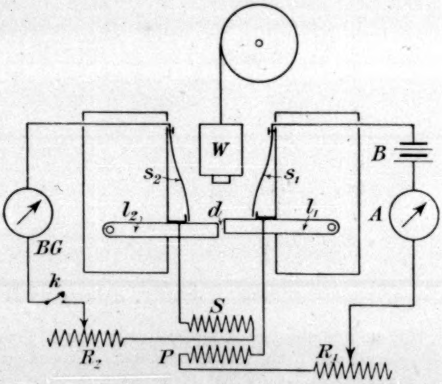

FIG. 1.—DIAGRAM OF EXPERIMENTAL ARRANGEMENT.

$$\text{Deflection}=\frac{\beta}{R}\,\varepsilon^{at_1/R}$$

where a and β are instrumental constants. The verification of this formula supplies a good test of the performance of an instrument, and of the accuracy with which experimental conditions involving a given value of the measured magnetic induction, can be reproduced. It is evident that considerable care is required, and that a determination of α and β is necessary, before deflections obtained with low values of R can be interpreted.

EXPERIMENTAL VERIFICATION OF THE THEORY.

In Fig. 1 P and S represent the primary and secondary windings of an air-core transformer whose mutual inductance was 0·067 henry. The current in the primary circuit is supplied by a battery, B, and measured by an ampere-meter A, and its magnitude is varied by an adjustable resistance R_1. Two levers, l_1 l_2, are held horizontally by contact pressure due to springs S_1 S_2. The primary circuit is interrupted by a falling weight W on striking the lever l_1, and after an interval of time (t_1) the secondary circuit is interrupted on the weight striking the lever l_2. The time-interval t_1 is determined by the difference in level d between the two levers l_1 l_2, and the velocity of the falling weight W. To check the accuracy of the small values of t_1, determined from measurements of d and the velocity of W, the proportion of charge lost by a condenser through a high resistance during the time t_1 was determined by a quadrant electrometer, and hence the value of t_1 was found. The weight was supported by a cord wound on a pulley, which was geared to a train of wheels terminating in a rapidly rotating vane. Thus the velocity was kept constant by air friction over a considerable distance, and its value was accurately determined. The difference of level d was capable of adjustment by raising or lowering the framework supporting the lever l_2.

The ballistic galvanometer *BG* was of the D'Arsonval type, and had a resistance of 467 ohms. The resistance of the secondary coil S was 48 ohms. A variable resistance R_2 and key k were included in the secondary circuit. The scale used with the galvanometer had $\frac{1}{2}$ mm. divisions, and was at a distance 1,340 mm. from the mirror. The constants of the instrument were: Periodic time, 8 seconds; amperes per scale-division steady deflection, $1{\cdot}5 \times 10^{-9}$; coulombs per scale-division, $1{\cdot}91 \times 10^{-9}$; decrement factor on open circuit, 1·03.

(*a*) TIME OF CONTACT t_1 CONSTANT.

Experiments were first made to verify the prediction of theory that for a given value of the time t_1 and for a given total resistance the first deflection θ is proportional to the quantity of electricity Q. Two sets of curves are given in Figs. 2 and 3, in which the primary current in arbitrary units is plotted horizontally and the deflections vertically; they indicate an exact agreement with theory. In the first set (Fig. 2) the value of t_1 is 0·094 second, and it is 0·397 in the case of Fig. 3.

(*b*) TIME OF CONTACT t_1 VARIED.

The second set of experiments consisted in keeping the primary current constant, and varying the resistance in the secondary circuit (R_2) for given intervals of time t_1. The deflections thus obtained are given in Table I., and Fig. 4 is a graph in which for a given total resistance the time t_1 is plotted horizontally and the first deflections vertically. When the total resistance is 515 ohms the deflections increase rapidly as the time t_1 diminishes, and after rising to a maximum they would fall to zero when $t_1=0$. If the time t_1 had the value 0·1 second in actual working, the sensibility on the first deflection would be increased 1·8 fold as compared with the usual method of keeping the secondary circuit permanently closed; and it would be increased 3·3 fold if, as is sometimes the case, the sum of the first and second deflections is employed.

TABLE I.

Time of Contact t_1 in seconds	$R=515$ $Q=1{\cdot}52\times10^{-6}$.		$R=1520$ $Q=0{\cdot}515\times10^{-6}$.		$R=5520$ $Q=0{\cdot}142\times10^{-6}$.		$R=10520$ $Q=0{\cdot}0745\times10^{-6}$.	
	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
∞	390	17	204	82	69	47	38	28
2·77	392	315	206	183	72	62	39	34
0·787	436	403	222	207	73	67	40	36
0·412	546	480	242	210	75	66	39	35
0·187	655	550	259	218	76	63	39	30
0·094	720	612	266	232	76	64	39	35

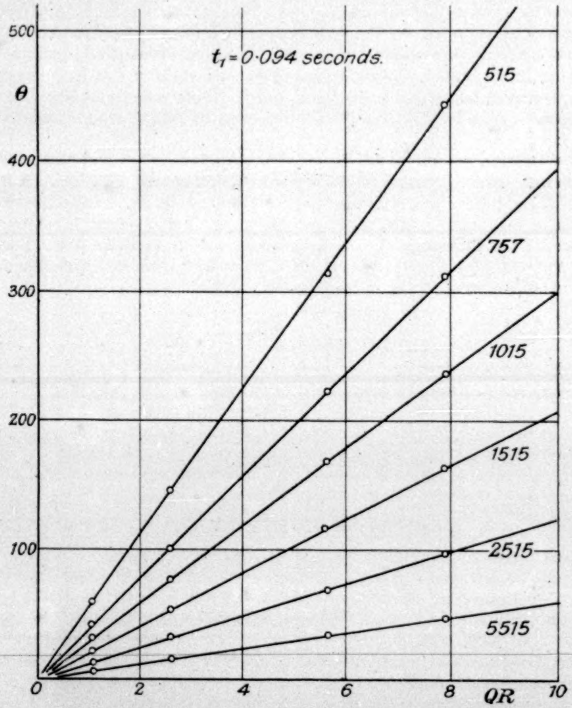

FIG. 2.—OBSERVED GALVANOMETER DEFLECTIONS—t_1 CONSTANT.

When swinging with its coil on open-circuit the decrement factor is 1·03, and this is to a near approximation true when the total resistance has the value 10,520 ohms. The calibration of the instrument with this factor gives $1{\cdot}91\times10^{-9}$ coulombs per scale division. Confining ourselves to the first deflection, now called θ_1 to distinguish it from the second deflection θ_2, the quantity discharged is $39\times1{\cdot}91\times10^{-9}=74{\cdot}5\times10^{-9}$ coulombs when R has the value 10,520 ohms.

If the primary current is kept constant then the electro-motive-force induced in the secondary coil is a constant, and the quantity discharged through the instrument on interrupting the primary circuit will be $10520\times74{\cdot}5\times10^{-9}/R$ coulombs for various resistances R. The theory states that the deflection θ_1 is equal to $\beta/R\varepsilon^{\alpha t_1/R}$, where α and β are instrumental constants. From the values of t_1 θ_1 and R given in Table I., the values of α and β have been obtained, and these are given in Table II. It will be seen that for values of R 515 and 1520, α varies from -403 to -730, and

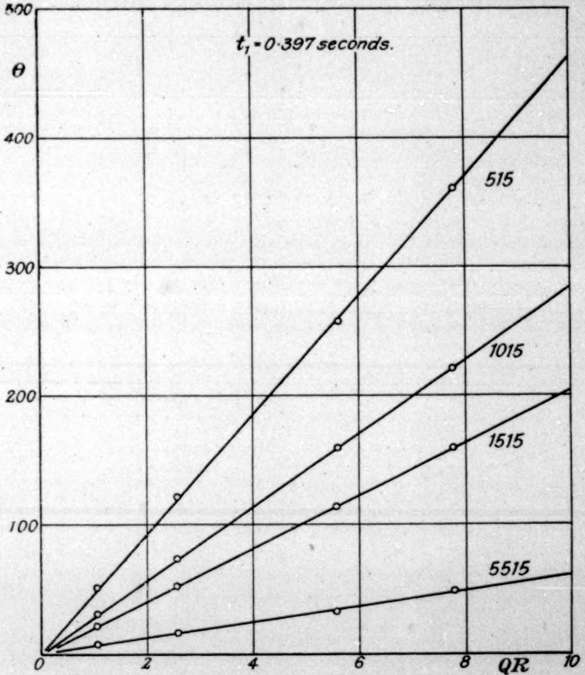

FIG. 3.—OBSERVED GALVANOMETER DEFLECTIONS—t_1 CONSTANT.

for values of R 515 and 5520 the variation is from -422 to -755. This shows primarily the effect of having t_1 important in comparison with the periodic time of the instrument.

TABLE II.

Time of Contact t_1 in seconds.	R 515 and 1520.		R 515 and 5520.	
	α	β	α	β
0·094	—730	415×10^3	—755	424×10^3
0·187	—635	422×10^3	—658	414×10^3
0·412	—510	425×10^3	—535	431×10^3
0·787	—403	422×10^3	—422	427×10^3
		1684		1696
		Mean 422×10^3		Mean 424×10^3

If the galvanometer coil were free to move without retarding forces, one would expect the deflection θ_1 to vary strictly as the quantity Q. Referring again to Table I., and increasing θ_1 observed with 10,520 ohms total resistance, in the ratio of the quantities, the values of θ_1 without serious decrement have been worked out. The differences $\delta\theta_1$ between the values calculated without decrement and the observed values are given in Table III., together with the percentage diminution. When the time of contact t_1 is 0·094 second, the deviation is less than 10 per cent. with a total resistance of 515 ohms, and it falls to zero as the resistance is increased. As the time of contact is increased the percentage variation increases until it is equal to 50 with 515 ohms total resistance.

TABLE III.

Time of Contact t_1 in seconds.	$R=515$		$R=1520$		$R=5520$	
	$\delta\theta_1$	Per cent.	$\delta\theta_1$	Per cent.	$\delta\theta_1$	Per cent
∞	405	51	66	24·4	7	9·2
2·77	403	50·5	64	23·7	4	5·26
0·787	359	45	48	17·8	3	3·95
0·412	249	31·3	28	10·3	1	1·32
0·187	140	17·6	11	4·07	0	0
0·094	75	9·45	4	1·5	0	0

When working with this method it is necessary to be quite certain that the time of contact t_1 is sufficiently long to allow of the complete change in the magnetism when measurements of permeability are in question. This can be checked by taking deflections with a high constant resistance in circuit for various times of contact. The deflections should be equal until t_1 reaches the minimum value below which it should not be allowed to go. Alternatively with low total resistance the time t_1 should be adjusted to give a maximum deflection.

APPLICATION OF THE METHOD TO THE TESTING OF IRON.

When the ballistic galvanometer is used in the testing of iron a switch is employed to reverse the primary current. To break the secondary circuit at known intervals of time after reversal, use has been made of a freely falling weight, which is released by the reverser at the instant when reversal is complete. On falling through a given distance the weight strikes a lever and severs the secondary circuit. The arrangement consists of a vertical tube, which acts as a guide for a small cylinder of brass. Initially the cylinder is supported in its position at the top of the tube by a wire which is threaded through a small hole in the cylinder, and which is attached to the reverser. The wire is so adjusted in length that at the instant the primary current is reversed the cylinder is released and falls as above stated. The time interval t_1 is varied by varying the distance of fall.

APPLICATION.

It only remains to describe experiments which were made with a cylinder built up of transformer iron stampings, the object being to discover if the permeability reaches a constant value when very small changes of the magnetic induction are in question. Previous experiments with these stampings indicated that between the limits of the magnetic force H 0·0356 and 0·000073 the permeability varied from 172 to 148.*

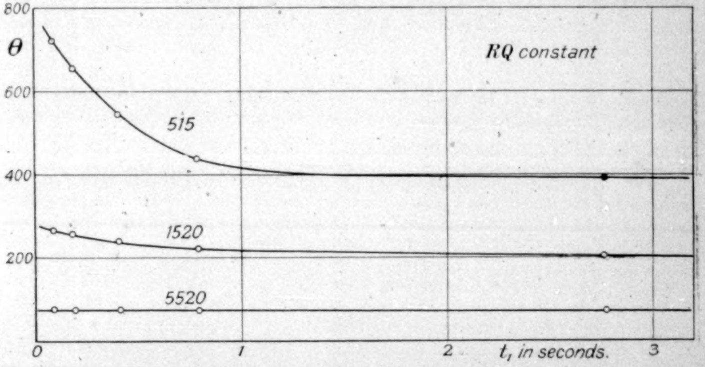

FIG. 4.—OBSERVED GALVANOMETER DEFLECTIONS—RQ CONSTANT.

The cylinder had a volume of 23,440 cubic cm., and the average thickness of the stampings was 0·0613 cm. Each stamping had internal and external diameters 30·5 and 40·6 cm. respectively, and the effective cross-section of the cylinder was 210 sq. cm. For use with the instrument used in the foregoing experiments it was decided to wind on three layers of insulated copper wire for the secondary coil, and calculation showed that the ratio of turns on the three layers to the resistance of the coil and galvanometer had a maximum value when the wire, which was insulated with a double layer of cotton, had a diameter of 0·061 cm.

At the time of making the present experiments only two of the layers had been wound, and the total turns were 1,838, and the resistance 102 ohms ; the results are not intended to be more than preliminary or tentative. The primary winding was wound over the secondary coil, and consisted of copper foil 7·5 cm. wide, and the total turns on the single layer were 16. The whole surface of the cylinder was covered by the primary coil, and to prevent overlapping the width of the copper foil had to be reduced on the inside surface of the cylinder.

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* Proc. Roy. Soc., A. Vol. XCIII., 1917, p. 409.

The deflections were observed when the key in the secondary circuit was continuously closed, and when it was opened 0·2 second after reversal had been completed. In order to eliminate the zero reading of the instrument the sum of the first and second deflections (θ_1 and θ_2) have been taken, and these are set out in Table IV. From the data supplied in Table I. it has been estimated that when the total resistance is 569 ohms, and when $t_1=0{\cdot}2$ second, the value of $\theta_1+\theta_2$ must be multiplied by the factor 1·20. Assuming this factor in conjunction with the ordinary constants the values of the magnetic induction B have been found, and hence the permeability μ. It will be seen that the values of $\theta_1+\theta_2$ when $t_1=\infty$ are very small, whereas when $t_1=0{\cdot}2$ second the values are roughly three-fold, and can be read to a greater degree of accuracy. The experiments are being continued with the completed secondary circuit, but so far as the present experiments go it would appear that when the magnetic force varies from 17×10^{-6} to $6{\cdot}84\times10^{-6}$ C.G.S. units the permeability is still variable.

TABLE IV.

H 10^{-6}.	$\theta_1+\theta_2$		B 10^{-6}	μ
	$t_1=\infty$	$t_1=0{\cdot}2$		
17·1	9·5	29·5	2480	145
11·4	6·5	19·0	1600	140
8·55	5·5	14·0	1180	138
6·84	3·5	11·1	935	137

DISCUSSION.

DR. MARSH: I have been greatly interested in Prof. Wilson's Paper on a method of increasing the sensitivity of a moving coil ballistic galvanometer, and the more so as I have been employing a very similar method for several years. A special form of key was used, by depressing which the primary current was closed, while further pressure released a spring which opened the galvanometer circuit. The time interval between the closing of the one and the opening of the other could be varied at will. When this interval is too short, the inductive discharge through the secondary is not complete, and the galvanometer throw is too small; if the interval is too long, the galvanometer coil moves sensibly from its initial position, and the electromagnetic damping reduces the throw. The *maximum* throw is taken (quickly obtained after a little practice), the throw being corrected subsequently for air damping in the ordinary way. In the "reversals" method the ordinary form of commutator was used, and the key described was employed solely to break the secondary circuit. It would appear that if a *definite* time interval is to be employed, as in Prof. Wilson's method, then it is of the highest importance that such an interval be measured with considerable accuracy. With regard to the graphs plotted in Figs. 2 and 3, they do not appear to me to be a verification of Prof. Wilson's theory of the damping effect; for reference to equation (2) shows that for a given t_1, the exponential factor is constant, and consequently the factor embodying his results disappears when ratios are taken. A better test of the theory would be to keep Q constant and determine the value of $\theta_{\text{max.}}$ for different values of t_1. Equation (2) would then suggest the relation $\theta_{\text{max.}} \propto \varepsilon^{-qt_1/2}$, the other factors being constant. On plotting the values of log θ_1 against t_1 (first and second columns of Table I.), the graph is very different from a straight line.

DR. RUSSELL: I have been much interested in the author's ingenious demonstration of the principle described in his Paper, and particularly by the results he has obtained in connection with the permeability of iron. It is too often assumed that the result given by Lord Rayleigh long ago, namely, that the permeability is constant for very small values of H, is correct beyond question. The author's investigation seems to throw doubt on this assumption, and the question is one which ought to be settled.

Mr. T. **Smith**: I should like to ask the author if he can give us any information as to the physical significance and the importance of the shape of the permeability curve in the neighbourhood of the origin, to which reference has been made in the Paper and by the last speaker.

Mr. C. C. **Paterson** remarked that the question touched upon by the last two speakers was of great importance in telephone and high-frequency problems.

Dr. D. **Owen** enquired whether the author had tried the measurement of small changes of magnetic flux, using a galvanometer of the moving-needle type, such as the Broca. Doubtless magnetic shielding would be involved, and his experiences in that direction would be of interest.

Prof. **Wilson**, in reply, said: I am much interested to learn from Dr. Marsh that he has been employing the method of obtaining increased sensitivity by breaking the current of a ballistic galvanometer attached to the secondary coil of an iron-cored transformer, and in reply to his remarks, I should like to state that the experiments described in the Paper were largely carried out with a view to their application in the testing of iron in fairly large masses. In the experiments at the end of the Paper, I was concerned with the total change of magnetism in a very large specimen, and not the instantaneous effect. I found that a time of about 0·2 of a second was necessary in order to obtain a maximum deflection on the instrument, and this was the time actually used. The figures given in Table II. were intended to show that, provided I kept this time of contact constant, I should be safe in assuming the observed deflections to be strictly proportional to the change of magnetism. I have not obtained the true value of α directly from the instrument itself, but this could be done if necessary. The variable value of this constant I attribute to the higher approximations which were made in the analysis leading up to the formula given in the abstract. It is explicitly stated that the time of contact must be very small in comparison with the periodic time of the instrument, if the formula is to be capable of general application; and this is clearly not the case in my experiment. When the value of the time of closure t_1 is allowed to vary through wide limits it would be necessary to go back to the fundamental formulæ involving no such approximations.

The remarks of Dr. Russell are exceedingly interesting to me, as I have felt for many years that the results of Lord Rayleigh's experiments merit further investigation. The constancy or not of magnetic permeability in iron at very low forces is of great importance at the present time, and it is worthy of note that ballistic galvanometer tests do not agree with those of Lord Rayleigh, who used the magnetometer. If I might make the suggestion, I feel that Lord Rayleigh's experiments could be repeated with advantage in order to discover if there was anything in his instrumentation which might possibly lead to other results.

Mr. T. Smith's remark with regard to the shape of the hysteresis loops when very low values of magnetic induction are in question, is an interesting one. It is well known now that these loops do not in any way resemble the ordinary hysteresis loops obtained in magnetic testing, as they ultimately resolve themselves into two parallel straight lines enclosing a very small and diminishing area.

Mr. Paterson quite rightly emphasises the importance of the subject in telephonic and high-frequency problems.

In reply to Dr. Owen, I may say that I have not worked with a Broca galvanometer, but I contemplate doing so.

Since the reading of my Paper, Professor Howe has called my attention to the current number of the "Annalen der Physik" in which a Paper by M. Wwedensky deals with the problem of opening the ballistic galvanometer circuit with a view to increasing its sensitivity. He refers to a Paper by M. Gildemeister (Annalen der Physik, 1907, XXIII., p. 405), who used this self-same method, and deserves all credit for his early publication. Gildemeister was concerned with the instantaneous change of magnetism which occurred in very fine iron wires at an early stage (under a thousandth of a second) after the breaking of a magnetising current. He used a Helmholtz pendulum, making the secondary circuit after a very small interval of time, and breaking it after a further very small interval of time, for the purpose of obtaining increased sensitivity in the galvanometer.

IX. *On the Determination of the Damping Decrement of a Tuning Fork.* By R. Ll. Jones, *M.A.*

Received October 10, 1921.

1. In Proc. Phys. Soc., Vol. XXXI., p. 87, 1919, A. Campbell describes a method of determining the damping decrement of a tuning fork. The vibrating prong of the fork causes the induction through a coil fixed near it to vary. An E.M.F. is, thereby, induced in the coil. The coil is connected to a sensitive vibration galvanometer tuned to the frequency of the fork. As the fork vibration dies down the galvanometer deflection falls off similarly. It is assumed that the amplitudes of the galvanometer vibrations are closely proportional to those of the fork, and hence that their rates of decay are the same. If θ_0 and θ_1 be the amplitudes of the galvanometer vibrations at times $t=o$ and $t=t_1$ respectively, the decrement c is given by

$$c=\frac{1}{t_1}\log_e\left(\frac{\theta_0}{\theta_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is suggested* that the relation between the vibrations of the galvanometer and the fork requires investigation, and the theory of the method does not appear to have been considered in detail. In this Paper the motion of the vibration galvanometer, when the E.M.F. in its circuit is of the form $E_1e^{-ct}\cos pt$, will be first given, and it will be seen that in this simplest case the decrement of the galvanometer vibration is equal to the decrement of the E.M.F. to which the galvanometer is tuned. The E.M.F. induced in the circuit by the fork will then be evaluated, and its harmonic components and their decrements indicated. The conditions necessary in order to obtain the true decrement of the fork by the method will follow.

2. Let E be the impressed E.M.F., i_1 the current in the circuit, and θ the deflection of the galvanometer at a time t. The equation of motion of the galvanometer coil is

$$\alpha\ddot{\theta}+\beta\dot{\theta}+\gamma\theta=HA\,.\,i_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where H is the strength of the field in which the galvanometer coil moves and A is the effective area of the coil.

If r be the total resistance of the circuit, the current is given by

$$i_1=\frac{E-HA\,.\,\dot{\theta}}{r}$$

Substituting in (2) we get

$$\alpha\ddot{\theta}+\left(\beta+\frac{H^2A^2}{r}\right)\dot{\theta}+\gamma\theta=\frac{HA}{r}\,.\,E$$

or

$$\ddot{\theta}+2\lambda\dot{\theta}+n^2\theta=g\,.\,E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Writing E_1e^{iqt} for E where E_1 is constant and $i=\sqrt{-1}$, the forced vibration is given by

$$\theta=gE_1\frac{e^{iqt}}{n^2-q^2+2i\,.\,\lambda q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Let $q=p+ic$ where p and c are real.

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* Campbell, loc. cit.

Substituting this for q in (4), the real part will give the forced vibration due to an impressed E.M.F., $E_1e^{-ct}\cos pt$.

Thus

$$\theta=\frac{gE_1}{R}e^{-ct}\cos(pt-\chi)$$

where

$$R^2=(n^2-p^2-2\lambda c+c^2)^2+4p^2(\lambda-c)^2 \text{ and } \tan\chi=\frac{2p(\lambda-c)}{n^2-p^2-2\lambda c+c^2}.$$

The complete solution of (3) is

$$\theta=\frac{gE_1}{R}e^{-ct}\cos(pt-\chi)+Ae^{-\lambda t}\cos(\sqrt{n^2-\lambda^2}\,.\,t+B). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

3. The second term on the right-hand side of (5) is the free vibration. In order to be able to deduce c from observations of the amplitude of θ by (1) the free vibration must be got rid of. The only practical way to do this is to give λ (which is at our disposal to a certain extent) such a value that this vibration dies away much more rapidly than the forced vibration—*i.e.*, λ must be greater than c. The actual value of λ with a given galvanometer in circuit can be determined by appropriate observations,* and the time required for the reduction of the free vibration to (say) 1 per cent. of its amplitude at any time can be calculated. If this interval be allowed to elapse after the galvanometer deflections, whatever be their initial magnitudes, are reduced to the compass of the scale, before observations for the deductions of c by (1) are begun, we may be sure that sufficient time has been given for the free vibrations to die away to insignificance, and that the amplitudes we are observing are due to the forced vibrations alone. If, for instance, $\lambda=1{\cdot}53$, then the interval is three seconds.

Assuming, then, that the free vibration has disappeared, the motion of the galvanometer coil will be given by the first term of (5). Tuning is effected by altering n. R is a minimum, and the tuning is exact when $n^2=p^2+2\lambda c-c^2$, or when $n=p$ practically, as c is very small compared with p in any tuning fork.

In this case $R=2p(\lambda-c)$.

Thus, when the galvanometer has been tuned to the frequency $p/2\pi$

$$\theta=\frac{gE_1}{2p(\lambda-c)}e^{-ct}\sin pt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

and the factors which determined the decay of θ and E are in this simple case the same—viz., e^{-ct}.

If, however, the value of E be

$$[E_1e^{-ct}+E_3e^{-3ct}+E_5e^{-5ct}+\quad . \quad . \quad . \quad]\cos pt$$

where E_1, E_3, E_5, &c., are constants (*see* §6), then θ will be given by

$$\theta=\frac{g}{2p}\left[\frac{E_1}{\lambda-c}e^{-ct}+\frac{E_3}{\lambda-3c}e^{-3ct}+\frac{E_5}{\lambda-5c}e^{-5ct}+\quad . \quad . \quad . \quad . \quad\right]\sin pt$$

and the rates of decay of θ and E are not the same.

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* A. Campbell, Proc. Phys. Soc., Vol. XXVI., p. 121 (1915).

4. The E.M.F. induced in the circuit by the vibrations of the fork can be evaluated as follows: Consider a magnetic pole of strength m near a circuit C in which a current i_1 is flowing. Let N be the number of lines of force due to m which pass through C, and let V be the potential energy of the system. Then

$$V=-i_1N \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Let F_y be the force on m due to unit current in C aiding any displacement δy of m. The work done during this displacement will be $i_1F_y\,.\,\delta y$. By this displacement N becomes $N+\delta N$, and the decrease in the potential energy of the system is by (7) $i_1\delta N$.

Hence
$$i_1F_y\delta y=i_1\delta N$$
and
$$F_y=\frac{\partial N}{\partial y}$$

Let E be the E.M.F. induced in the circuit C by the motion of m. Then

$$E=-\frac{\partial N}{\partial t}=-\frac{\partial N}{\partial y}\,.\,\dot{y}=-F_y\,.\,\dot{y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This expression for the E.M.F. may also be deduced by considering the power required to maintain the motion of m.* Let m be the polarised prong of a tuning fork, and y its displacement from the position of rest at a time t, and let $y=be^{-ct}\sin pt$. Its velocity is $\dot{y}=bpe^{-ct}\cos pt$.

Let F_y be the force on m due to unit current in the coil, when in the displaced position y, and in the direction of y. Then the E.M.F. induced in the circuit by the prong is

$$E=F_y\,.\,bpe^{-ct}\cos pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the value of F_y be the same at all positions of m in the course of a vibration, F_ybp is a constant, and the rate of decay of E in (9) is determined by the factor e^{-ct} alone, and is the same as that of the fork. Hence the value of c determined by (1) is the damping decrement of the fork.

If the initial amplitude of the fork be such that F_y varies sensibly over the range through which m moves, the rate of decay of E may depend, in the initial stages at any rate, on other factors besides e^{-ct}. This case then requires further consideration.

5. Suppose that F_y varies uniformly through the range of vibration of m. Let its value in one extreme position of m be F_0+f, and in the other F_0-f, b being the amplitude of the vibration, and F_0 the value of F_y in the position of rest. Then the value of F in the position defined by the displacement y is

$$F_y=F_0+\frac{f}{b}y=F_0+fe^{-ct}\sin pt$$

By (8)
$$E=(F_0+fe^{-ct}\sin pt)bpe^{-ct}\cos pt$$
$$=F_0bpe^{-ct}\cos pt+\tfrac{1}{2}fbpe^{-2ct}\sin 2pt \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The galvanometer is tuned to the frequency $p/2\pi$; its motion will therefore be wholly due to the first term on the right hand side of (10), the effect of the second term of frequency corresponding to $2p$ being insignificant. The rate of decay of the effective term in E is determined by the factor e^{-ct} alone, and is the same as that of the fork. Hence in this case also the value of c determined by (1) is that of the fork.

---

* Butterworth, Proc. Phys. Soc., Vol. XXVII., p. 411 (1915).

6. Generally we have

$$F_y = F_0 + y \,.\, F'_0 + \frac{y^2}{2} \,.\, F_0'' + \ldots\ldots \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

where $F_0' = \left(\frac{dF}{dy}\right)_{y=0}$, &c., and y is the displacement, and

$$E = F_y \,.\, \dot{y} = \left(F_0 + y \,.\, F_0' + \frac{y^2}{2} \,.\, F_0'' + \ldots\right) . \dot{y}.$$

The value of the first two terms has just been considered in §5. The contribution of the third term $\left(\frac{y^2}{2} F_0'' \,.\, \dot{y}\right)$ to the value of E is

$$\tfrac{1}{2} b^2 e^{-2ct} \sin^2 pt \,.\, F_0'' \,.\, bpe^{-ct} \cos pt$$

$$= E_3 e^{-3ct} (\cos pt - \cos 3pt),$$

where $\qquad E_3 = \tfrac{1}{8} b^3 p \,.\, F_0''.$

The only part of this which is effective in influencing the value of θ is $E_3 e^{-3ct} \cos pt$, because its frequency is that to which the galvanometer is tuned. Its initial magnitude depends on b^3 and F_0'', and is generally not large. Its decrement is three times that of the principal term.

Similarly it can be shown that the 4th term of (11) makes no effective contribution to E, while the contribution of the 5th term has e^{-5ct} as a factor. Hence in the general case where E_3, E_5, &c., are appreciable, the effective value of E is given by

$$E = [E_1 e^{-ct} + E_3 e^{-3ct} + E_5 e^{-5ct} + \ldots] \cos pt \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

The rate of decay of this is *not* that of the fork, and as shown in §3 the rate of decay of θ in this case is different from that of E, and the fork.

The following example gives an idea of the relative values of E_1 and E_3 in (12) in a case of practical importance. The ratio $\frac{E_3}{E_1} = \frac{1}{8} b^2 \frac{F_0''}{F_0}$ and $\frac{F_0''}{F_0} = -3 \frac{r^2 - 4y^2}{(r^2 + y^2)^2}$ for a coil of radius r with m at a distance y on the axis from the centre of the coil. The radius r being given, $\frac{F_0''}{F_0}$ is a maximum when $y = 0$ and its value is $-\frac{3}{r^2}$. Taking $r = 1$ cm. and $b = 0{\cdot}1$ cm., $E_3/E_1 = -0{\cdot}004$ approx. in this case. If y be not zero E_3/E_1 is less than this; it is zero of course for $y = \frac{r}{2}$, and very nearly zero for a considerable range round this value of y. In practice the coil will have an appreciable depth and width of winding, and m will be a distributed and not a point pole as assumed above. The effect of all these factors is to diminish the ratio E_3/E_1, which is thus seen to be generally small.

7. These considerations show that the decrement determined by (1) *is* the decrement of the fork, provided the field defined by F_y and over which m moves is not very irregular. If the field varies uniformly the effect on the tuned galvanometer is the same as if the field were uniform, and its strength the mean strength of the uniformly varying field, and the decrement of the fork is correctly determined by the method in these cases. It is only when the rate of change or the slope of the field changes rapidly within the range of movement of m that anything appreciable of frequency $p/2\pi$ is contributed to E whose rate of decay is different from that of the fork. In this case the method may be said to fail. When the coil is that of an electro-magnet employed to maintain the fork in vibration, contributions such as E_3, E_5, &c., are usually very small compared with E_1. In any case their decrements being 3, 5, &c., times the decrement of the principal component their values become evanescent, while the deflection of the galvanometer is still finite. Thus by working with small amplitudes of the fork or in a field whose slope does not change or does not change rapidly, conditions are readily secured for the correct application of the method.

THE PHYSICAL SOCIETY OF LONDON.

A DISCUSSION

ON

HYGROMETRY.

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AT THE

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THE MEASUREMENT OF ATMOSPHERIC HUMIDITY.

BY

SIR NAPIER SHAW, *F.R.S.*

So far as meteorological practice is concerned, hygrometry, the measurement of water-vapour in the atmosphere, has to be approached from the point of view of the daily observer, and therefore any practical process must be capable of repetition daily, once, twice or thrice, year in and year out, without fail. The whole subject of hygrometry bristles with points of interest for the student of physics who has a laboratory at his disposal. Many of the questions involved may be, or become, of practical importance in the measurement of vapour ; but the practical determination of humidity must be of such a nature that for regular observations it can be entrusted to an attendant who has no experience of work in a laboratory, who can follow a prescribed routine. He has to make his observations alone, his records reported to and used by people who may never have seen him or the conditions under which he is working. If a particular reading seems unreasonable, it may be rejected, but it cannot be corrected. Any symptoms of unreasonableness that appear in a schedule of observations of humidity render the whole series unusable on account of general discredit.

Hence the mode of procedure must be to regard the observer as a tool-user, not as a tool-minder, still less as a tool-maker.

For that reason, meteorological practice in the measurement of humidity is limited to the wet and dry bulb thermometers, which are used almost everywhere, and the hair hygrometer, which is used for the term hour observations in Norway and for self-recording hygrometers everywhere.

There is little difficulty about the use of the wet and dry bulb so long as the temperature of both remains above the freezing point. The formula for reduction depends to some extent on wind velocity, and in dry climates, like that of Egypt, a small error in formula may lead to ridiculous results in practice ; if anything vital turned on the results, there is little doubt that a system would be evolved without difficulty. But as soon as the freezing point is passed the wet bulb ceases to be an effective instrument. The water has no definite freezing point, but freezes at all sorts of temperatures, and when the water is frozen the capillary action, upon which the moistening depends, ceases altogether. The directions for the treatment of the wet bulb during frost are palliatives designed to carry on in an occasional emergency. Nobody in London could possibly satisfy himself from the evidence at his disposal whether a reading of humidity during frost entered in a schedule from, say, Yarmouth, or on board a ship, represented frozen bulb, super-cooled wet bulb, semi-dry bulb or dry bulb. In these circumstances, equality of reading of wet bulb and dry bulb, the most probable occurrence in the circumstances, has no effective meaning.

In explanation of the vagaries of freezing wet bulbs, I refer to some experiments made for me by Sir James Dewar, and noted in the " Computer's Handbook," and for some general remarks on the uncertainty of freezing at the freezing point to some remarks by Dr. J. Aitken in a Paper on " Frazil-ice " in the Journal of the Scottish

Meteorological Society, Vol. XVIII. Moreover, below the freezing point the amount of water-vapour in the atmosphere and its variations become very small. There is very little to go upon when dealing with absolute measures. It is, however, remarkable that the shrinkage and slacking of cordage and other results upon animal and vegetable fibres are apparently as effective as at any other temperature.

The conclusion at which I have arrived is that the wet bulb is useless for temperatures just below freezing, and should be discarded. Descriptions of the endeavour to find a suitable formula abound in meteorological literature ; but practically the subject has not advanced at all in my time.

So far as I know, the hair hygrometer is the only instrument which is available in those circumstances to give a reading which can be interpreted. Since the publication of Regnault's classical memoir, physicists have been very shy of it, because it offends the first principles of a physical laboratory ; it does not always repeat the same reading in the same physical conditions. It suffers from age and misuse and other infirmities. But in view of the general failure of more orthodox methods, the prejudice of the Physical Laboratory against the instrument is a little overdone and unreasonable. I suppose that there is a limit beyond which even the most accurate physical experiment does not repeat itself—that is to say, unrecognised causes supervene. And in an atmosphere where rejoicing in the triumphant accuracy of one part in a hundred thousand is a common experience, it is annoying to be brought up by a matter that cannot keep steady within 5 per cent. Yet hair is apparently a more definite physical quantity than wet muslin.

Certain it is that the story which the recording hair-hygrometer tells of the variation of the humidity of the atmosphere at any temperature is an extremely romantic and interesting one ; and, even when its tabulated figures are erroneous, its sequences are valuable evidence of the course of events.

I have little personal experience of its working. I do not much like covering it with a damp cloth or wetting the hairs and calling the reading 95 per cent. ; and yet, considering that the atmosphere is always in turbulent motion, and a process of mixing is always going on, it is really doubtful whether the actual humidity of a small portion of the atmosphere is a physical quantity to be measured by any instrument freely exposed to the passing air.

Knowing what hair can do, we ought to find some substance which has corresponding structure and is less subject to change than a part of a living organism. Chemists may know of such substances. There is a transparent stuff called " celliphane," which is certainly hygroscopic, and may be more stable than hair ; but, if no such substance is available, then, it seems to me, some competent physicist ought to live for a while with two (or more) recording hair hygrometers, use one (or two) as a current recorder and experiment with the others, not for the purpose of showing that it may have serious errors, but to find out how they may best be reduced to manageable limits. We might then learn that, if the hairs are treated in a specified way, they will, or will not, recover their tone, that changing the hairs every week and treating them as prescribed, the reading will be accurate to within 5 per cent.

The uncertain physical condition of water in the region just below the normal freezing point is an obstacle to the introduction of many other devices for indicating the dew-point by a deposit of moisture. Perhaps the most definite hygroscopic indication is to be obtained from the same process which gave most conclusive evidence of the vagueness of the thermometric " fixed " points—namely, the formation of cloud. I suppose a sample of air in a thermos flask properly supplied with nuclei cannot fail as an index of " below the dew-point " at any temperature; the flask might easily be boxed in so as to make an incipient cloud visible. It is not suitable for self-recording, but as an eye-method I think it must succeed.

The purpose of this note is to indicate that while investigations of various aspects of humidity and condensation from the point of view of an accuracy of one-tenth per cent. are always interesting, in the practical measurement of humidity below the normal freezing point a measure which could be relied upon within 5 per cent. would be of some advantage, and could probably be realised with existing apparatus by anyone who has access to air in varying conditions of moisture with a temperature below the freezing point. Such conditions occur in the open air of our observatories; but so infrequently that continuous observations cannot be arranged. Artificial conditions on a large scale are required.

SOME MODIFIED FORMS OF HYGROMETERS.

BY

EZER GRIFFITHS, *D.Sc.* (*National Physical Laboratory*).

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1. APPARATUS FOR OBTAINING VARIABLE HUMIDITY.

2. STANDARD OF REFERENCE IN CALIBRATION WORK ON HYGROMETERS.

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   - (*a*) *Determination of velocity of air stream past the wet bulb.*
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6. THERMAL EFFECT ON COTTON HYGROMETER.
   - (*a*) *Previous work on heating of cotton by water-vapour absorption.*
   - (*b*) *Electrical-resistance type.*
   - (*c*) *Apparatus utilising the effect for hygrometric measurements.*

7. THE HOT-WIRE TYPE HYGROMETER.

ABSTRACT.

The Paper describes experiments made with the object of developing suitable apparatus for the measurement of humidity at low temperatures. The three classical methods—the wet and dry bulb hygrometer, the dew-point apparatus, and the hair hygrometer—were studied and so modified that they could be used in a low temperature room inaccessible to the observer.

The common form of wet and dry bulb hygrometer is notoriously unreliable, but the ventilated form is quite satisfactory provided the wind velocity past the bulb exceeds

about 3 meters per second. A tubular form of wet and dry bulb hygrometer was developed with an electric fan for aspiring the air past the bulbs. A continuously-recording, resistance thermometer, type was also made, which had a cam device for rapidly dipping and withdrawing the thermometer bulb at periodic intervals from a reservoir of water.

The dew-point apparatus was modified into a form suitable for permanent installation; the cooling of the polished metal surface being effected by pipe connections to the cold brine circulation of the stores.

In another form a silver thimble, cooled by the evaporation of ether, was placed at one end of a long aluminium tube and a telescope at the other. By suitable arrangements of mirrors and lamps both the silver surface and the thermometer column were thrown into the field of the telescope. The end containing the thimble was inserted into the enclosure through a circular aperture. The instrument was portable and self-contained.

A distant reading type of dew-point apparatus was constructed, in which the formation of dew was indicated by the change of reflecting power of the cooled surface. A beam of heat radiation was reflected from the polished surface and concentrated on to a minute thermopile. The thermo-junctions were connected to a pivoted indicator situated at the observation station where the observer controlled the flow of cooling fluid by means of a solenoid valve. The formation of dew resulted in a sharp falling-off of the reflecting power and was indicated by the movement of the pointer of the instrument.

The hair hygrometer was studied under a variety of conditions and the changes in the calibration of this type of instrument, caused by exposure to low and high temperature, &c., were determined.

A very simple form of distant reading hair hygrometer was developed. In this the clockwork drum of the ordinary recorder was replaced by a tubular rheostat. The pointer was pressed into contact with the drum when a reading of the humidity was desired. The two portions of the resistance winding then constituted the two arms of a Wheatstone's bridge.

A number of other forms of hygrometers were also studied, notably the one utilising the heating effect on dry cotton when exposed to a humid atmosphere.

A convenient form of apparatus for the calibration of hygrometers over a wide range of humidities is described. It consists essentially of a drum fitted with a heavy glass front making an air-tight joint. The drum contains an electric fan which circulates the air over a dish of strong sulphuric acid. This method of reducing the humidity is more effective than bubbling and does not produce acid spray. The acid dish can be cut out of communication with the chamber by a ground glass cover operated by a lever projecting out of the drum.

INTRODUCTION.

HUMIDITY and temperature are two variables which have a profound influence on the physical properties of materials of organic origin. One has only to study such industrial processes as the seasoning of timber, the drying of raw rubber, or the preservation of foodstuffs, to appreciate the necessity for control over both the humidity and the temperature of the surrounding atmosphere.

As regards the measurement of temperature, there are available at the present time a variety of reliable instruments which can be adapted to meet most requirements. Such a satisfactory state of affairs does not exist as regards the measurement of humidity. Yet the importance of a knowledge of the hygrometric state of the atmosphere appears to have been recognised from an early date. As far back as 1783 De Saussure devised the hair hygrometer, while Daniell invented the dew-point hygrometer in 1827.

The origin of the familiar wet-and-dry bulb hygrometer is not quite certain. It is stated that Sir John Leslie in 1813 converted his differential air thermometer into a form of hygrometer by keeping one bulb wet and the other dry, and observing the temperature difference.

The simplicity of the wet-and-dry-bulb mercury hygrometer has led to the adoption of this instrument to the exclusion of practically all other types of hygrometers in meteorological and industrial work. The conservative attitude of scientific workers in this respect is greatly to be deplored, for it is now generally admitted that the instrument in its elementary form is very inaccurate.

It is a curious fact that no fundamentally new method of hygrometry has since been introduced, and the above instruments have remained the sole physical appliances available for routine tests. It is true that innumerable proposals have been made concerning novel hygrometric devices. Usually, however, such instruments are put forward without receiving adequate experimental investigation, and it often happens that humidity is but one of the many factors in operation.

The investigation described in the present Paper arose from an inquiry for suitable apparatus to use in measuring the humidity of the air in cold stores. It was considered that the design of the standard form of laboratory apparatus was unsuitable for this purpose, particularly when it was desired to measure the humidity without actually entering the room.

With the object of meeting these special requirements a study was first made of the three standard methods of hygrometry :—

(1) The ventilated wet and dry bulb hygrometer.
(2) The dew point apparatus.
(3) The hair hygrometer.

In the course of the work other methods of measuring humidity were also studied.

But before passing on to the description of the various appliances for the measurement of humidity it is necessary to consider the means available for adjusting to any desired value the humidity within a closed space. For it is essential in calibration work to be able to vary the humidity over a wide range without being dependent upon climatic conditions.

1. APPARATUS FOR OBTAINING VARIABLE HUMIDITY.

The problem of humidity control is not difficult if only comparatively small volumes of air have to be considered, but when it becomes necessary to vary the humidity in a space of, say, 10 cubic metres in capacity it becomes a troublesome matter, especially if an observer has to enter the room.

Since the experiments were primarily concerned with temperatures near 0°C., a sheet iron enclosure was constructed within a cold stores room. In this enclosure an electric fan was fitted to keep the air in rapid circulation. If a humidity value below normal was required the air was circulated over granulated calcium chloride. This salt was spread out on lead trays so inclined that the liquid drained off to reservoirs beneath.

With this arrangement it was possible to maintain a humidity as low as 45 per cent. with the observer in the room. The presence of the observer is a serious hindrance to the maintenance of a low humidity at a low temperature, for the quantity of water required to saturate a cubic metre of air at 0°C. is only 4·84 grams, and the average human being gives off 63 grams of water per hour when in repose and considerably more when exercising.* Hence it will be realised that very efficient methods of water vapour absorption must be employed if the humidity is to be maintained at a low value.

When it is practicable to carry out experiments in a closed vessel of moderate dimensions, then the problem becomes decidedly simpler. For instance, with some types of hygrometers it is possible to make observations without the necessity of manipulating the instruments or aspirating considerable volumes of air out of the enclosure. Such is the case with the hair hygrometer, the dew-point apparatus, and the refractive index of glycerine method. For some experiments the apparatus shown in Fig. 1 has been found to work admirably.

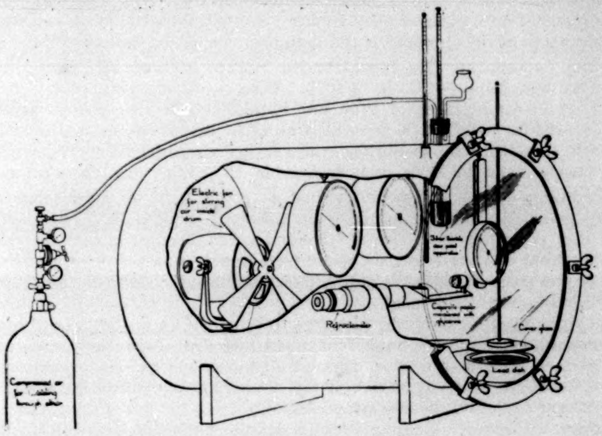

FIG. 1.

Description of Apparatus.

The apparatus consists of a sheet metal vessel 14 inches in diameter by 18 inches in length, closed by a piece of plate glass clamped down on a soft rubber packing so as to give an air-tight joint. A small electric fan is fixed in the drum which is switched on when it is desired to mix the air in the vessel before an observation. The humidity of the atmosphere inside the drum is varied by the aid of a lead dish containing strong sulphuric acid. The fan drives the air over the exposed acid surface and the humidity can be rapidly lowered. When the required point has been reached the dish is closed by a weighted glass cover carried from a rod. This is operated from without the enclosure. The humidity of the atmosphere can be raised by blowing in a little steam or merely by breathing into the drum through a piece of rubber tubing.

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* About 1 gram per kilogram of adult human body per hour in repose. This quantity is extremely variable; the slightest exercise increases it enormously.—*Recueil de Constantes Physiques*, Société Française de Physique, p. 740.

Driving the air by means of the fan over the exposed acid surface is a far more effective method of changing the humidity than by pumping the air round through wash bottles containing acid. This latter method was tried, and abandoned on account of its extreme slowness.

With the simple arrangement illustrated in Fig. 1 humidities down to 5 per cent. can be obtained with ease. To determine the actual value of the humidity a dew-point apparatus is installed as shown in the diagram. This consists of a silver thimble soldered on a piece of glass tubing. Air is bubbled through the ether and the temperature of the appearance and disappearance of the dew observed.

In our experiments this apparatus has worked satisfactorily over the range of temperatures from −8°C. to +40°C. When studying the behaviour of the instruments in atmospheres of very low humidities the dew-point was lowered as far as −28°C. (−20°F.), a depression of 21°C. below the air in the enclosure. Even with these extreme conditions the method was workable ; and under the circumstances it would have been impossible to employ the chemical method.

In addition to the dew-point method a special form of Abbé refractometer was tried, so as to obtain the humidity on a direct reading scale. This instrument will be seen projecting in from the left-hand side. The prism and the telescope are rigidly fixed and the refractive index read from a scale in the eye-piece. The surface of the piece of cigarette paper, moistened in glycerine,* is illuminated by the small bull's-eye electric lamp.

The procedure in making the tests was to bring the air to the desired humidity approximately, close the sulphuric acid vessel (the top of the lead vessel and the plate had been ground to a good fit), and allow the fan to run for some time to mix up the contents of the drum thoroughly. After an interval the fan was stopped, the instrument read, and the dew-point observed.

Another method of obtaining air of a definite humidity is to bubble through H_2SO_4 solution of appropriate density. The advantage of this procedure is that it is not necessary to determine the humidity, but only to ensure that the moisture content of the air is that of the partial vapour pressure corresponding to the strength of sulphuric acid solution employed. The usual procedure is to bubble the air at the rate of 50 c.c. to 100 c.c. per minute through two or three wash bottles in series.

An essential precaution in the use of this method is to pass the air through a tube rather tightly packed with glass wool, so as to remove the particles of sulphuric acid carried over by the air stream.

This tendency of the acid spray to be carried over when bubbling air through the solution is troublesome to overcome. For example, in attempting to obtain air of definite humidity in a large cylindrical vessel by bubbling air through a layer of acid in the bottom, it was found that the air near the top of the vessel and at a distance of 18 in. away from the liquid surface was acid in reaction to litmus paper. This feature rendered the method impracticable for the experiments described in this Paper.

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* **A solution of glycerine will evaporate or condense water vapour from the surrounding atmosphere until the concentration of the solution is such that it is in equilibrium with the aqueous vapour pressure prevailing.—(Giraud).**

For studying the humidity equilibria of various substances which can be packed into U-tubes of small capacity the above method appears to be quite satisfactory, and it might be observed that accurate control of the temperature is not very important, since the moisture content of most organic materials varies but little with moderate changes in temperature, providing the relative humidity is maintained constant.

TABLE I.—*Strength of* H_2SO_4 *Required to give Definite Humidities.*

Relative Humidity.	Per cent. Sulphuric Acid required at			
Per cent.	0°C.	25°C.	50°C.	75°C.
10	63·1	64·8	66·6	68·3
25	54·3	55·9	57·5	59·0
35	49·4	50·9	52·5	54·0
50	42·1	43·4	44·8	46·2
65	34·8	36·0	37·1	38·3
75	29·4	30·4	31·4	32·4
90	17·8	18·5	19·2	20·0

NOTE.—The values given in the above table were computed by R. E. Wilson from the experimental data of Regnault, Sorel, Helmholtz, Bronsted and Dieterici. *See* Journ. of Industrial and Engineering Chemistry, Vol. XIII., No. 4, p. 326 (1921).

Reference to Table I. shows that for a temperature change of about 10 deg. the relative vapour pressure of most of the solutions is practically constant ; it must not, however, be supposed that constant humidity conditions will be maintained in a closed space when subject to temperature change, unless the temperature of the entire apparatus is uniform throughout. Local irregularities of temperature produce very serious errors if diffusion is depended upon for equalising the humidity. Hence in precision work it may be assumed that thermostatic control over the temperature of the space is necessary.

An analogous method for use at higher temperatures is that employed in the Freas oven. This oven is designed with shelves on which suitable crystals containing water of crystallisation are placed. An air-circulating device is arranged so as to sweep air over these crystals, thus maintaining a humidity content corresponding to the dissociation vapour pressure of the water of crystallisation. An essential condition is that the chemical must be present in both the anhydrous and crystallised forms.

When the moisture content of the air becomes too great, owing to the material in the oven giving off a great deal of moisture, then the anhydrous salt will remove some of it, and be converted to the crystalline form. On the other hand, if the air is too dry the crystallised form will furnish the moisture required. Adjustment of the salt to the required state is effected by drawing unsaturated air over the chemical when the crystal form is in excess, and saturated air when the anhydrous form is in excess. It is stated that such treatment for an hour or so will prepare a mixture of the chemical in the proper proportion of crystal and anhydrous form for rendering it successful for maintaining constant humidity.

By this method it is possible to obtain a range of humidity up to temperatures as high as 80°C., but, of course, it is advisable to control the temperature by a thermostat arrangement.

In Table II. some figures are quoted from the catalogue of Messrs. Eimer & Amend to indicate the possibilities of the method.

TABLE II.—*Dissociation Tension with Loss of Water of Crystallisation.*

Temperature Degrees C.	Tension of Water.	Tension of Water Vapour.			
		$CuSO_4-5H_2O$	$MgSO_4-7H_2O$	$ZnSO_4-7H_2O$	$FeSO_4-7H_2O$
		mm.			
20	17·4	6·0	...	10·1	10·9
30	31·5	11·6	27·0	20·2	20·3
40	54·9	22·4	47·2	44·2	40·1
50	92·0	42·0	75·7	73·1	74·8
60	148·9	...	122·5	113·9	131·3
70	233·3	...	186·0	170·8	205·2
78	327·0	234·0	...	258·5	321·5

2. STANDARD OF REFERENCE IN CALIBRATION WORK ON HYGROMETRY.

In calibration work it is generally accepted that the chemical method is the ultimate standard of reference.

In our early experiments at low temperatures we found the chemical method exceedingly tedious to operate. There was also the fact that the absolute weight of water, for a given percentage of humidity, was considerably reduced at low temperatures; and difficulties were encountered owing to dew deposition on the weighing tubes, &c. So, although it was possible with care to obtain a reasonable degree of accuracy by this method, we were soon forced to seek for more expeditious appliances for calibration work.

Of the three physical methods of hygrometry, the dew-point apparatus stands out conspicuously as being the one instrument which has a sound theoretical foundation and, further, it has the practical advantage that it is as easy to manipulate at low temperatures as at room temperatures. After a lengthy experience with the various methods under the most diverse conditions we have come to the conclusion that the dew-point is the most convenient of all methods for low temperature work; and, consequently, it has been employed as standard of reference for most of the work recorded in this Paper.

3. WET AND DRY BULB HYGROMETER OF THE VENTILATED TYPE.

Whilst the unventilated type of wet and dry bulb hygrometer is a notoriously unreliable instrument, it does not appear to be sufficiently well known that by so modifying the construction that air at a definite velocity is drawn past the thermometer bulbs, it can be converted into an instrument of satisfactory precision.

This important fact was demonstrated by the Italian physicist Belli in 1830, and in view of the simplicity of the device it is somewhat surprising that the stationary form of wet and dry bulb hygrometer is tolerated at all at the present day.

In the familiar equation* for the wet and dry bulb hygrometer,

$$p=p_w-AP(t-t_w)$$

---

* Attempts have been made by various scientists to deduce from theoretical considerations the relation between the vapour pressure of the aqueous vapour in the air and the difference of temperature between the dry and the wet bulbs. Not one of these is satisfactory, and the simplest plan is to use an empirical equation of the above form.

where t=air temperature.

t_w=wet bulb temperature.

p=pressure of aqueous vapour in the air.

p_w=saturation vapour pressure at the temperature of the wet bulb.

P=barometric pressure.

A=a quantity which for the same instrument and certain conditions is constant, or a function depending in a small measure on t_w.

The value of A differs materially according to whether the wet bulb is in quiet or in moving air; it is in this connection that the use of a forced air-stream past the bulb becomes of fundamental importance. In theory the value of A depends upon the size and shape of the thermometer bulb, size of stem, and velocity of the ventilation; so that different formulæ are required for different forms of instruments. But using an air velocity exceeding about three metres per second the differences in the results given by different instruments vanish, and the same tables can be adapted to any kind of thermometer, and to all changes of velocity above that which gives sensibly the greatest depression of the wet bulb temperature; and with this arrangement there is no necessity to measure or estimate the velocity in each case further than to be certain that it does not fall below the assigned limit.

It is not difficult to obtain sufficient velocity of the air past the bulbs, since all that is necessary is to fix the two thermometers securely on a rod and swirl them round. Espy at a very early date noted the advantage of this procedure, for he states: "When experimenting to ascertain the dew-point by means of a wet bulb, I always swung both thermometers moderately in the air, having first ascertained that a moderate movement produced the same depression as a rapid one."*

The most popular form of ventilated wet and dry bulb hygrometer is the Assmann type. In this the two thermometers are fixed alongside each other in two tubes, and a clockwork-driven fan sucks air past the bulbs. Before each series of observations the instrument requires winding up, and the muslin sack around the wet bulb thermometer needs moistening with distilled water.

Tables † are published covering the temperature range from −30°C. to +40°C., for converting the readings into relative humidity; the same operation can be performed graphically.

We have made a large number of observations with this form of hygrometer and compared its indications with those of a dew point apparatus. At room temperature its readings agreed with those of the dew-point apparatus within 1 to 2 per cent., but at low temperatures the divergence was greater on account of the disturbance caused by the observer when reading the thermometers at close quarters. The actual data will be found in Columns 4 and 5 of Tables IX. and XIII.

The following observations may also be quoted as illustrating the degree of concordance obtained with three ventilated psychrometers obtained from various sources (Continental and British) when compared with a dew-point apparatus. In each case, of course, corrections were applied to bring the mercury thermometer readings to true temperature.

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* **Smithsonian Meteorological Tables, 4th edition (1918).**

† **Aspirations—Psychrometer—Tafeln. Published by Vieweg Braunschweig.**

TABLE III.—*Psychrometer Readings.*

Instrument No.	Wet bulb temperature.	Dry bulb temperature.	Relative humidity.
1	13·3° 13·5°	17·3° 17·8°	64 per cent. 62 ,,
2	13·6° 13·8°	17·2° 17·9°	67 ,, 64 ,,
3	13·6° 13·9°	17·5° 18·1°	65 ,, 63 ,,

Dew-point Apparatus.

Air temperature.	Dew appears.	Dew disappears.	Mean reading.	Relative humidity.
17·2 17·8	10·15° 10·55°	10·35° 10·65°	10·25° 10·6°	63·7 per cent. 62·7 ,,

(*a*) *Determination of Velocity of Air Stream Past the Wet Bulb.*

It is always advisable to test an instrument to ensure that the velocity of the air stream past the bulb is of the order of 3 metres per second. This can be effected by connecting on to the tube surrounding the wet bulb a glass tube, and determining the speed of a puff of smoke along the glass tube. An alternative method is to measure the velocity in the extension tube by means of a Pitot tube and sensitive gauge, making corrections for the relative cross-sections of the extension tube and annular space around the bulb. Since only an approximate check is usually necessary, the smoke method is quite effective.

One practical defect of the Assmann design is the possibility of water being squirted over on to the dry bulb when saturating the sack around the wet bulb. This, of course, vitiates the readings.

(*b*) *Tubular Psychrometer.*

We constructed a modified form of instrument which permitted of observations being taken without the observer entering the room. In this arrangement an electric fan aspired the air past the bulbs contained in a steel tube 2·7 cm. in diameter and 70 cm. long. The complete instrument is shown in Fig. 2. The motor was directly connected to the 100-volt supply, and took 0·1 of an ampere. After once moistening the sack around the wet bulb, the instrument would continue to operate for 40 minutes without attention.

(*c*) *Experiments below* 0°C. *with the ordinary form of Assmann Psychrometer.*

At temperatures below 0°C. the water freezes on to the muslin bag, but the instrument still operates fairly satisfactorily.

In order to obtain the data necessary for the design of a distant-reading wet and dry bulb instrument, which would act automatically and could be placed at a considerable distance from the recording station, some observations were made with a standard mercury thermometer type of Assmann psychrometer. In these experiments the instrument was set up in the cold stores room and observations taken at intervals for a period of 3½ hours, the observer entering the room occasionally to wind up the clockwork and read the thermometers. Reference to Fig. 3 will show the curves obtained.

Initially, both bulbs were dry, and at approximately the same temperature (about—3·5°C.). The muslin around the wet bulb was moistened with water, whose temperature was above 0°C., and the fan started. The reading of the thermometer then dropped quite rapidly to —4·4° under the influence of the air current. It will be observed from an inspection of the data in Fig. 3 that the temperature fell to this value in 3½ minutes. At this point, apparently, the under-cooled water froze and the temperature rose sharply to 0°C. Then the frozen mantle cooled down and acquired an equilibrium temperature about 1° below the dry bulb after an interval of 10 minutes from the time of the initial moistening. From this point forward for

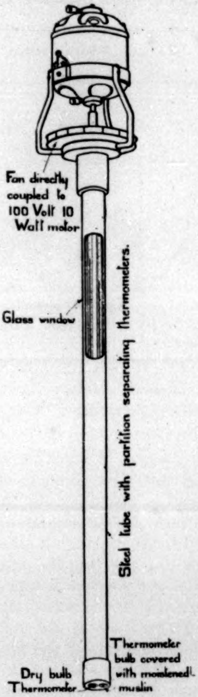
Fan directly coupled to 100 Volt 10 Watt motor
Glass window
Steel tube with partition separating thermometers.
Thermometer bulb covered with moistened muslin
Dry bulb Thermometer

FIG. 2.

a period of about two hours the difference between the two thermometers was practically constant. In Fig. 3 the time scale after the first 10 minutes is contracted up so as to include the respective temperatures over a long interval of time. The violent oscillations of temperature were due to the observer going in and out of the cold stores room and disturbing the thermometers when winding the clockwork at intervals of about ten minutes.

(*d*) *Low Temperature Distant Reading Psychrometer.*

From the results of the above experiment it appeared that saturating muslin in water at intervals of from 1 to 2 hours would suffice, so the problem resolved itself to devising a mechanical device by means of which the wet bulb could be periodically saturated with water. It was, of course, necessary to maintain the water into which the bulb was dipped at a temperature above 0°C. by the use of a small heating coil. It would have been easy to perform the dipping by a clockwork arrangement, but since an electric fan was required for producing an air stream past the bulbs, it was decided to utilise the same motor for operating the dipping mechanism.

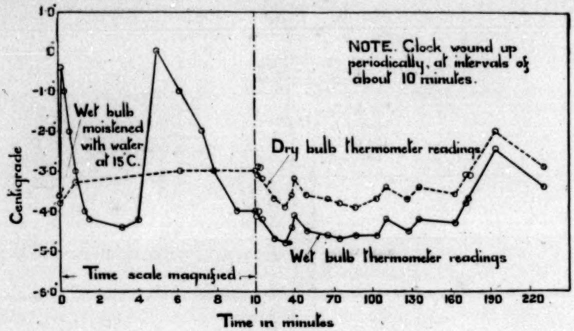

FIG. 3.

The essential condition to be complied with was that, at the desired instant, the thermometer should be plunged into the water fairly rapidly, kept there for a period sufficient to ensure that any residual ice was melted, then withdrawn sharply and restored to its normal position in front of the fan. To effect this motion the arrangements shown in Figs. 4 and 5 were constructed. The resistance thermometer is carried from the end *A* of the arm *AB*. This arm prolongs to *C*, where it terminates in a small pin projecting out at right angles to the arm. Parallel to this extension *BC* is a spring *S*, with a pin projecting at its end. *BC* and *S* are situated in front of the face of a disc. This disc is rotated at the rate of one revolution per two hours by means of the reduction gearing from the motor. A square section ring projects out from the plane of the disc, which has two slots cut in it. In the ordinary way, the pin *C* rides on the inner or outer circumference of this projecting ring. On the face of the disc are two inclined planes *O* and *P*. In the course of the rotation the pin at the end of the spring comes into contact with the inclined plane *O*. It travels up the plane, and the spring is thereby depressed towards the centre of the disc. The compression of the spring tends to rotate the arm *AB* and, consequently, when the pin *C* comes opposite the slot it slips through, and end *A* of the arm *AB* is raised. Referring to Fig. 4, the arm is shown in the position when the pin *C* has just slipped through the slot.

During the next period of rotation the pin *C* rides upon the inner circumference of the ring and the thermometer remains stationary in the air breeze. When the disc has nearly made a complete revolution, the pin projecting from the spring *S* comes into contact with the inclined plane *P*, set at such an angle as to pull away from the centre of the disc. Consequently, when the pin *C* reaches the slot, it passes through from the inner to the outer circumference of the projecting ring. In so doing the point *A* is depressed, and the thermometer bulb plunges into the water.

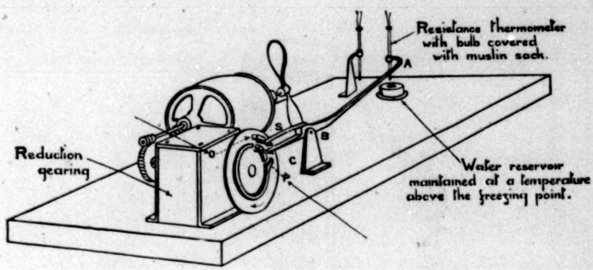

FIG. 4.

For the comparatively short period that the pin *C* is travelling on the outside circumference of the projecting ring the bulb remains immersed in the water. As the rotation of the disc continues, the pin at the end of the spring travels up the inclined plane *O*, and the compression of the spring forces the pin *C* through when it arrives opposite the slot, thus withdrawing the thermometer from the water.

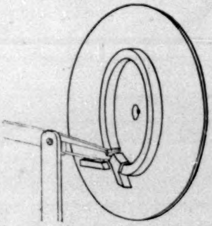

FIG. 5.

In Fig. 5 the same management is shown in a slightly modified form. Here the pin *C* rides on the outer circumference of the ring when the thermometer is in the "up" position.

The rest of the apparatus needs no detailed description. The resistance thermometers were constructed with very small compact bulbs, as the amplitude of the motion was limited. The condensation of moisture within the sheaths on the thermometer coils had to be guarded against by careful sealing. In our experiments the wet and dry bulbs were connected up differentially to a thread recorder.

A distant reading hygrometer of this character does not involve much more apparatus than is necessary for the recording of temperature by the use of a resistance thermometer outfit, since the expensive items of both equipments are identical.

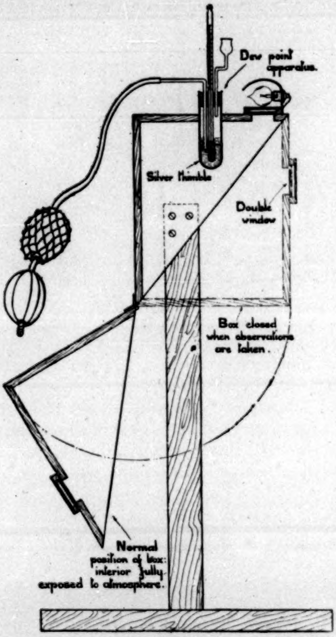
Dew point apparatus.
Silver thimble
Double window
Box closed when observations are taken.
Normal position of box: interior fully exposed to atmosphere.

FIG. 6.

4. THE DEW-POINT HYGROMETER.

Although numerous attempts have been made to formulate some consistent theory for the wet-and-dry-bulb hygrometer, the nature of the assumptions which have to be made are such that the resulting formulæ will not stand experimental test. The dew-point method, on the other hand, is based on straightforward physical facts and the theory is so simple that it inspires confidence in the results obtained with the instrument.

For our requirements it was necessary to be able to operate the hygrometer in a somewhat confined space, and we developed, after a few preliminary experiments, the form shown in Fig. 6. The polished-silver thimble contains the thermometer and tubes for bubbling air through the ether. This is situated in the lead-lined box cut diagonally as shown. By oscillating the lower half of the box it is possible to ensure that an average sample of air is obtained. The box is then closed and the observer sets the air bubbling and watches the formation of the dew through the double windows in front. The use of a box around a dew-point apparatus has the great advantage that it practically eliminates the disturbing influence of the observer in the vicinity of the apparatus, and this is a most important

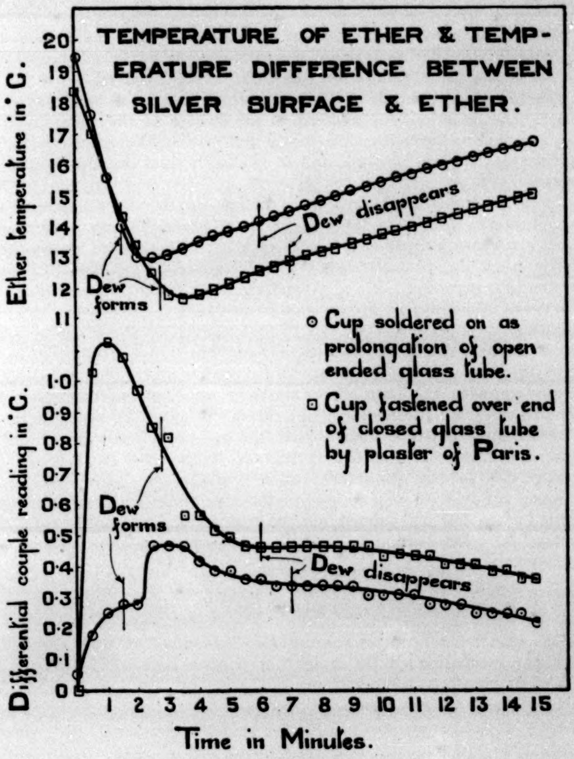

FIG. 7.

factor in low-temperature work, for it will be remembered that less than 5 grams of water will saturate a cubic metre of dry air at 0°C.

The box method has the additional advantage that it eliminates the disturbances of convection currents and draughts in the room.

(*a*) *Temperature Difference between Thermometer and Silver Surface.*

A point of importance in connection with the method is the measurement of the difference in temperature between the thermometer in the ether and that of the external surface of the thimble on which the dew is deposited.

To ascertain the magnitude of this a differential couple was made up of copper constantan. One junction was soldered to the surface of the silver thimble and the other attached to the bulb of the thermometer. The difference of temperature between the two junctions was observed at successive time intervals by means of a potentiometer.

Two forms of apparatus were tested; in the one the silver cup was soldered on as a prolongation of a glass tube, whilst in the other the cup was cemented on to a closed glass tube by means of plaster of Paris. This second method of fixing is, of course, not to be recommended, but we have found it employed in apparatus sold for teaching purposes. The glass and plaster are poor heat conductors and often the defect is aggravated by the plaster not being a continuous layer but full of air bubbles.

The results obtained in the two experiments are shown graphically in Fig. 7. The upper curves refer to the temperature of the ether during the period of cooling and warming up, while the lower curves are the readings of the differential couples. It will be observed that in the case of the thimble attached by plaster of Paris the surface of the silver is 0·8°C. warmer than the ether when dew forms and about 0·4°C. when it disappears. The soldered thimble, on the other hand, is only 0·28°C. warmer when dew appears and 0·35°C. when it disappears.

We found it advisable in practice to bubble air through the ether at intervals in the period when dew had formed and the temperature was slowly rising until it disappeared. By a little practice it was possible to mix the ether without causing appreciable evaporation.

(*b*) *Estimate of Weight and Thickness of Dew Deposit.*

Another question of considerable interest in connection with the dew-point method is the estimation of the weight of deposit required to give a visible indication of its presence on the surface.

Such estimates were arrived at by several independent methods and observers.*

(1) *By Weighing a Film.*

(*a*) A silver thimble of 17·4 sq. cm. area was weighed, cooled down, exposed to the atmosphere of the room until a deposit appeared, sealed in a weighing-jar and again weighed.

---

* I am indebted to my colleague, Mr. A. H. Davis, for most of the observations on weight of dew deposition.

The following results were obtained in a series of experiments :—

TABLE IV.

Experiment.	Weight of Deposit.
1	0·0016
2	0·0013
3	0·0011
4	0·0009
5	0·0008
6	0·0012
	Mean 0·0011

Hence the weight of deposit per square centimetre$=6{\cdot}3\times10^{-5}$ gms.

Results of the same order were obtained by another method. In this a film was deposited directly on the balance pan by breathing on it. The change of the equilibrium position of the pointer of the balance was observed by the usual method of oscillations. The pointer deflection scale was calibrated by noting the effect of changes in position of the rider. It was found that with no film the calculated zero would be in a certain position. A film altered the zero and, as the film gradually disappeared, the pointer gradually returned to its old zero. As the day was a wet one the rate of evaporation was not unduly great, being about 0·5 milligram per complete oscillation.

About four-fifths of the balance-pan area was covered with deposit equivalent to an area of 55 sq. cm.

The following results were obtained :—

TABLE V.

Experiment.	Weight of Deposit.
1	0·005 gms.
2	0·006
3 Extra heavy film.	0·008
4 Slight film.	0·002

Thus the mass of water deposit per square centimetre lies between $3{\cdot}6\times10^{-5}$ gms. and 14×10^{-5} gms. for slight and thick deposits respectively.

(2) *By Interference Colours.*

A film was obtained by breathing on a piece of platinised glass. With relatively thick films rather impure colours were visible, such as reddish brown.

Assuming these to be due to the usual interference effects with thin films, it appears that the thickness in this case is of the order of three or four wavelengths of light.

So the mass of water per square centimetre of surface would be about

$$24\times10^{-5}\text{ gms.}$$

(3) *From Water Vapour in one Breath.*

In normal breathing a person takes 16 breaths per minute, or $2{\cdot}3\times10^4$ per day.

The total weight of water breathed out per day on the average is about 10 oz. = 300 gms. Hence the weight of water in one *normal* respiration is

$$\frac{300}{2{\cdot}3\times10^4}=1{\cdot}3\times10^{-2}\text{ gms.}$$

One ordinary breath will dew over an area of 100 sq. cms. Hence assuming all the water in the breath is deposited the weight per square centimetre becomes

$$13\times10^{-5}\text{ gms.}$$

An alternative method of obtaining a similar estimate is to consider a *full* breath.

A full breath includes

Tidal air	30 cubic in.
Complementary air	100 „ „
Supplementary air...	100 „ „
Total	230 „ „

This total together with the 100 cubic in. of residual air which always remains in the lungs makes up the total lung capacity of 330 cubic in.

In a normal respiration a person breathes out 30 cubic in. which as shown above contains $1{\cdot}3\times10^{-2}$ gm. of water.

Assuming a *full* breath to contain a correspondingly larger weight of water, we have for the weight in a full breath

$$1{\cdot}3\times10^{-2}\times\frac{230}{30}=0{\cdot}1\text{ gm.}$$

The area dewed over by a full breath is roughly 1,000 sq. cm. Hence, if all water vapour is deposited the thickness of the film is 10^{-4}, and the mass of water vapour per square centimetre is 10×10^{-5} gm.

Now 1 c.c. of air saturated with water vapour at 20°C. contains $1{\cdot}7\times10^{-5}$ gm., and at 0°C. $0{\cdot}48\times10^{-5}$ gm. So that we can calculate from the above estimates the volume of dry air that the weight of water in the film would saturate.

TABLE VI.

Method.	Magnitude of dew film expressed as :—		
	Gms per sq. cm.	C.c.s of dry air that dew on 1 sq. cm. of the surface would saturate.	
		At 20°C.	At 0°C.
Weighing :—			
(*a*) Normal film	$6{\cdot}3\times10^{-5}$	4	13
(*b*) Slight film	3·6	2	7
(*c*) Heavy film.........	14	8	29
Interference colours ...	24×10^{-5}	14	50
Respiration :—			
(*a*) Full breath.........	10×10^{-5}	6	21
(*b*) Ordinary breath	13×10^{-5}	7	27

(*c*) *Industrial Forms of Dew-Point Apparatus.*

(*i*) *Wall Fixture Type.*

The apparatus shown in Fig. 6 is hardly suitable for routine use in cold stores, so for this purpose we developed the form illustrated in Fig. 8. Here the silver thimble is replaced by a nickel-plated block of copper which can be cooled by the brine circulation used for cooling the stores. The temperature of the copper is determined by means of a thermometer with its bulb fixed as close as possible to the surface of the copper. The manipulation of this instrument is very simple. When an observation has to be taken the observer closes the glass front of the instrument and turns on the brine supply. The temperature at the instant of dewing over is observed and also of the disappearance.

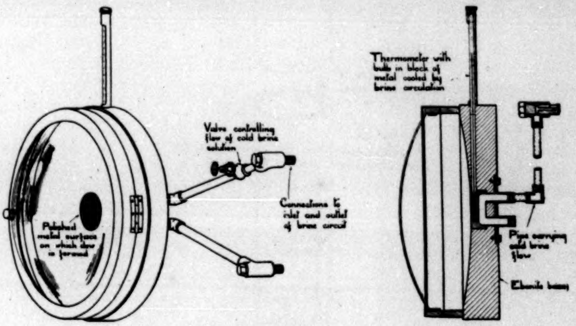
Polished metal surface on which dew is formed
Valve controlling flow of cold brine solution
Connections to inlet and outlet of brine circuit
Thermometer with bulb in block of metal cooled by brine circulation
Pipe carrying cold brine flow
Ebonite base

FIG. 8.

It may be of interest to quote some of the results obtained with this instrument.

TABLE VII.—*Test of " Wall Fixture " Type Dew-Point Apparatus.*

Experiment No.	Psychrometer readings.			Dew-point apparatus.		
	Dry bulb thermometer.	Wet bulb thermometer.	Humidity.	Dew forms.	Dew disappears.	Humidity.
			Per cent.			Per cent.
1	15·7	12·9	73	9·5	11·7	72
2	15·9	13·0	72	10·5	11·3	72
3	16·0	13·2	73	11·0	11·3	73
4	16·0	13·2	73	10·9	11·2	72·6
5	14·1	12·1	79	9·7	11·9	80
6	14·7	13·2	85	12·2	12·3	85

(*ii*) *Portable Form.*

It was found desirable to have an instrument which could be taken from place to place and inserted through the circular aperture built in the walls to permit of the insertion of the thermometers. For this purpose the hygrometer shown in Fig. 9 was constructed.

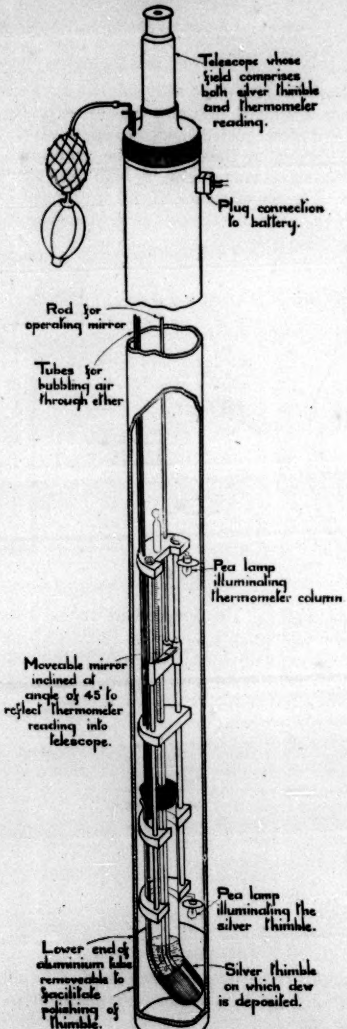
Telescope whose field comprises both silver thimble and thermometer reading.
Plug connection to battery.
Rod for operating mirror
Tubes for bubbling air through ether
Pea lamp illuminating thermometer column
Moveable mirror inclined at angle of 45° to reflect thermometer reading into telescope.
Pea lamp illuminating the silver thimble.
Lower end of aluminium tube removeable to facilitate polishing of thimble.
Silver thimble on which dew is deposited.

FIG. 9.

It consists of an aluminium tube 8 feet in length by 2½ inches in diameter. At one end of the tube the dew-point thimble is placed while a telescope is fixed to the other end.

The silver surface is an angle of about 45° to the axis of the tube, so that it is possible to view it directly from the telescope. This arrangement of the silver thimble has the additional advantage of permitting the apparatus to be operated with the axis of the tube either vertical or inclined with the thimble pointing downwards. The thermometer scale is read by the aid of a small mirror moving along guides, so as to maintain the meniscus of the mercury column in the field of view. Both the mercury column and the silver surface are illuminated by minute "pea" lamps set out of the field of view; a mirror being fixed opposite the silver surface to throw the light on to it.

In the vicinity of the dew-point surface a wide slot is cut in the aluminium sheath to permit of free access of air to the silver surface.

(*iii*) *Distant Reading Dew-point Apparatus.*

It will be observed that the dew-point method depends primarily upon the acuteness of vision of the observer to detect the formation of a trace of dew, and whilst it is possible, by the aid of a telescope, to operate the apparatus at distances up to 10 feet, for considerably greater distances it becomes necessary to find some other method of dew detection.

Now the dew-point method is essentially a discontinuous one and a prescribed cycle of operations has to be gone through for each determination of humidity. If these operations could be performed mechanically, without human intervention at any stage, we should have an industrial form of hygrometer possessing a sound theoretical basis.

It was anticipated that the conversion of the dew-point apparatus into a distant reading form would present unique practical difficulties and our anticipations were fully realised when the attempt was made to produce a continuously recording hygrometer working on this principle.

Some preliminary experiments indicated that a promising line to follow in the design of such an instrument would be to utilise the change of reflecting power of a metallic surface on the formation of dew.

The apparatus illustrated in Fig. 10 shows how this idea was applied in the construction of a distant-reading hygrometer.

The source of radiant energy is a gas-filled tungsten lamp. The energy is reflected from the plane surface on to the concave mirror* and thus concentrated on two minute thermo-junctions connected in series.

The indicator is located at a point convenient for observation and the instrument is worked in a similar manner to the standard form of apparatus; the only difference is that the observer watches the movement of the pointer instead of the surface on which dew forms.

The valve controlling the brine flow is operated by means of a solenoid. Until the dew formation the deflection of the pointer is steady, but simultaneously with the deposition of dew the reflecting power is diminished and the pointer moves rapidly toward zero. The temperature of the surface on which dew forms is then read by means of a thermo-junction.

---

* We are indebted to Mr R. S. Whipple for presenting us with the concave metallic mirror and thermo-junctions which are standard parts of Fery pyrometers.

This apparatus functioned satisfactorily and was tested against a psychrometer with the results shown in Table VIII.

TABLE VIII.—*Test on Distant Reading Dew Point Apparatus.*
(Typical Experiment.)

Time.		Deflexion of radiant heat indicator.	Temp. of metallic surface in °C.	Remarks.
Min.	Sec.			
	0	45·5	14·0	
	30	45·7	14·1	
1	0	45·8	14·2	
	30	45·7	14·0	
2	0	45·5	11·1	
	30	45·0	9·1	
3	0	45·0	7·5	
	30	45·0	6·5	← Dew forming.
	45	41·5	....	
4	0	40·5	6·2	
	30	39·0	6·5	
5	0	39·5	6·8	
	30	44·5	7·1	← Dew disappearing
6	0	45·0	7·5	
	30	45·3	7·7	

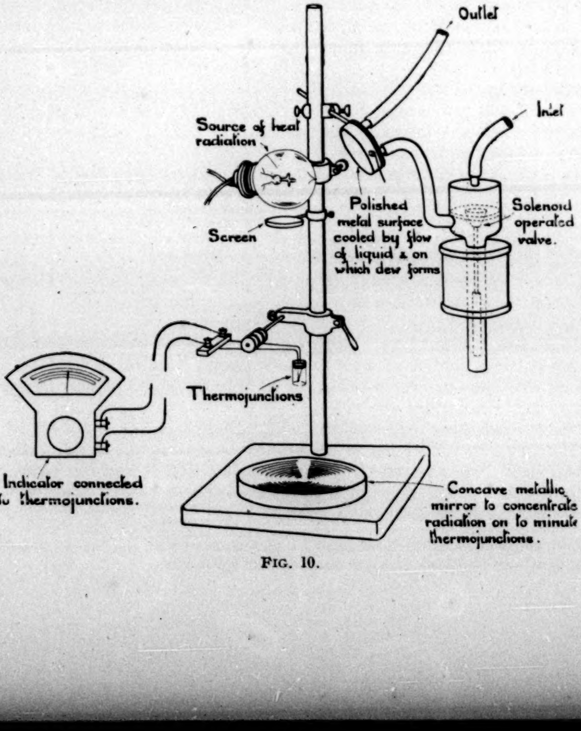
Outlet
Inlet
Source of heat radiation
Polished metal surface cooled by flow of liquid & on which dew forms
Solenoid operated valve.
Screen
Thermojunctions
Indicator connected to thermojunctions.
Concave metallic mirror to concentrate radiation on to minute thermojunctions.

FIG. 10.

If it were possible to work the indicator as a relay for operating the solenoid valve, then it might be possible to maintain the surface always at the dew-point temperature and thus obtain a continuous record of the dew-point in the room.

Hence taking the mean of the temperatures of appearance and disappearance the value found for the humidity is 59·5 per cent.

Taking the temperature of appearance only the value becomes 58·3 per cent.

Assmann psychrometer readings during the above series :—

Dry bulb temperatures = 14·6°, 14·7°.
Wet ,, ,, = 10·2°, 10·4°.

Hence humidity by psychrometer = 58 *per cent.*

Experiments have also been made on another method of indicating the occurrence of the dew point—namely, the change of electrical resistance of a surface on the deposition of dew. The apparatus used consisted of a test tube, on the outside of which were deposited two rings of platinum film with a small gap of one or two millimetres between them. The platinum films were connected to a "megger" or to the secondary of an induction coil with a telephone in series. The test tube was cooled down in the usual way by bubbling air through ether.

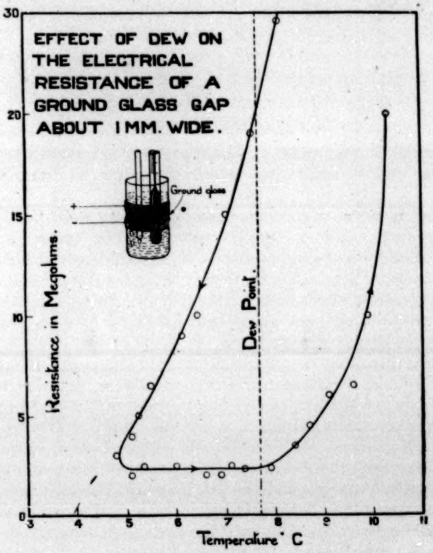

FIG. 11.

When the glass between the films was quite clean the resistance of the gap remained at infinity after the deposition of the dew. With the glass dipped in a solution of an efflorescent salt like sodium carbonate and dried out the resistance even when dry was too low. Finally, the method was adopted of grinding the glass before deposition of the platinum films. In this case breathing on the surface gave a very rapid fall and rise in resistance as the moisture was deposited and evaporated. When, however, the test tube was slowly cooled down to the dew point, apparently no very sharp change of resistance occurred, and consequently it was difficult to detect the point at which the dew appeared. One set of observations are shown in Fig. 11.*

The true dew point obtained by independent methods was at a temperature of 7·7°C. The tube was cooled down to 4·8C. and then allowed to heat up. It will be noticed that the resistance temperature curve did not return on itself, but that the resistance remained practically constant till the temperature rose to 7·9°C. (just above the dew point), when a rise of resistance occurred which became increasingly rapid. In another experiment where the tube was cooled *just* below the dew point, the curves of rising and falling temperature were much closer.

The reason for the effect obtained with a ground glass surface is not clear; possibly slight chemical decomposition is produced giving traces of soluble salts.

5. THE HAIR HYGROMETER.

The two types of hygrometers already described require a certain amount of manipulation and computation before the value of the humidity can be ascertained.

This may not be of much consequence in laboratory work, but for every-day use a direct reading instrument of simple construction is much to be desired. A hygrometer possessing these characteristics is tolerable even if the accuracy of its indications is somewhat below that demanded in a laboratory instrument.

The hair hygrometer comes within this category. Besides the fact that it is direct reading, it has the additional advantage that it can be used at low temperatures, and so is applicable to cold storage work.

It utilises the principle that most substances consisting of organic tissue are hygroscopic, and change their length when they absorb or part with moisture.

Hair, free from grease, increases its length when moistened, and De Saussure's hygrometer works on this principle.

His original instrument has been greatly modified in mechanical detail since its invention, and modern forms of the instrument are illustrated in Figs. 12 and 13.

The hair hygrometer seems to have lost its reputation as a reliable instrument, but the evidence as to its unreliability is very meagre, and, so far as we have been able to ascertain, no serious scientific study of it has been made since the time of Regnault, three-quarters of a century ago.

For the purpose of this inquiry, five instruments were studied:—

One was of the form illustrated in Fig. 12, and will be referred to as type A.

Two were of the form shown in Fig. 13, which will be referred to as type B.

One was a recording hair hygrometer, while the other was an instrument which we had arranged for distant reading work (*see* Fig. 16).

---

* These observations were taken by Mr. F. H. Schofield, to whom our thanks are due.

It seemed desirable to have quantitative information on the following points, and experiments were made with this object in view :—

(1) The reliability of the instruments over a long interval of time when subject to moderate ranges of temperature and humidity (humidities from 45 per cent. to 95 per cent.).

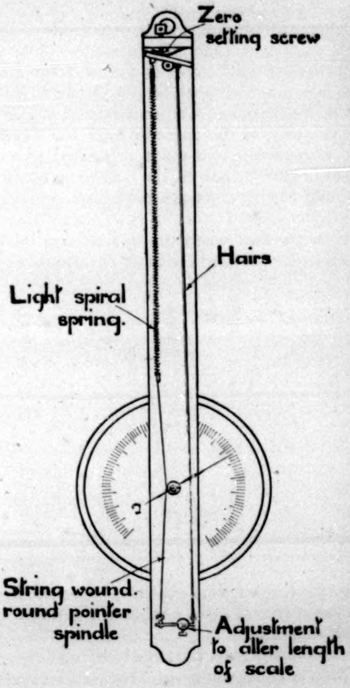
Zero setting screw
Hairs
Light spiral spring.
String wound round pointer spindle
Adjustment to alter length of scale

FIG. 12.

(2) The behaviour of the instrument when taken through a complete cycle of humidity changes such as from 7 per cent. to 90 per cent.

(3) The influence of temperatures below freezing point of water on the calibration curves.

(4) The effect of temperature changes above that of the room (for a nearly constant relative humidity of the atmosphere).

(a) *Reliability over Long Intervals of Time.*

Systematic observations were made over a period of three months at room temperatures (17°C.) and at temperatures about 0°C.

A range of humidity from 45 per cent. to 95 per cent. was covered, and it was considered that this was sufficient for the range of climatic changes met with in this country.

The dew-point apparatus was supplemented by the use of an Assmann psychrometer as standard of reference for obtaining the true humidity. A comparison of the data given in columns IV. and V. of Table IX. shows the close agreement between the results obtained with these two instruments, although fundamentally different in principle. The agreement is within 2 per cent. for practically all the observations taken at room temperature. The divergence at low temperatures is greater, and is probably due to the fact that the presence of the observer near the psychrometer disturbed the humidity conditions, whereas the dew-point apparatus was shielded by its box.

In the reduction of the observations on the hair hygrometers mean calibration curves have been drawn, and the divergencies of the individual readings from these curves are given.

A study of the data shows that when the instruments have settled down the divergencies are of the order of 4 per cent., with an occasional departure amounting to 10 per cent. The bigger divergencies are possibly due to the lag of the instrument in getting to equilibrium value after a rapid change of humidity, and they generally occurred in all three simultaneously.

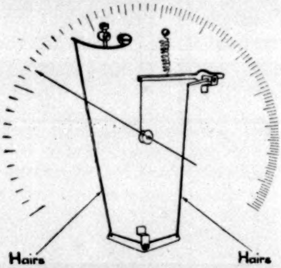

FIG. 13.

At low temperatures it was observed that the hair type of instrument was slow in settling to its final reading, particularly when it was taken from room temperature (15°C.) down to 0°C. For instance, when the instruments were taken into the cold stores it required 2½ hours for the readings to settle to their equilibrium values.

It is, however, somewhat surprising that the calibration at room temperatures holds so satisfactorily at lower temperatures, for the absolute weight of water vapour per unit volume of the atmosphere for a given humidity varies considerably. For example, 85 per cent. humidity at +15°C. corresponds to 11 milligrammes of water per litre, whilst at 0°C. it corresponds to only 4 milligrammes of water per litre.

It would appear that hair behaves somewhat similarly to flannel, for which the moisture absorption depends on the relative humidity only, and varies but little with temperature.

TABLE IX.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Date.	Time.	Room temperature.	Humidity by psychrometer.	Humidity by Dew-point apparatus.	Mean value of humidity.	Correction to mean calibration curve.			
						Hair hygrometer Type A.	Hair hygrometer. Type B.		Recording hair hygrometer.
							No. 1.	No. 2.	
Feb. 5	12 p.m.	14°	55	55	55	0	...	...	—3
,, 7	4 ,,	13°	52	53	53	0	...	...	—2
,, 16	1 ,,	15°	62	62	62	+9	...	...	+1
,, 17	1 ,,	16°	65	...	95	+11	...	...	+4
,, 18	...	17°	95	95	95	+11	...	...	+7
,, 19	10 a.m.	10°	73	74	74	+6	...	...	+5
,, 21	4 p.m.	15°	62	62	62	+6	...	...	+2
,, 22	11 a.m.	11°	69	69	69	+4	...	...	0
,, 22	3 p.m.	15°	63	62	63	+6	+5	...	+3
,, 24	3 p.m.	15°	66	68	67	+2	+2	...	...
,, 26	11 ,,	9°	63	64	64	0	0	...	+3
,, 28	5 ,,	9°	56	57	57	0	0	...	+1
Mar. 2	12 ,,	6°	52	50	51	—2	0	...	0
,, 3	12 ,,	6°	49	47	48	—1	—2	...	—1
,, 3	5 p.m.	7°	49	48	48	—1	—2	...	—1
,, 4	12 ,,	12°	46	46	46	0	—1	...	—1
,, 4	5 ,,	13°	46	46	46	0	—1	...	0
,, 5	12 ,,	14°	45	44	44	+2	+1	...	0
,, 7	12 ,,	12°	45	45	45	+1	0	...	0
,, 7	5 p.m.	13°	44	44	44	+1	0	...	0
,, 8	12 ,,	9°	46	44	45	0	—1	...	0
,, 8	5 ,,	7°	47	47	47	+1	—1	...	0
,, 9	5 ,,	4°	52	51	51	+2	+2	...	+1
,, 10	12 ,,	5°	47	47	47	+1	0	...	0
,, 10	5 p.m.	4°	48	47	47	+1	0	...	—5
,, 11	12 ,,	4°	52	51	51	—3	—4	...	—5
,, 11	5 ,,	0°	55	52	53	—4	—5	...	—7
,, 12	12 ,,	0°	54	51	52	—7	—8	—4	—10
,, 12	4 p.m.	0°	89	89	89	0	—2	—2	—2
,, 14	12 ,,	10°	83	84	83	+2	+2	+1	+2
,, 14	12 ,,	12°	79	79	79	+2	+2	+2	0
,, 15	12 ,,	12°	72	72	72	+2	+1	+1	+1
,, 15	5 ,,	13°	73	73	73	+3	+1	+2	+1
,, 16	12 p.m.	12°	72	74	73	+3	+1	+2	+1
,, 16	5 ,,	14°	71	71	71	+3	+1	+1	0
,, 17	12 ,,	12°	71	71	71	+2	+1	+1	0
,, 17	5 ,,	14°	84.	84	84	0	0	0	—1
,, 18	12 ,,	14°	79	80	79	—1	0	+1	0

(*Continued on page* xxxiv.)

TABLE IX.—*Continued.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Date.	Time.	Room temperature.	Humidity by psychrometer.	Humidity by Dew-point apparatus.	Mean value of humidity.	Correction to mean calibration curve			
						Hair hygrometer Type A.	Hair hygrometer. Type B.		Recording hair hygrometer.
							No. 1.	No. 2.	
Mar. 18	5 p.m.	15°	88	87	87	+1	+1	+1	—1
„ 19	12 „	15°	87	87	87	+3	+3	+33	+3
„ 21	5 „	15°	80	80	80	+1	0	0	0
„ 22	12 „	16°	82	80	81	0	—1	0	—1
„ 22	5 „	15°	68	67	67	+1	—1	+2	0
„ 31	5 p.m.	—1°	63	61	62	—5	—6	—3	—7
Apr. 1	12 „	0°	78	75	76	—3	—5	—4	—7
„ 1	5 „	0°	92	90	91	0	—1	0	—3
„ 2	11 a.m.	—1°	87	82	84	—4	—6	—4	—7
„ 2	12 p.m.	—2°	72	69	70	—3	—2	—1	—3
„ 11	12 p.m.	15°	71	68	69	—2	—3	—2	—2
„ 14	5 „	14°	66	...	66	—1	—4	—2	—2
„ 20	5 „	12°	62	60	61	—1	—4	—2	—3
„ 21	12 „	12°	63	62	62	—2	—4	—2	—3
„ 21	5 „	14°	60	57	58	—2	—5	—3	—4
„ 22	5 „	15°	67	66	66	0	—3	—1	—3

A study of the table brings out the interesting fact that the hair hygrometer tends to read higher as time goes on, as is evidenced by the negative sign to the corrections during the latter period of the comparisons. Hence the hair appears to take up a permanent stretch.

In these experiments there were three variables : time, temperature and humidity. The main object of the experiments was to determine the magnitude of the changes to be anticipated for such fluctuations in these variables as would be encountered in practice, leaving for separate experiment the determination of the effect of temperature alone, etc.

It is interesting, however, to note that the results summarised in Table IX. clearly indicate that at lower temperatures the instruments read high, and this is in agreement with subsequent observations when temperature and humidity were the only variables.

(*b*) *Behaviour of the Instrument when Taken through a Complete Cycle of Humidities.*

For this part of the inquiry a more expeditious method than that adopted by Glaisher had to be found. In his comparison of the dry and wet bulb hygrometer with Daniell's hygrometer he obtained his high temperature points in India, his low and medium temperatures in Toronto, and the remaining odd thousand or so observations at Greenwich during the period 1841 to 1854.

The apparatus illustrated in Fig. 1 was employed for this part of the investigation, and by means of it a wide range of humidity could be covered in a comparatively short interval of time.

TABLE X.

Experiment No.	Date.	Time.	Room temperature.	Humidity by dew-point apparatus.	Correction to mean calibration curve.		
					Hair hygrometer Type A.	Hair hygrometer Type B. No. 1.	Hair hygrometer Type B. No. 2.
1	May 7th	12.45 p.m.	16°	21	— 5	—10	— 9
2	,, 7th	3.0 p.m.	19°	7	+ 2	— 7	— 5
3	,, 7th	3.15 p.m.	18°	8	+ 3	— 7	— 5
4	,, 7th	...	18°	47	— 6	—99	— 7
5	,, 9th	9.45 a.m.	13°	57	— 7	—12	— 9
6	,, 9th	2.30 p.m.	19°	31	— 8	—11	— 9
7	,, 9th	4.45 p.m.	19°	77	— 5	—10	— 7
8	,, 9th	4.50 p.m.	19°	76	— 4	— 9	— 6
9	,, 10th	10.30 a.m.	14°	62	— 1	— 5	— 3
10	,, 10th	11.0 a.m.	15°	51	— 1	— 5	— 4
11	,, 10th	11.30 a.m.	17°	35	— 2	— 5	— 4
12	,, 10th	12.30 p.m.	19°	23	— 3	— 7	— 6
13	,, 10th	2.30 p.m.	18°	45	— 2	— 4	— 4
14	,, 10th	3.0 p.m.	15°	50	— 4	— 7	— 7
15	,, 10th	4.0 p.m.	18°	63	— 5	—11	— 8
16	,, 10th	5.15 p.m.	19°	81	— 6	—10	— 8
17	,, 10th	5.35 p.m.	19°	83	— 4	— 8	— 6
18	,, 10th	9.15 p.m.	13°	72	— 3	— 7	—11
19	,, 13th	2.25 p.m.	20°	88	— 7	— 8	— 7
20	,, 13th	3.15 p.m.	20°	77	— 2	— 6	— 6
21	,, 13th	3.45 p.m.	20°	52	+ 1	— 2	— 2
22	,, 13th	4.30 p.m.	21°	30	+ 2	— 5	— 5
23	,, 13th	5.0 p.m.	21°	50	— 2	— 6	— 6
24	,, 13th	5.25 p.m.	21°	79	— 5	—10	— 9
25	,, 14th	9.45 a.m.	15°	83	— 5	—12	—10
26	,, 14th	10.10 a.m.	15°	85	— 5	—11	—10
27	,, 14th	10.35 a.m.	17°	70	— 3	— 7	— 6
28	,, 14th	11.20 a.m.	19°	52	+ 2	— 7	— 6
29	,, 14th	12.0 noon	20°	36	+ 3	— 8	— 7
30	,, 14th	12.30 p.m.	21°	50	+ 1	— 6	— 5
31	,, 17th	9.30 a.m.	13°	73	— 5	—12	— 9
32	,, 19th	11.5 a.m.	18°	89	— 3	—10	— 8
33	,, 19th	11.35 a.m.	18°	67	—11	— 7	— 5
34	,, 19th	12.5 p.m.	19°	45	— 1	— 6	— 5
35	,, 19th	12.30 p.m.	19°	35	— 2	— 6	— 5
36	,, 19th	1.15 p.m.	20°	87	— 3	—10	— 9

The following low temperature readings were taken in Smithfield Cold Stores.

Experiment No.	Date.	Time.	Room temperature.	Humidity by dew-point apparatus.	Hair hygrometer Type A.	Hair hygrometer Type B. No. 1.	Hair hygrometer Type B. No. 2.
37	May 20th	3.45 p.m.	—6°	67	—14	—23	—16
38	,, 20th	3.50 p.m.	—6°	67	—12	—22	—16
39	,, 20th	4.10 p.m.	—6°	49	—13	—22	—18
40	,, 20th	4.15 p.m.	—6°	52	—13	—20	—15
41	,, 20th	5.10 p.m.	—5°	31	—11	—13	—13
42	,, 20th	5.20 p.m.	—5°	32	—13	—14	—14
43	,, 20th	6.10 p.m.	—5°	21	—11	—13	—13
44	,, 21st	8.20 a.m.	—9°	28	—15	—17	—17
45	,, 21st	8.25 a.m.	—9°	27	—15	—17	—17
46	,, 21st	9.10 a.m.	—7°	15	—12	—16	—10
47	,, 21st	9.50 a.m.	—5°	30	—12	—13	—14
48	,, 21st	10.0 a.m.	—5°	31	—12	—13	—14
49	,, 21st	10.45 a.m.	—6°	48	—16	—21	—18
50	,, 21st	11.30 a.m.	—5°	60	—12	—20	—15
51	,, 21st	11.35 a.m.	—5°	61	—12	—20	—15

(*Continued on page* **xxxvi.**)

TABLE X.—*Continued.*

The following low temperature readings were taken in Smithfield Cold Stores—(Continued).

Experiment No.	Date.	Time.	Room temperature.	Humidity by dew-point apparatus.	Correction to mean calibration curve.		
					Hair hygrometer Type A.	Hair hygrometer Type B.	
						No. 1.	No. 2.
52	May 21st	12.0 noon	—6°	59	—14	—19	—14
53	,, 23rd	9.30 a.m.	—7°	87	— 9	—18	—14
54	,, 23rd	9.35 a.m.	—7°	86	— 9	—10	—13
55	,, 23rd	10.25 a.m.	—5°	49	—11	—17	—13
56	,, 23rd	10.30 a.m.	—5°	50	—11	—16	—13
57	,, 23rd	11.10 a.m.	—4°	37	—14	—16	—14
58	,, 23rd	11.15 a.m.	—4°	38	—12	—16	—14
59	,, 23rd	11.50 a.m.	—3°	21	—11	—14	—13
60	,, 23rd	1.45 p.m.	—6°	20	—13	—15	—14
61	,, 23rd	2.30 p.m.	—4°	34	—14	—14	—14
62	,, 23rd	2.40 p.m.	—4°	35	—13	—14	—13
63	,, 23rd	3.30 p.m.	—5°	54	—13	—19	—16
64	,, 23rd	3.35 p.m.	—5°	54	—13	—19	—15
65	,, 23rd	4.20 p.m.	—5°	65	—15	—24	—18
66	,, 23rd	4.50 p.m.	—5°	75	—14	—24	—18
67	,, 25th	9.0 a.m.	—9°	65	—13	—21	—16

Experiments at Room Temperature.

68	May 25th	5.0 p.m.	+21°	84	— 5	— 9	— 7
69	,, 25th	5.20 p.m.	+22°	65	— 3	— 7	— 6
70	,, 26th	10.20 a.m.	+20°	47	— 3	— 8	— 7
71	,, 26th	11.10 a.m.	+22°	39	— 4	— 7	— 8
72	,, 26th	12.0 noon	+23°	26	— 2	— 6	— 7
73	,, 26th	12.45 p.m.	+23°	37	— 4	— 0	— 8
74	,, 26th	2.30 p.m.	+21°	49	— 8	—12	—12
75	,, 26th	3.25 p.m.	+23°	64	— 8	—14	—14
76	,, 26th	4.10 p.m.	+22°	76	— 8	—16	—13
77	,, 26th	4.55 p.m.	+21°	78	— 8	—16	—13
78	,, 27th	10.15 a.m.	16°	62	— 7	—14	—12
79	,, 27th	11.10 a.m.	20°	82	— 6	—14	—12
80	,, 27th	12.30 p.m.	21°	62	— 4	— 9	— 9
81	,, 27th	2.20 p.m.	22°	54	— 5	—11	— 9
82	,, 27th	3.10 p.m.	22°	44	— 4	— 8	— 9
83	,, 27th	4.5 p.m.	22°	28	— 4	— 9	— 9
84	,, 27th	4.50 p.m.	22°	45	— 5	— 9	— 9
85	,, 27th	5.15 p.m.	22°	60	—10	—17	—13
86	,, 28th	10.15 a.m.	19°	88	— 3	—10	— 9
87	June 1st	9.45 a.m.	16°	65	— 5	—13	—11
88	,, 1st	10.15 a.m.	18°	52	— 5	—11	— 9
89	,, 1st	11.10 a.m.	20°	46	— 4	—10	— 9
90	,, 1st	11.50 a.m.	21°	31	— 4	— 9	— 9
91	,, 1st	12.30 p.m.	21°	42	— 5	—10	— 8
92	,, 2nd	9.50 a.m.	17°	55	— 6	—13	—12
93	,, 2nd	10.20 a.m.	19°	65	— 8	—17	—14
94	,, 2nd	11.0 a.m.	19°	81	— 5	—13	—11
95	,, 2nd	12.0 noon	22°	61	— 3	—10	—12
96	,, 2nd	12.50 p.m.	23°	45	— 3	— 9	— 9
97	,, 2nd	2.50 p.m.	23°	31	— 3	—10	—10
98	,, 2nd	4.0 p.m.	24°	26	— 4	— 9	—10
99	,, 2nd	5.0 p.m.	25°	45	— 5	—11	—11
100	,, 3rd	9.30 a.m.	18°	55	— 5	—13	—12
101	,, 3rd	10.10 a.m.	19°	67	— 7	—16	—15
102	,, 3rd	10.35 a.m.	20°	75	— 7	—17	—14
103	,, 4th	...	16°	57	— 6	—14	—13

When the hair hygrometers were placed in the apparatus, and subjected to very low humidities of the order of 5 per cent. a permanent change occurred and the calibration curve shifted in a direction which indicated that the hairs had stretched permanently. The same change, but not to the same extent, occurred in all the hair hygrometers tested.

One of the curves for an instrument type B are reproduced in Fig. 14. The change in the instrument type A (illustrated in Fig. 12) was 3 to 4 per cent. throughout the scale. The calibration curves of the two instruments of type B (illustrated in Fig. 13) changed by 5 and 8 per cent. respectively.

The calibration points repeated themselves, and there was no evidence of appreciable hysteresis effect. The data for these experiments are summarised in Table X. (May 7th to 19th).

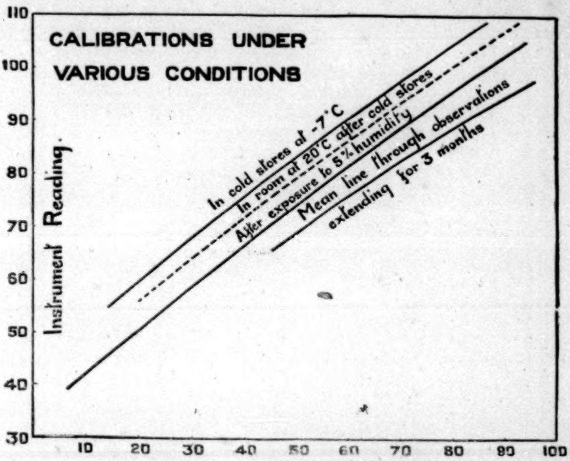

FIG. 14.

(c) Calibration at Low Temperatures.

The next series were made in a cold store at a temperature of about −7°C., the same apparatus being employed. This was then taken to the Central Meat Markets Store at Smithfield, and the calibration repeated at low temperatures.

Comparing the results obtained in the cold stores with the previous ones at room temperature it will be seen that the calibration curve at low temperature for type A has shifted by 8 to 9 per cent.; for type B, No. 1, 10 to 13 per cent., and for No. 2 9 per cent.

On the return of the apparatus to the laboratory the experiments were repeated at room temperature. The results show that instrument type A repeated its original room temperature calibration within the limits of the error of experiment. But a permanent alteration had occurred on exposure to low temperatures with instruments type B resulting in their calibration curves being displaced (*see* Fig. 14). These results indicate that subjection to extremes of temperature or humidity causes a permanent "set," but for moderate variations of the conditions the instruments behave fairly satisfactorily.

So far as we could ascertain the only difference between types A and B was that the hairs in B were under greater tension than in A.

(*d*) *Influence of Change of Temperature from* 16°C. *up to* +38°C. (60°F. *to* 100°F.).

In these experiments the temperature was varied in steps up to 38°C., while the indications of the instruments were kept at a fixed value by adjusting the moisture content of the atmosphere. The corresponding humidity was determined by the dew-point apparatus. The data obtained are summarised in Table XI., and graphically in Fig. 15. The first exposure of the hairs to higher temperatures on June 8th gave results which indicate a slight contraction of the hairs with increasing temperatures. That the change is in this direction will be seen by consideration of the fact that the humidity of the surrounding atmosphere had to be raised as the temperature was raised, and the effect of an increase in the humidity of the surrounding atmosphere is to cause a lengthening of the hairs. In the present experiment the lengthening due to increasing the humidity counteracted the contraction due to temperature.

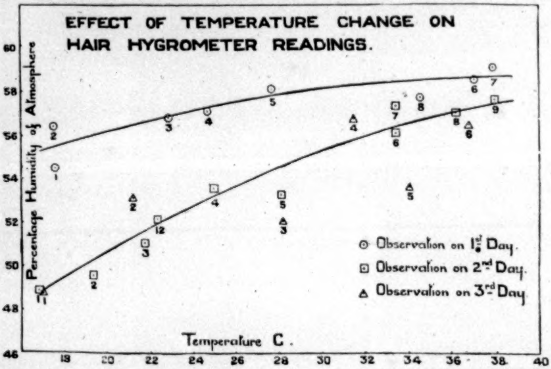

FIG. 15.

When repeating the observations the next day, June 9th, the surprising result was obtained that the hairs had taken a permanent set overnight in the direction of expansion; the change amounting to 6 per cent. since the period of exposure to the higher temperatures. It now required only 49 per cent. relative humidity of the atmosphere at a temperature of 16°C. to give the same instrument reading as on the previous day was equivalent to 55 per cent. relative humidity of the atmosphere. The results on this day and the succeeding day show a well-marked coefficient of ***contraction*** with temperature, the magnitude of which is 0·4 per cent. relative humidity per 1°C. It is interesting to note that the *expansion* on exposure to cold stores' temperature works out to be practically the same in numerical value as the contraction on exposure to higher temperatures—*i.e.*, an average change of 10 per cent. in the humidity scale for a lowering of 25 deg. in temperature. This gives a coefficient of 0·4 per cent. relative humidity per 1°C.

TABLE XI.—*Influence of Temperature on Indications of Hair Hygrometers.*
(Humidity of atmosphere varied to maintain instrument readings at steady value.)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	
No. of Experiment.	Temperature of atmosphere.	Dew Forms.	Dew disappears.	Dew-point temperature.	Humidity.	Hair Hygrometer. Type A.	Hair Hygrometer. Type B.	
							No. 1.	No. 2.
June 8, 1921.								
1	17·6	8·0	8·5	8·2	54·3	63·5	77·5	**85·1**
2	17·6	8·6	8·9	8·7	56·2	64·5	78·0	**86·0**
3	22·8	13·6	13·7	13·6	56·5	63·5	77·5	**85·1**
4	24·7	15·3	15·7	15·5	56·8	63·5	77·3	**85·1**
5	27·7	18·4	18·7	18·6	57·8	63·5	77·5	**85·0**
6	37·2	27·6	27·8	27·7	58·5	63·5	77·5	**85·2**
7	37·9	28·5	28·6	28·5	58·9	64·5	77·5	**86·0**
8	34·6	25·0	25·1	25·0	57·5	63·5	77·0	**85·6**
June 9, 1921.								
1	16·7	5·5	6·2	5·8	48·8	63·5	77·8	**85·8**
2	19·4	8·5	9·2	8·8	49·5	63·4	77·7	**85·5**
3	21·8	10·8	11·5	11·1	50·9	63·5	77·8	**85·6**
4	25·0	14·6	15·0	14·8	53·4	63·5	77·9	**85·6**
5	28·2	17·7	17·8	17·7	53·2	63·5	77·7	**85·5**
6	33·4	23·4	23·5	23·4	55·9	63·5	77·5	**85·4**
7	33·4	23·8	23·8	23·8	57·1	63·5	77·2	**85·2**
8	36·3	26·3	26·5	26·4	56·7	63·5	77·7	**86·0**
9	38·2	28·2	28·4	28·3	57·4	63·9	76·3	**84·5**
	The following three observations were taken directly after cooling down.							
10	24·2	14·8	15·0	14·9	56·3	63·6	77·8	**85·6**
11	22·9	12·7	13·0	12·8	56·4	63·5	76·8	**85·0**
12	21·7	11·2	11·6	11·4	52·1	63·5	77·2	**85·3**
June 10, 1921.								
1	16·5	5·3	6·0	5·6	48·7	63·5	77·5	
2	22·2	11·1	11·3	11·2	49·9	63·5	77·5	
3	28·3	17·3	17·5	17·4	51·8	63·5	77·2	
4	31·5	21·7	22·0	21·8	56·6	63·5	77·5	
5	35·1	24·3	24·3	24·3	53·6	63·6	77·5	
6	36·8	26·5	26·9	26·7	56·3	63·5	77·3	

The experiments show that the instrument has a true thermal contraction or expansion with increasing or decreasing temperatures.

The most serious defect of this type of hygrometer is the permanent changes on exposure for the first time to big fluctuations of temperature or humidity resulting in elongation of the hairs. It is very curious that the change should always occur in this direction, and it may possibly be due to the fact that the hairs are always under tension by the control springs.

(*e*) *Accuracy of the Engraved Scale Supplied with the Instruments.*

It was somewhat disconcerting to find the scales supplied with the instruments of types A and B to be so much in error. Even when the pointers were adjusted to read correctly at one point of the scale the divergence at the extreme end amounted to as much as 30 per cent. (*See* Table XI.)

This wide discrepancy was traced to the fact that the maker had attempted to obtain the scale by comparison with a wet and dry bulb thermometer of the non-ventilated type.

Recently we obtained from another source an instrument somewhat similar to type A (Fig. 12), but with gravity instead of spring control. On testing this instrument at room temperature over the range 50 to 100 per cent. humidity, the maximum error only amounted to 3 per cent.

(*f*) *The Relation between the Extension of the Hairs and the Relative Humidity of the Atmosphere.*

The relation between the linear expansion of the hairs and the relative humidity is given in Table XII.

TABLE XII.—*Relation between Extension of Hairs and Relative Humidity of Atmosphere.*

Relative humidity of atmosphere.	Percentage extension of hairs.
10	0·40
20	0·67
30	0·91
40	1·11
50	1·29
60	1·45
70	1·58
80	1·70
90	1·81

These data have been obtained from the measurement of the lever system of one of the instruments (type B) and the experimental data connecting the angular deflection of the pointer with the relative humidity.

In view of the complex structure of the hair it is hardly to be expected that the same relationship will rigorously apply to all instruments of this type.

(*g*) *Summary of Conclusions Concerning Hair Hygrometers.*

From the results of the experiments one arrives at the conclusion that, under normal working conditions and subjected to ordinary variations of humidity and temperature, the hair hygrometer shows a tendency to give high readings with lapse of time. This may be due to the fact that the hairs are always under tension, and consequently a permanent set takes place.

In our experience the instruments which behave most satisfactorily are those in which the tension force is small and whose bearings and pivots are so well made that friction is a minimum. There is no doubt that good performance from the hair hygrometer can only be obtained when the design of the instrument is such that the hairs are safeguarded from any undue strain.

The instrument should not be subjected to large variations of temperature, and the calibration should be checked at frequent intervals under conditions identical with those under which it is used.

The presence of oil vapour in the atmosphere and of dust is stated to reduce with time the sensitivity of the hairs by preventing free access of the aqueous vapour to the surface, and precautions should therefore be taken to eliminate these disturbances as much as possible.

(*h*) *Distant Reading Form of Hair Hygrometer.*

The hair hygrometer is the simplest of all instruments to convert to a distant reading type. We constructed a distant reading instrument of the form shown in Fig. 16. It is a standard form of recording hair hygrometer with the clockwork drum removed and replaced by a stationary drum wound with oxidised wire giving a total resistance of 1,000 ohms. A longitudinal strip is cleaned on the surface of the drum so that electrical contact can be effected with the wire. The pointer has a silver contact piece instead of the pen. Above the pointer is fixed a " chopper bar " arrangement operated by an electro-magnet. When a reading of the humidity is desired the observer switches on the electro-magnet which brings the pointer into contact with the drum immediately beneath the spot where it is situated.

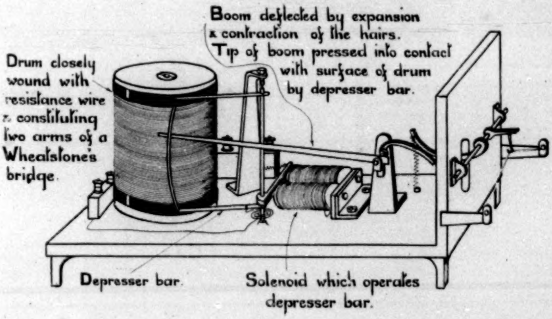
Boom deflected by expansion & contraction of the hairs. Tip of boom pressed into contact with surface of drum by depresser bar.
Drum closely wound with resistance wire & constituting two arms of a Wheatstone's bridge.
Depresser bar.
Solenoid which operates depresser bar.

FIG. 16.

The resistance drum then constitutes the two arms of a Wheatstone's bridge and the pointer is the contact to the galvanometer circuit ; the other galvanometer contact is on a similar drum at the observation station. The observer balances the bridge in the usual manner and thus determines electrically the position of the pointer on the drum which is situated at the hair hygrometer.

The behaviour of the instrument was found to be quite satisfactory over a long period of test, but it suffers, of course, from all the inherent defects associated with the use of hair as a fundamental element in the design of a physical instrument.

6. Thermal Effect on Cotton Hygrometer.

Orme Masson* in 1904 investigated the change in temperature produced in dry cotton wool when exposed to a damp atmosphere. He employed a mercury thermometer with about 1 gram of cotton wool wound around the bulb and observed the temperature rise when the instrument was withdrawn from a desiccator to a humid atmosphere.

His experiments showed that the heat production was proportional to the amount of hygroscopic absorption. He suggested the possibility of utilising the phenomenon as a method of measuring humidity, but did not give this aspect of the subject detailed investigation.

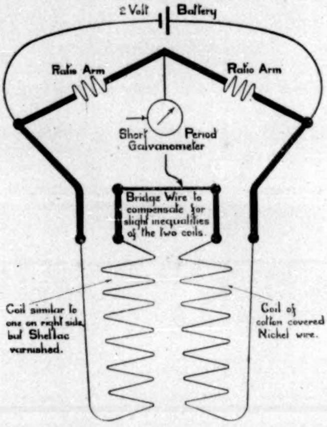
2 Volt Battery
Ratio Arm
Ratio Arm
Short Period Galvanometer
Bridge Wire to compensate for slight inequalities of the two coils.
Coil similar to one on right side, but Shellac varnished.
Coil of cotton covered Nickel wire.

FIG. 17.

Drs. Chattock and Tyndall modified Masson's arrangement by employing thermo-junctions instead of mercury thermometers. By this device they were able to reduce the time of exposure required for attaining the maximum temperature to 15 seconds and the period of drying out to half an hour. In Masson's experiments the maximum temperature was not attained below a period of about 5 minutes and the drying out occupied about 24 hours.

Drs. Chattock and Tyndall found that the "constant" of their instrument varied in an inexplicable manner and the changes were of such a magnitude as to render the instrument of little value as a hygrometer. They brought the subject before the Engineering Committee of the Food Investigation Board, who requested that the matter should also be investigated further at the Laboratory as the method seemed to have possibilities as the basis of the design of a distant-reading hygrometer.

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* Proc. Roy. Soc., Vol. LXXIV., p. 230 (1904). Masson and Richards, Proc. Roy. Soc., Vol. LXXVIII., p. 412 (1906).

(*b*) *Electrical Resistance Type.*

After consideration of the magnitude of the temperature changes to be observed we decided to abandon the use of thermo-junctions in favour of a resistance thermometer method. This had the additional advantage that the absorbent cotton was distributed evenly over a length of wire instead of being concentrated in a blob around the thermo-junction.

Accordingly a differential resistance thermometer was made of cotton covered nickel wire wound on a large ebonite rack similar to a resistance thermometer. On one of the coils the cotton covering was left in its natural state, whilst that on the other was shellac varnished. These coils were connected up in the form of a Wheatstone's bridge and the arrangement of the coils is shown in Fig. 17. Each coil was composed of about 15 feet of wire of approximately 22 ohms resistance. The mean temperature coefficient of resistance between 0°C. and 100°C. was 0·00516 per 1°C.

The function of the varnished coil is to compensate for changes of resistance of the cotton covered coil due to changes of room temperature, so that the bridge is balanced at all temperatures. If the coils are placed in a dry atmosphere, then on exposure to a humid atmosphere a large deflection of the galvanometer spot is obtained, due to the ready absorption of water vapour by the untreated cotton on the one coil.

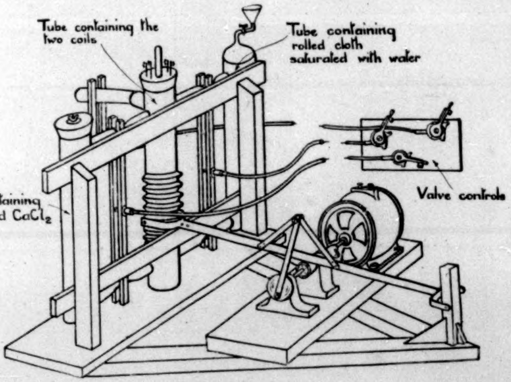
Tube containing the two coils
Tube containing rolled cloth saturated with water
Valve controls
Tube containing granulated $CaCl_2$

FIG. 18.

The thermal effect on absorption of water vapour occurs with extraordinary rapidity, the maximum deflection being attained within 6 seconds of the exposure. In order to slow down the effect six additional layers of cotton were wound on the wire, bringing up the external diameter to approximately one millimetre, the diameter of the nickel wire being only one tenth of a millimetre. This increased the time taken to reach a maximum to about 12 seconds.

With the means at our disposal it was not possible to wind the additional layers of thin cotton so closely and well spaced as to give a perfectly even surface, and consequently it presented a slightly corrugated appearance. This may be an advantage, as it helps to expose more surface to the atmosphere. Having satisfied ourselves that the instrument would work at temperatures below 0°C. experiments were made at room temperatures so as to obtain some information as to the characteristics of the phenomenon.

In the early experiments the coil was carried from a rubber stopper which could be inserted into wide mouthed bottles containing a dry or a saturated atmosphere. The coil when transferred from the dry to the saturated atmosphere would give consistent readings, but these were quite valueless owing to the fact that the coil absorbed the moisture in its vicinity, and the effect of such absorption was to upset completely the hygrometric conditions of the atmosphere in the vessel.

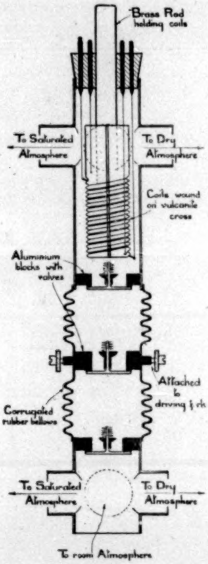
Brass Rod holding coils
To Saturated Atmosphere
To Dry Atmosphere
Coils wound on vulcanite cross
Aluminium blocks with valves
Attached to driving jack
Corrugated rubber bellows
To Saturated Atmosphere
To Dry Atmosphere
To room Atmosphere

FIG. 19.

(*c*) *Apparatus utilising the Effect for Hygrometric Measurements.*

To overcome these difficulties the form of apparatus, shown diagrammatically in Fig. 18, was devised. The coil is contained in the central tube and past it is pumped either dry air from the tower on the left, saturated air from the tower on the right, or air from the room through an inlet and outlet in front. The pumping device is a corrugated rubber tube made from a motor tyre and it contains three disc valves of large diameter as shown in Fig. 19. This method of pumping was adopted in preference to the usual type of piston in order to avoid the necessity of introducing any lubricant between a piston and cylinder which might be carried on to the coil by the air stream, also all generation of heat by friction between moving parts is eliminated. Connections between the towers are made by 1¾-inch bore rubber tubing and both inlet and outlet to a tower can be closed by one movement of a spring-operated crossbar. The general features of the apparatus will be understood from the diagrams. The drying tower contains granulated calcium chloride, while the saturation tower is filled with a roll of brass gauze and muslin interwoven, which is periodically saturated with water.

The procedure in taking the observations is as follows: The tower containing the coil is placed in communication with the drying tower and air pumped around for a few minutes. The inlet and outlet to the atmosphere of the room is then opened quickly and the drying tower cut off. A typical time-deflection curve for the galvanometer spot is shown in Fig. 20, the maximum deflection corresponding to a temperature change of 2·7°C. When the galvanometer spot has returned to nearly zero the communication with the saturation tower is quickly opened and that to the atmosphere of the room closed. A similar form of curve is obtained.

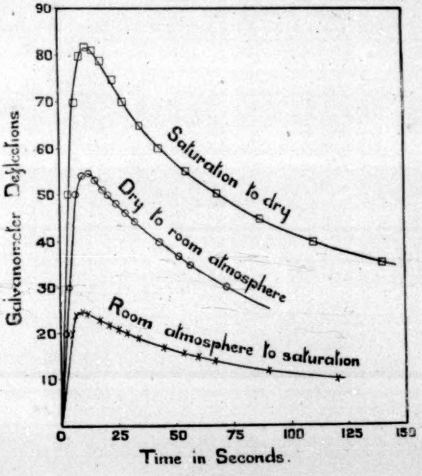

FIG. 20.

Finally the saturation tower is cut off and communication made to the dry tower. The deflection will now be in the opposite direction since the cotton is being subjected to a dry atmosphere following a saturated one, and is consequently liberating moisture. Any one of these operations can be carried out in the reverse direction with practically identical results.

Since the natural period of the moving coil system of the galvanometer is a fifth of a second, it closely follows the change of resistance of the coil. It will be observed that the sum of the maximum deflections obtained on exposure from " dry to room " and " room to saturation " successively is nearly equal to the maximum deflection obtained on going direct from " saturation " to " dry."

The direct transfer gives a slightly larger deflection, owing presumably to the smaller loss of heat during the time it is rising to a maximum. The actual magnitude of the deflection obtained depends upon several factors besides the relative humidity, such as the temperature of the air, velocity of the circulating air stream, galvanometer sensitivity, and the battery current. It was found, however, that the percentage deflection, *i.e.*, ratio of {(Dry to Room)/[Dry to Room+Room to Saturation]} was independent of all the above variables, and (within the limits of the errors of experiment) a function of the relative humidity alone (*see* Table XIII. and Fig. 21).

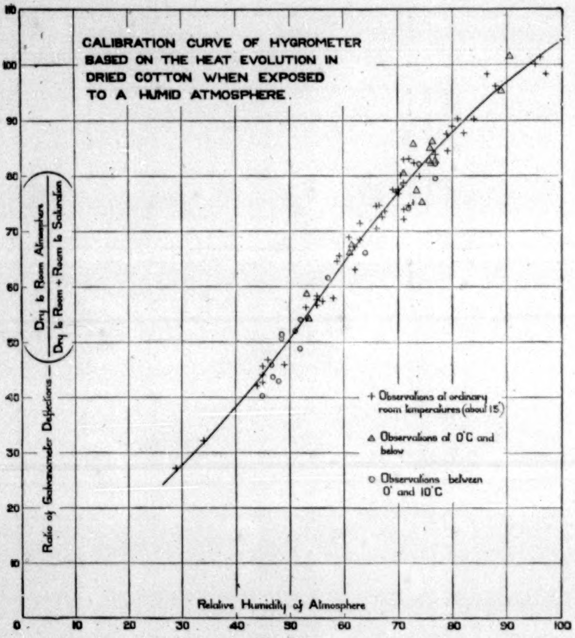

Fig. 21.

This procedure in taking the observations was found to be much more convenient

TABLE XIII.—*The Thermal (Cotton) Hygrometer.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Date.	Time.	Temp. room.	Humidity by psychrometer.	Humidity by dew-point apparatus.	Mean humidity.	Deflection :— Dry to room atmosphere.	Deflection :— Room atmosphere to saturation.	Sum of dry to atmosphere + atmosphere to saturation.	Deflection :— Saturation to dry.	Ratio :— Dry to atmosphere / Dry to atmosphere + atmosphere to saturation.	Difference from Mean curve through observations.
1920.											
Oct. 15	10.42 a.m.	15°	82	...	82	67·8	9·5	77·3	71·5	87·7	—2·3
„ 20	9.31 „	12°	68	...	68	56	...	...	82	...	...
„ 20	11.50 „	13·7°	63	...	63	54·5	25	79·5	82	68·5	0
„ 21	10.26 „	10·5°	72	...	72	53·8	18·8	72·6	75	74·1	—5·4
„ 25	4.44 p.m.	15·6°	70	72	71	52	20	72	71	72·2	—5·8
„ 25	9.15 a.m.	15°	57	59	58	40	29	69	71	58	—3·5
„ 26	10.54 „	17°	54	58	56	44	32·5	76·5	80	57·5	—1·5
„ 26	2.37 p.m.	23·5°	96	98	97	110	18	128	96	98·4	—4·1
„ 27	3.33 „	15°	69	71	70	56·5	17	73·5	71	76·9	—0·1
„ 28	11.40 a.m.	15°	65·5	66·5	66	48	20	68	69·8	70·6	—1·4
„ 28	4.34 p.m.	23°	100	100	100	110	—5	105	88	104·7	+0·2
„ 29	11.45 a.m.	14°	67	67	67	45·5	17·2	63	66·2	72·6	—0·9
„ 29	3.49 p.m.	16°	62	...	62	42·8	25	67·8	69·3	63·1	—3·9
„ 30	7.30 „	14°	55	55	55	40	30·5	70·5	68·8	56·7	—0·8
„ 31	6.30 „	12°	71	71	71	50·6	13·7	64·3	63·1	78·7	+0·7
Nov. 1	3.30 „	10°	79	75	77	50·4	13	63·4	59	79·5	—5·5
„ 1	8.30 „	9°	79	75	77	46·6	10·1	56·7	54·5	82·2	—2·8
„ 2	4.55 „	9°	...	46·7	...	23·2	27·3	50·5	54·7	45·9	—0·6
„ 3	12.30 „	13°	...	28·8	...	13·6	36·5	50	53·5	27·1	+0·1
„ 3	4.50 „	15°	...	34	...	19	40	59	62·5	32·2	+0·2
„ 6	11.30 a.m.	0°	76	73	74·5	22·8	7·5	30·3	31	75·3	—6·7
„ 6	12.45 p.m.	0°	75	72	73·5	24	7	31	29·7	77·4	—3·6
„ 9	3.0 „	—1°	75	79	77	24·3	5	29·3	29·4	82·9	—2·1
„ 15	2.30 „	0°	77	75	76	27·8	4·9	32·7	32·5	85	+1
„ 25	11.0 a.m.	—3°	78	74	76	24	5	29	26·5	82·8	—1·2
„ 27	10.30 „	—4°	74	72	73	20·8	3·5	24·3	24	85·6	+5·1
Dec. 2	12.30 p.m.	—1°	76	73	74·5	25·2	8	33·2	29·8	75·9	—6·1
„ 3	9.45 „	—1°	79	74	76·5	25	4	29	27·5	86·2	+2·2
„ 6	...	—2°	78	75	76·5	22·4	4·8	27·2	27	82·3	—1·7
1921.											
Jan. 24	...	15°	71·7	71	71	51	10·4	61·4	60	83	+5
„ 25	2.30 p.m.	17°	59	58·5	58·7	42·2	23	65·2	69	64·6	+1·6
„ 25	4.45 „	18°	54·4	55·5	55	39·5	29	68·5	69	57·7	+0·2
„ 26	2.30 „	18°	53	53	53	39·3	30·5	69·8	71·5	56·3	+1·8
„ 26	4.55 „	19°	45	45	45	29·2	39·4	68·6	71·8	42·6	—1·4
„ 27	12.30 „	17°	49	48	49	29	34	63	69	46	—3
„ 28	4.0 „	16°	75	73	74	53	13·3	66·3	66	80	—1·5
„ 29	12.0 a.m.	17°	77·5	76	77	57·5	11	68·5	67	83·9	—0·6
„ 31	4.20 p.m.	15°	73	71	72	49	16·7	65·7	63	74·6	—4·9
Feb. 1	4.55 „	13°	69·5	70	69·7	42·5	12·5	55	55·7	77·3	+0·3
„ 2	4.30 „	14°	71	71	71	46	14	60	58·4	74	—4
„ 5	12.35 „	14°	55	55	55	35	25	60	61·7	58·3	+0·8
„ 7	4.35 „	13°	52·5	53·5	53	32	27	59	61	54·2	—0·3
„ 16	1.10 „	16°	62	61·7	62	43	21	64	64·6	67·2	+0·2
„ 17	1.0 „	16°	96	...	96	73·5	—1	72·5	66·5	101·4	—0·1
„ 18	...	17°	95	95·4	95	74	0	74	68·5	100	—1
„ 19	10.30 a.m.	10°	73·5	74·4	74	41·2	9	50·2	51	82·1	+0·6
„ 22	11.30 „	12°	69	68·6	69	43	12·5	55·5	56·6	77·5	+1·5
„ 26	11.20 „	10°	63·5	64·1	64	33·2	17	50	49·5	66·1	—3·9
„ 28	5.0 p.m.	9°	56·5	57·3	57	29	18·1	47	49	61·6	+1·1
Mar. 2	12·40 „	6°	52·5	51	52	23	19·5	42·5	42·3	54·1	+0·6
„ 3	12·35 „	6°	49·5	47·5	48·5	21	20·5	41·5	42·5	50·6	+1·9
„ 3	5.0 „	7°	49	48	48·5	22	20·8	42·8	42	51·4	+2·7
„ 4	5·0 „	13°	46	46	46	24·3	27·5	51·8	52	46·9	+1·4
„ 5	12.30 „	14°	45	45	45	24	30·6	54·6	53·5	44	0
„ 7	12.30 „	12°	45	45	45	22·3	26·6	48·9	49·2	45·6	+1·6
„ 7	5.0 „	13°	44·1	44	44	22·5	31	53·5	51·2	42·1	—0·9
„ 8	12.45 „	9°	46	44	45	18·9	28	46·9	47	40·3	—3·7
„ 8	5.0 „	7°	47	47	47	18·2	23·4	41·6	41	43·8	—3
„ 9	5.0 „	4°	52·5	51	52	18·2	19	37·2	36·7	48·9	—4·5
„ 10	5.0 „	4°	48	48	48	16	21·2	37·2	37·5	52	0
„ 11	12.30 „	3°	52	50	51	18·5	17	35·5	34	54·4	—1
„ 11	5.0 „	0°	55	52	53·5	16·7	14	30·7	31·7	66·3	+0·3
„ 12	9.30 a.m.	—1°	64	58	61	17·7	9	26·7	28·7	58·8	+4·3
„ 12	12.45 p.m.	0°	54	52	53	16·4	11·5	27·9	29	95·3	—0·7
„ 12	4.0 „	0°	89	89	89	30·2	1·5	31·7	28·8	87·5	+0·5
„ 14	5.0 „	12°	79	79	79	53·8	7·7	61·5	56·4	83·1	+3·6
„ 15	12.30 „	12°	72	72	72	46	9·3	55·3	54·5	82·4	+1·9
„ 15	5.0 „	13°	73	73	73	47	10	57	56·5	81	+0·5
„ 16	12.40 „	12°	72	74	73	46·8	11	57·8	84·3	78·5	+0·5
„ 16	5.10 „	14°	71	71	71	49	13·4	62·4	59·6	80·2	+2·7
„ 17	12.40 „	12°	70·5	70·7	70·6	43·5	10·7	44·2	52	90·3	—1·4
„ 17	5.0 „	14°	84	84	84	56	60	116	57	84·5	—2·5
„ 18	12.35 „	14°	78·5	79·8	79·2	51·8	9·5	61·3	57·7	96·2	+1·2
„ 18	5.0 „	15°	88	87·4	87·7	63·5	2·5	66	64	98·3	+4·3
„ 19	12.35 „	15°	86·5	86·5	86·5	60	1	61	59·5	89	+1
„ 21	5.0 „	15°	80	80	80	55	6·8	61·8	58·5	90·4	+1·4
„ 22	12.30 „	16°	82	80	81	56·4	6	62·4	60	73·7	—0·3
„ 22	5.0 „	15°	68	67	67·5	45·8	16·3	62·1	61·5	67·5	+1
„ 31	4.50 „	—1°	63	60	61·5	22	10·6	32·6	30·4	93·9	+9·1
Apr. 1	12.30 „	0°	78	76	77	21·3	1·9	23·2	31·2	101·5	+4
„ 1	5.0 „	0°	92	89	90·5	33·7	—5	28·7	31·2	80·4	+2·4
„ 2	12.30 „	—2°	72	70	71	23	5·6	28·6	28·5	77·5	+0·5
„ 11	5.0 „	15°	71·5	68·5	70	48·3	14	62·3	63·2	74·6	+2·3
„ 14	5.0 „	14°	66	...	66	42·7	14·5	57·2	48·2	69	+3
„ 20	5.0 „	12°	62	60	61	36	16	52	52·2	71·5	+3
„ 21	12.30 „	12°	63·5	62·5	63	36·5	14·5	51	51·3	65·7	+2·7
„ 21	5.0 „	14°	60	58	59	36·5	19	55·5	56	74·6	+2·1
„ 22	5.0 „	15°	67	65·6	66·3	44	15	59	57		

[*To face page* xlvi.

than attempting to find a "constant" for the instrument, since it reduces the work to merely observing deflections of the galvanometer spot, and the instrument is self calibrating once the form of the curve connecting deflection ratios to true humidity has been determined.

Several features of the curves shown in Figs. 20 and 21 deserve attention. An analysis of the data in Fig. 20 indicates that the rate of cooling is not proportional to the temperature excess; the heat loss is very rapid initially, and then slows down considerably. In fact it seems that the cotton continues to absorb moisture, thus generating heat, for a considerable time after the maximum effect has been observed. The same applies to the drying out when exposed to air which has circulated through the $CaCl_2$ tower. Hence it is difficult to calculate the correction for the heat loss during the period the deflection is rising to a maximum. The problem is further complicated by the fact that the observed temperature is that of the wire core, and this may differ appreciably from that of the cotton surface in contact with the air stream. Cotton is a complex organic material concerning which very little is known.

It is probable that the form of the curve shown in Fig. 21 is determined to some extent by the mechanical details of the apparatus. In the ideal case one would expect that the quantity [(Dry to atmosphere)/(Dry to atmosphere + Atmosphere to saturation)] would be proportional to the relative humidity.

When the atmosphere of the room is near saturation one finds that the deflection obtained on exposure of the dry cotton to the atmosphere is equal to or slightly greater than that obtained when the dry cotton is exposed to the air circulated from the "saturation" tower. This is probably due to the fact that the air coming into contact with the resistance coil from the "saturation" tower is below room temperature. When the humidity of the room atmosphere is raised by blowing steam into the air, the temperature rises quite considerably, and it is to be expected that the temperature of the layer of muslin within the saturation tower would lag behind that of the room.

Further the action of the pumping arrangements tends to produce rarefaction of the air being circulating if there is any resistance to its flow. The conclusion one arrives at after an extended experience with the apparatus is that it is very difficult to obtain results with it when the humidity of the atmosphere is near saturation.

It is conceivable that by further improvements in construction these defects may be remedied.

7. THE HOT WIRE HYGROMETER.

Our attention has frequently been drawn to the possibility of applying the hot-wire method for measuring humidity.

Instruments utilising the fact that the thermal conductivity of the surrounding gas determines the heat loss from a wire, under appropriate conditions, have found extensive application as hydrogen and carbon dioxide recorders.

The method is an extraordinarily sensitive one for hydrogen-air mixtures since the thermal conductivity of hydrogen is nearly six times that of air. Koepsel* states that his apparatus is capable of easily detecting the presence of 0·001 per cent. of hydrogen without the use of a very refined measuring apparatus.

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* Ber. physik. Ges. **10**, p. 814 (1908).

During the war Dr. Shakespear* made a portable instrument termed the "Katharometer" for measuring the permeability of balloon fabrics to hydrogen, and his instrument has been applied to a variety of uses.

The katharometer and CO_2 recorders are manufactured commercially by the Cambridge & Paul Instrument Co.

We are not acquainted with the results of any experiments on the thermal conductivity of water vapour air mixtures, but from some data we estimated that the effect of the change from absolute dryness to saturation of air at —10°C. would be to increase its thermal conductivity by 8 parts in 10,000.

As it is desirable to measure humidities to an accuracy of 1 per cent. this will necessitate the measurement of thermal conductivity changes to a few parts in a million.

We inquired of the Cambridge & Paul Instrument Co., what order of deflections they considered possible with their hot-wire instruments in hygrometric measurements. They stated that at room temperature with a totally suspended type of indicator it would be possible to obtain full scale deflection for 100 per cent. humidity, but at —10°C. the deflection would be about 14 millimetres, and with a recorder half this amount.

This decrease in sensitivity is due to the fact that the deflection depends upon the actual weight of vapour present in the air, and not its relative humidity.

A full scale range of 14 millimetres would not give sufficient accuracy in hygrometric measurements, so it would be necessary to increase it either by the use of a more sensitive indicator or by raising the temperature of the hot wire.

Increasing the galvanometer sensitivity usually results in unstable zero, and increasing the temperature of the wire gives rise to convection currents. So the problem resolves itself into making very accurate resistance measurements when the wire and its insulation are exposed to a humid atmosphere.

One of the troubles of low-temperature work is the frequent deposition of dew, due to temperature changes causing supersaturation, and the formation of a moisture film on the electrical insulation of the wire would be fatal to reliable measurements.

There is also the additional complication of the presence of varying amounts of CO_2 in the atmosphere of the stores. It is a well-known fact that apples, &c., give off CO_2, and the modern method of storage is to seal the room and allow the CO_2 content to rise to 10 per cent. at the expense of the oxygen. Now it appears that volume for volume CO_2 has a greater effect on the thermal conductivity of the atmosphere than H_2O vapour. So to eliminate this variable it would be necessary to employ an absorbent for CO_2 leaving the water vapour.

The writer is not acquainted with any absorbent for CO_2 which will not absorb H_2O at the same time. The alternative method is to estimate by difference: determine the conductivity of the air containing CO_2 and the water vapour; then saturate and again find the conductivity.

It must not, however, be concluded that the hot-wire method has no applications in hygrometry. It is conceivable that it will prove to be of great service for humidity measurements at temperatures beyond the range of the classical methods, since the instrument measures the actual percentage of vapour present, and its sensitivity as a hygrometer will increase with temperature at about the same rate as the saturation vapour pressure.

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* **Proc. Phys. Soc., Vol. XXXIII., p. 168 (1921).**

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The work described in this Paper constitutes a part of a more extensive research carried out on behalf of the Food Investigation Board of the Department of Scientific and Industrial Research.

To the Chairman of the Engineering Committee, Sir Alfred Ewing, and the Chairman of the Insulation Sub-committee, Sir Richard Glazebrook, the author wishes to record his thanks for their continued interest in the work.

The Director of the Laboratory, Sir Joseph Petavel, has furthered the work by placing every facility at our disposal, and by constructive criticism of the various designs.

Mr. W. B. Hardy, Sec. R.S., Director of Food Investigation, has kindly arranged for the blocks for illustrating the Paper to be placed at the disposal of the Society. (The permission of the Controller, H.M. Stationery Office, has been obtained for the reproduction of these illustrations, which are taken from Special Report No. 8 of the Food Investigation Board of the Department of Scientific and Industrial Research, now in preparation.)

Mr. A. Snow, Junior Observer, has rendered valuable assistance with the experimental work, and Mr. Edgar A. Griffiths has suggested many features embodied in the instruments.

Discussion on

THE THEORY OF THE HAIR HYGROMETER.

BY

F. J. W. Whipple, *M.A.*, *F.Inst.P.*

As a practical instrument for the measurement of humidity, the hair hygrometer has many advantages over its rival the psychrometer. Hair hygrometers are in general use in countries where temperature is below the freezing-point for a large fraction of the year, in conditions in which it would be impossible to obtain satisfactory results with the ordinary psychrometer. Moreover, the hair hygrometer lends itself to adaptation as a recording instrument,and as such it is frequently seen in this country. The discrepancies between the humidities shown by hygrometers of different types are often serious, however, and encourage scepticism as to the value of any of them.

The hair hygrometer was first made as an instrument of precision by de Saussure, whose memoir on the subject* appeared in 1783 and is still of great interest. Apparently the theory of vapour pressure had not been developed at the time de Saussure wrote. The "degrees" of his scale were made proportional to the extension of the hair, 0 corresponding with the perfectly dry hair and 100 with the saturated hair, and he investigated the relation between this scale and the vapour density.

The relation between de Saussure's scale and the relative humidity of the air was investigated experimentally by Gay-Lussac. No account of the research is mentioned in lists of Gay-Lussac's works, but the results were communicated to Biot, who incorporated them in his Traité de Physique.† The method adopted by Gay-Lussac was to determine the vapour pressure due to salt solutions or to mixtures of sulphuric acid and water, and to take readings of the hair hygrometer in a close space in presence of such liquids.

Relative Humidity.	Centesimal Degrees of the Hygrometer.		
Per Cent.	No. 44.	No. 5.	No. 68.
0·0	0·0	0·0	0·0
2·1	2·4	2·6	4·6
9·2	14·2	14·5	19·3
13·3	25·3	27·1	30·6
18·9	35·5	36·3	40·0
31·8	51·2	52·9	56·0
35·6	58·5	59·9	61·3
43·7	65·4	66·7	68·0
47·8	71·3	72·1	72·3
54·1	76·8	77·6	78·0
61·9	83·7	83·9	83·2
67·1	87·3	87·7	86·7
77·8	93·4	93·2	91·3
100·0	100·0	100·0	100·0

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* Essas sur l'Hygrometrie. Neuchâtel, 1783.

† Paris, 1816. The experiments are described in Vol. II., p. 109; tables are given in Vol. I., p. 532, and in Vol. II., p. 200.

The best-known laboratory experiments on the subject are those of Regnault,* who used solutions of sulphuric acid of various strengths, and developed a normal method in which the air was extracted from the vessel in which the hygrometer was hung up above the acid.

From Regnault we quote the table on preceding page.

This table shows that the results obtained with the two hairs Nos. 44 and 5 were very consistent. The figures referring to hygrometer No. 68 in which the tension applied was greater show considerable departures from those for the other two instruments.

Regnault points out that the zero of the hygrometer scale is not to be trusted, since the reading of the instrument in a perfectly dry atmosphere depends on the time of exposure.

A conspicuous feature of the table is that a variation of humidity of, say, 10 per cent., gives a small change in the length of the hair when it is near saturation, and a large change in a comparatively dry atmosphere. This point is brought out by plotting the results on semilogarithmic paper as in Fig. 2. The change in length corresponding with the change in humidity from 70 to 100 is equal to that corresponding with the change from 20 to 30. The graphs (Figs. 1, 2) refer to hygrometer 68, which seems to have been the most satisfactory of the three.

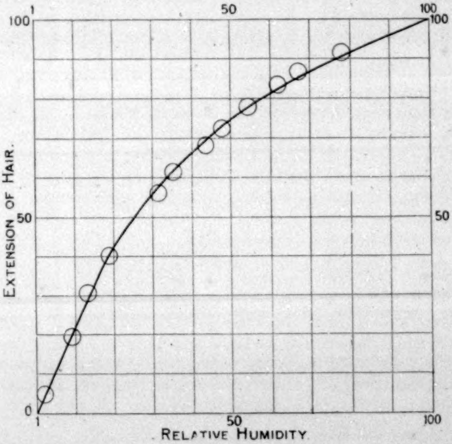

Fig. 1.

Attempts to express Gay-Lussac's results by empirical formulæ were made by Biot and by Klinkerfuess, but the only theoretical investigation of the phenomena appears to be that of Prof. B. Sresnevsky,* of Moscow. As stated by this author the theory is in some respects incomplete, and the argument has been modified considerably in the following summary.

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* Etudes sur L'Hygrometrie. *Comptes Rendus*, xx., pp. 1127, 1220, Paris, 1845. Reprinted in *Annales de Chimie et de Physique*, xv., p. 129, Paris, 1845. English translation, Taylor's *Scientific Memoirs*, iv., p. 606, London, 1846.

Microscopic examination of a hair shows that it contains numerous cells; some are filled with colouring matter, whilst others contain more or less water. According to Sresnevsky, when the hair is in a saturated atmosphere these pores are filled to overflowing. On the other hand, in a comparatively dry atmosphere, the water in each cell will be bounded by a curved meniscus. The tendency to reduce the radius of curvature of the meniscus strains the cell walls. (When the hair is saturated, there is no such strain.) It may be expected that, if the cells are elastic and are deformed in accordance with Hooke's law, the contraction of the hair as it dries will be a simple function of the relative humidity.

To determine the conditions in the cells, Sresnevsky makes use of a well-known discussion by W. Thomson (Lord Kelvin) of the way in which vapour pressure in the neighbourhood of a curved meniscus differs from that in the neighbourhood of a plane one. Thomson considered a capillary tube standing in a basin of liquid. In the steady state, there is a difference in vapour pressure at the top of the tube and at the surface of the liquid in the basin. This difference of pressure is due to the weight of the vapour.

In the case dealt with by Thomson the height is small. The pressures are related by the equation

$$p_0 - p_1 = gh\sigma, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where p_0 is the vapour pressure over the liquid in the basin, p_1 the corresponding vapour pressure over the meniscus in the capillary tube, h the elevation in the tube, σ the density of the vapour and g the acceleration due to gravity.

If the height is so great that the differences in the vapour density must be taken into account, Thomson's formula requires modification. If the absolute temperature θ is assumed uniform, and if R is the constant in the equation $p = R\sigma\theta$, then in this more general case it is the equation

$$h = \frac{R\theta}{g} \log \frac{p_0}{p_1}$$

which determines the height to which the column must extend so that the ratio of the vapour pressures at the bottom and top may have a specified value p_0/p_1.

Now if r be the radius of curvature of the meniscus in the capillary tube, T the surface tension of the liquid, P_1 and P_0 the atmospheric pressure on the upper and lower surfaces, and ρ the density of the liquid, assumed uniform, then

$$\frac{2T}{r} = g\rho h - (P_0 - P_1) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

The density of the liquid is high compared with that of air, and therefore, $P_0 - P_1$ is small compared with $g\rho h$, so that we have as a good approximation

$$\frac{2T}{r} = g\rho h = R\rho\theta \log (p_0/p_1) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

Provided that $g\rho h$ is less than the atmospheric pressure P_0 the whole column in the capillary tube is under hydrostatic pressure, but if $g\rho h$ exceeds this limit the upper part of the column is in tension, or subject to a negative pressure. This must happen when the capillary is so fine that the rise in the tube exceeds the height of a barometric column of the liquid in question. In the case of water the limit is reached when the radius of the meniscus is about 0·0015 mm., *i.e.*, $1{\cdot}5\mu$, so that the diameter of the tube is about 3μ.

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* Théorie de l'hygrometre à cheveu. Jwijev. Sci. Rep. Imp. Univ., 1895, No. 3.

For tubes of much smaller microscopic dimensions the height of the column would be far greater than the height of the barometer, and the liquid would be subject to considerable tension or negative pressure. The magnitude of this negative pressure just below the meniscus is given by the equation

$$Q=R\rho\theta\log(p_0/p_1)-P_0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

The values of Q and r as determined by equations (4) and (5) are shown for water at 283a in the following table, the values assumed for the constants being $\rho=1$, $\theta=283$a, $T=74$ dyne/cm., and $R=4{\cdot}61\times10^6$ C.G.S. The units adopted for the tabulation of r and Q are the millimicrometre (denoted by $\mu\mu$) which is equal to 10^{-9} metre; and the C.G.S. atmosphere or bar, which is equal to 10^6 dyne/cm.^{2}

In these units $Q=3{,}004\log_{10}(p_0/p_1)-P_0$, whilst $r=1{,}480/(Q+P_0)$.

Relative humidity p_1/p_0.	Negative pressure in the cells Q.	Radius of meniscus r.
99 per cent.	12 bars.	113 $\mu\mu$
95	66	22
90	136	11
80	290	5·1
70	464	3·2
60	665	2·2
50	903	1·6
40	1190	1·2
30	1570	0·9
20	2100	0·7
10	3000	0·5
0	∞	0

The legitimacy of the use of the conception of surface tension in considering a tube whose bore approaches molecular dimensions is more than doubtful, so that too much weight must not be attached to the figures given for r. The values of Q would seem to be more trustworthy.

The very long tube of fine bore is only brought into the argument so that the relations between the curvatures of the meniscus, the internal pressure and the relative humidity may be discovered. The same relations are assumed to hold good when the liquid in a small cell is in equilibrium with the vapour in its neighbourhood.

Sresnevsky applies equation (4) to the problem of the hair hygrometer. His argument is as follows. In the case of a cell with elastic walls the strain exerted by the surface tension of the meniscus tends to contract the cell until the surface tension is in equilibrium with the elastic forces developed by the deformation. Thus the change of volume of the hair depends on the curvature of the meniscus and consequently on the relative humidity of the air. Precision would seem to be gained, however, by basing the argument on equation (5), which indicates that the walls of the cells are subject to a negative pressure proportional to log (p_0/p_1). If Hooke's law holds, the strain of each cell, and therefore the contraction of the hair as a whole, will be proportional to the same logarithm; so that the theory leads to the simple result that the contraction of the hair from its length when saturated is proportional to the logarithm of the relative humidity of the atmosphere. The theory should only hold good provided that the drying is not carried so far that many of the cells lose all their water. It is also certain that Hooke's law will not hold in extreme cases.

The agreement of experiment with Srevensky's theory may be judged by the diagram (Fig. 2) in which the contraction of the hair and the relative humidity are plotted on a semi-logarithmic chart, and for humidities above 10 per cent. is as good as could be expected.

As an additional verification that the theory is substantially correct we may compare the stresses which are supposed to be developed in the hair with those which occur when it is stretched by a suspended weight.

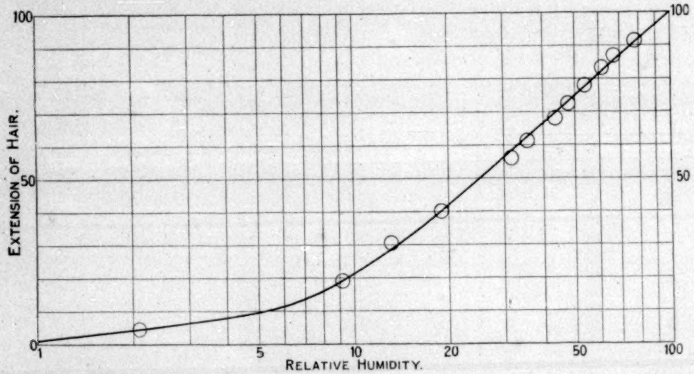

FIG. 2.

To obtain the necessary data* a hair about 22·5 cm. long was mounted vertically so that a lever attached to the lower end could move over a graduated scale, and so that weights could be hung from it. It was found that the extensions due to weight 2 gm., 4 gm. and 10 gm. corresponded with 11, 18 and 36 divisions of the scale respectively, whilst the contraction due to change of relative humidity from 100 to 10 per cent. was 34 divisions. The diameter of the hair was measured and found to be 0·03 mm. It will be seen that the contraction due to drying the hair to relative humidity 10 per cent. was nearly equal to the extension due to a tension of 10 gm. weight, or an average stress of $1{\cdot}4 \times 10^9$ dynes per square centimetre, or 1,400 bar. On the other hand, according to equation (5) the internal stress on the cell walls to produce this contraction should have been about 3,000 bar.

The agreement is not as good as might be hoped for, but it appears that the forces concerned are of the same order of magnitude. The strains in the two cases are evidently of very different characters ; when the hair is stretched by a weight, the cells are elongated, whereas when it shrinks owing to the negative pressure in the cells it is to be presumed that they contract transversely as well as longitudinally.

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* The observations were made by Mr. H. Fahmy.

It is of interest to notice that according to equation (5) the stress in the cells is proportional to the absolute temperature, and the resulting contraction should presumably obey the same rule. Laboratory experiments to decide this point are desirable.

An important application of the theory is that hair hygrometers should not be used in dry atmospheres, where the contents of the hair-cells may completely evaporate. It is well known that in such circumstances the readings are apt to be unreliable. No attempt should be made to carry the calibration of a hygrometer down to very low humidities ; the range from 100 per cent. to 20 per cent. should suffice for almost all purposes.

It is customary to graduate hygrometers so that humidities may be read off at once. For scientific purposes it would, I think, be better to revert to de Saussure's scale, on which equal intervals correspond with equal extensions of the hair ; if the logarithmic law may be assumed, two observations then suffice to calibrate the instrument, and scale readings may be converted to relative humidities by a straight line graph on semi-logarithmic paper. A similar remark applies to hygrographs. The adjustment of the Richard hygrograph is notoriously difficult, and there is, I think, an opening for an instrument in which the displacement of the pen is proportional to the contraction of the hair ; with such an instrument it should be possible to eliminate the zero and scale errors comparatively easily. A station where hair hygrometers are used, *e.g.*, an aeroplane base, should have a pair of Regnault's calibrating vessels giving humidities 100 per cent. and say 30 per cent., and should standardise the instruments as often as necessary. There can be little doubt that for flights above the clouds into regions where temperature is below the freezing point the hair hygrograph is much to be preferred to the psychrometer.

THE RATIONALE OF GLAISHER'S SYSTEM OF HYGROMETRY.

BY

F. J. W. WHIPPLE, *M.A.*, *F.Inst.P.*

BRITISH Meteorology owes a debt to James Glaisher, of the Royal Observatory at Greenwich, for the organisation of climatological observations.

Glaisher attached much importance to hygrometric observations and utilised observations of Daniell's hygrometer to prepare tables for the reduction of dry and wet bulb readings. Most of the observations were taken at the Royal Observatory, Greenwich, from 1841 to 1854 ; but these were combined with others taken at high temperatures in India, and at low and medium temperatures at Toronto. These tables* of Glaisher's have been in use in the British Meteorological Service for the last 50 years.

If one may judge by the preface to the tables, Glaisher was not guided by theory in the choice of a method of reduction of the material at his disposal. He had to correlate three sets of readings, and adopted the simplest possible type of formula for connecting them. The formula may be written

$$t-t''=k(t-t_w)$$

t being the dry bulb temperature, t_w the wet bulb temperature, t'' the dewpoint, and k a certain factor. Glaisher made the assumption that k was a function of t and independent of t_w. As the observations he utilised in obtaining the appropriate values of k do not appear to have been published in a suitable form we must test them by the work of other investigators, and the simplest way to do this is to refer to tables with a more logical basis that have been published. For this purpose the tables prepared by Hann† are appropriate. Hann uses three formulæ of the type associated with the names of Apjohn, August and Regnault,

$$p_w-p=AP(t-t_w)$$

Here p_w is the saturation pressure at temperature t_w, and p is the actual vapour pressure, whilst P is the atmospheric pressure.

Following Regnault, Hann adopts different values of A according to the strength of the wind blowing on the wet bulb ; in the part of his work in which slowly changing factors, which are intended to allow for variations in the latent heat of evaporation, are ignored the values of A are given for the centigrade scale as follows, the values in the first column being used when the "wet-bulb" reading is above the freezing-point, those in the second column when it is below :—

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* *Hygrometrical Tables adapted to the use of the Dry and Wet Bulb Thermometers*, by James Glaisher, F.R.S., &c., London, 1847. No essential changes appear to have been made after the second edition 1856. For the use of computers *Hygrometrical Tables based on Glaisher's Hygrometrical Tables* (*5th Edition*) were printed by authority of the Meteorological Committee "for private circulation" in 1876, and reprinted in 1904. These tables are designed to reduce the amount of interpolation required with Glaisher's. The Glaisher factors are also given in Marriott's *Hints to Meteorological Observers*, issued by the Royal Meteorological Society in 1881, and in the *Observer's Handbook* of the Meteorological Office, the first edition of which appeared in 1908. The use of Glaisher's method is almost confined to the British Isles.

† Jelineks Psychrometer-Tafeln erweitert und vermehrt von J. Hann ; neu herausgegeben und mit Hygrometer-Tafeln versehen von J. M. Pernter. Fünfte erweiterte Auflage, Leipzig, 1903. English authors have usually attributed the psychrometer tables to Pernter, but it is clear from the introduction that Hann should have the credit.

Strong wind	0·000656	... 0·000579
Moderate wind	0·0008	... 0·000706
Calm	0·0012	... 0·001060

There is a natural tendency to regard the formulæ for strong wind, moderate wind and calm as on the same footing. It should be emphasised, however, that the strong wind formula has a firm theoretical basis, the constant being easily derived from the specific heat of air, the ratio of the densities of air and vapour and the latent heat of evaporation ; and, moreover, that the validity of this constant has been verified over a great range. The moderate wind and calm formulæ are admittedly empirical ; they are mere compromises between more or less inconsistent observations.

The several curves in Fig. 1 refer to specified dry bulb temperatures, and the dew point is given as a function of the wet bulb temperature. On Glaisher's hypothesis the functional relation is linear, whilst Hann's three hypotheses give three curves for each dry bulb.

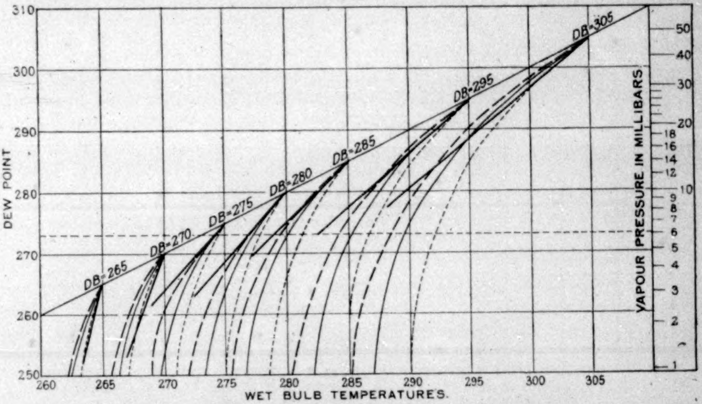

FIG. 1.—DEWPOINT AS A FUNCTION OF WET BULB READING.

For each dry bulb reading there are three graphs.
The straight line ———— gives the dewpoint by Glaisher's rule.
The highest curve — — — corresponds with the strong wind formula.
The continuous curve ——— with the moderate wind formula.
The lowest curve --------- with the no wind formula.

The table of Hann's in which atmospheric pressure is taken as 755 mm. (1007 *mb*) has been utilised. The values of Glaisher's factor corresponding with the dry bulb temperatures in question are as follows :—

Dry bulb ...	265a	270a	275a	280a	285a	295a	305a
Factor ...	8·55	5·77	2·54	2·17	1·99	1·75	1·63

The absolute centigrade scale is in general use in the Meteorological Office for theoretical investigations. Tables for finding humidities from observations on the absolute scale by the use of Glaisher's factors are included in the Computer's Handbook, Section I., so that the adoption of absolute temperature in the diagram is not to be regarded as an anachronism.

It will be seen from the diagram that for temperatures above the freezing point, and for air which is not too dry the Glaisher lines are in fair agreement with the run of the curves for moderate or strong winds. This agreement is such as would be expected if values of $(t-u)/(t-t_w)$ were computed from the observations and then averaged.

With large depressions of the wet bulb the Glaisher line is in each case above the strong wind curve, and there is every reason to believe that for such large depressions Glaisher's rule gives definitely false results. Thus taking the dry and wet bulb readings 305a and 290a respectively (89·6°F. and 62·6°F.), our diagram shows that according to Glaisher the vapour pressure is 10mb, whereas it cannot possibly be more than 9mb, and on the assumption of a moderate wind it is probably only 7 *mb*. The corresponding difference in the estimate of relative humidity as between 21 and 15 per cent. is serious, or at any rate would be serious if such low humidities were frequently met with in this country.

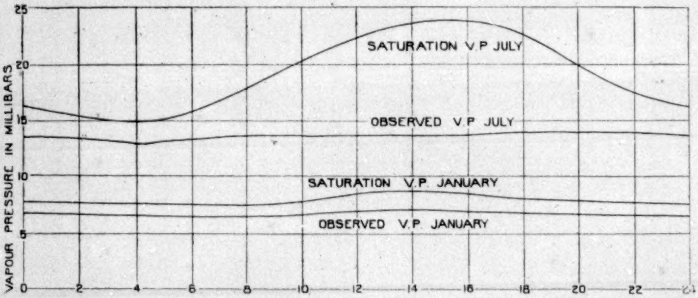

FIG. 2.—DIURNAL VARIATION OF VAPOUR PRESSURE: KEW OBSERVATORY, RICHMOND.

With wet-bulb temperatures below the freezing point the contrast between Glaisher's system and its rivals is more striking.

The calmest air occurs in frosty weather, and no doubt the Canadian observations on which Glaisher relied were taken in such circumstances; so that the fact that for this part of the range the lines representing his formula in Fig. 1 run closely with the lines representing Hann's calm formula is satisfactory confirmation of the observations.

The change in the value of the Glaisher factor in the neighbourhood of the freezing point is very rapid. The dependence of the factor on the dry bulb temperature is obviously illogical. What matters is whether the wet bulb is ice-covered or not.

I believe that the use of Glaisher's tables should be abandoned, but my diagram brings out the great uncertainty that is involved in the use of a thermometer with unknown ventilation, by whatever system the readings may be reduced. This uncertainty is more troublesome at low temperatures than elsewhere.

It has long been realised that no system of hygrometry which depends on the readings of a wet-bulb thermometer with indeterminate ventilation can be satisfactory. In America the whirled psychrometer has been in use for many years, and it would be well if it could be introduced here. At the same time we should have forced ventilation for the thermographs from which the standard readings at the observatories are taken.

At present the uncertainty attached to the method of observation is reflected in the results. This is especially the case with regard to the diurnal variation of vapour pressure.

In Fig. 2 the diurnal variation of pressure at Kew Observatory in January and in July is compared with the variation in saturation pressure, the observations having been worked up* by Glaisher's method. The figure serves to show that Glaisher's tables are not very seriously at fault for dealing with the average conditions met with in this country ; but it also shows that a very high order of accuracy is required if the true form of the diurnal variation curve is to be determined. This high order of accuracy will not be secured unless wet-bulb thermometers with continuous forced ventilation can be installed.

As to the ordinary routine of observations, the Meteorological Office has for many years realised that much improvement in its practice is to be obtained only by the adoption of a new method of observation, and that a change from one system of computation to another to be applied to the readings of unventilated psychrometers will be only a palliative. The provision of ventilated psychrometers means, however, the substitution of a complicated procedure for a simple one at every climatological station, and is a step for which abundant justification is required. We are still wanting information as to the magnitude of the errors which are involved in the use of the unventilated psychrometer in the conditions prevailing in this country, but from investigations now in progress, some definite conclusion should be arrived at before long. In the meantime it is, I think, desirable to make general the use of the "moderate wind" formula for the unventilated psychrometer and to give up Glaisher's method of reducing the observations.

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* The saturation-vapour-pressures have been taken as corresponding with the average temperatures, and the "observed" vapour-pressures have been derived from the saturation-vapour-pressures by utilising the average relative humidities. The data will be found in *Hourly Values from Autographic Records.*

THE WET AND DRY BULB HYGROMETER.

BY

PRINCIPAL S. SKINNER, *M.A.*, *F.Inst.P.*

THE wet and dry bulb thermometers form the simplest instrument for ascertaining the humidity of the air, and in some form are in general use. In the autumn of 1917 I began an experimental inquiry into the reduction of their readings. There are two main conditions under which they are used, *i.e.*, in still air or in moving air. Starting with Clerk Maxwell's formula for still air (" Ency. Brit.," 9th Edition, Article Diffusion),

$$p=p_w-\frac{PS_p}{L\sigma}\left(\frac{K}{\gamma D}+\frac{AR}{4\pi CS_p\rho D}\right)(t-t_w) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where :—

p is the pressure of aqueous vapour in the air at a temperature t ;
p_w the maximum pressure of aqueous vapour at the temperature of the wet bulb t_w ;
P the whole pressure of the air ;
S_p the specific heat of air at constant pressure ;
L the latent heat of water at t_w ;
σ the specific gravity of aqueous vapour compared with air ;
K the thermometric conductivity of air ;
γ the ratio of the specific heats $=1{\cdot}41$;
D the diffusivity of aqueous vapours in air ;
A the area of the wet bulb ;
R the radiation constant for the wet bulb ;
C the electric capacity of the bulb ;
ρ the density of air.

I tested its validity by replacing the dry air by dry hydrogen ; and by data at reduced pressure in air. The results showed that Maxwell's formula satisfied the data obtained in such a dry still gas as that inside a porous pot soaked in strong sulphuric acid.

The more important condition, *i.e.*, in moving air, was submitted to experiment, and was discussed theoretically. One theoretical method was to take Maxwell's formula for dry air and consider how it should be modified for air in motion. As soon as the velocity is great enough the diffusion effect will be very large compared with the radiation effect ; moreover, the conduction effect would increase *pari passu* with the diffusion effect. Arguing in this way and using the results of experiment, the formula in practice reduces to

$$p=p_w-\frac{PS_p}{L\sigma}(t-t_w) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2$$

This view was put forward at a meeting at the Meteorological Office, and in the discussion Major G. I. Taylor outlined a theory based on the passage of heat and water-vapour through the eddy-free layer next the wet thermometer to the eddy layer beyond. In the eddy-free layer his theory leads to the formula (k=the calorimetric conductivity and p_e the pressure of aqueous vapour at the junction of the two layers and t_e its temperature)

$$p_e=p_w-\frac{P}{L\sigma}\cdot\frac{k}{D\rho}(t_e-t_w) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

Now, since $\frac{k}{D\rho}$ for air is nearly numerically equal to S_p the formula in effect reduces to

$$p_e=p_w \ \frac{PS_p}{L\sigma}(t_e-t_w) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

In the second eddy layer the formula for the exchanges is

$$p=p \ -\frac{PS_p}{L\sigma}(t-t_e) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

When the effects of the two layers are combined we have

$$p=p_w-\frac{PS_p}{L\sigma}(t-t_w)$$

a formula the same as (2) above.

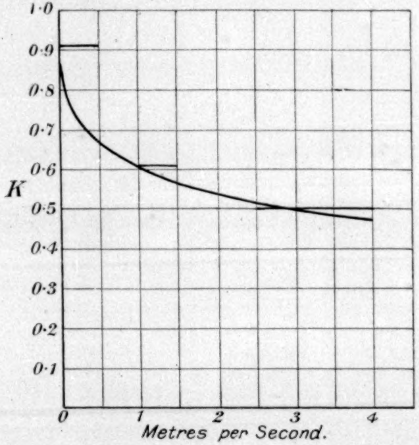
The formula was tested experimentally. A short test tube, 2·3 cm. diameter and 10 cm. long, was closed by a cork with three holes in it, through which passed an entrance glass tube leading to the bottom, an exit tube from near the top, and a thermometer graduated in 1/10ths of a degree. The wet bulb was made by wrapping one or two layers of linen gauze about the bulb of the thermometer and tying this in place with cotton thread. The wet thermometer was moistened by immersing it in a beaker of water before each experiment. To prevent heat from coming from the outside to this apparatus it was enclosed in a larger vacuum-jacketed test tube, and the intervening space was plugged with cotton wool. In some experiments this vacuum-jacket was dispensed with, for the complete experiment only lasts about four or five minutes, and not much heat can get in in that short time. The exit tube was connected to the suction nozzle of a Lennox electrical blower, so that a rapid draught could be maintained through the apparatus. The entrance tube was connected to a tall tower of pumice saturated with strong sulphuric acid in order to give a supply of dry air.

With dry air the following readings were obtained :—

Dry Air °C.	Wet Bulb °C.	Vapour Pressure at the Temp. of Wet Bulb divided by the depression of Wet Bulb.
15·15	3·1	0·477
15·1	2·8	0·472
18·2	4·7	0·460
16·7	4·0	0·474
17·9	4·7	0·483
18·2	4·9	0·476
		0·474 Mean.

The pressure in the apparatus during the passage of the air stream was determined by substituting for the thermometer a pressure gauge containing mercury. This showed that the pressure inside was 25 mm. less than atmospheric when the motor was working at its highest speed. Therefore, the theoretical constant is $735 \times 0{\cdot}2375/603 \times 0{\cdot}622$, or 0·473, which is in good agreement with the experimental result.

The velocity of the stream of air was ascertained by means of an air meter (Negretti & Zambra), fitted to the side of a box from which the air was drawn. The velocity of the air was found to be 86 feet per minute, and this multiplied by the ratio of the square of the diameter of the aperture of the air meter to the square of the diameter of the tube gave 3·92 metres per second for the velocity of the air passing over the wet bulb. Further experiments were made with hydrogen and with carbon dioxide, to which we have not space to refer, but the results were in agreement with the theory.



Variation with air-speed of the coefficient K in the convection formula.

Some experiments were made with the same apparatus to measure the amount of aqueous vapour present in air. For this purpose the apparatus for delivering dry air was removed and the air from the room drawn in with the same velocity. At the same time observations were made with Regnault's dew-point apparatus to determine the dew-point and thus obtain the true aqueous pressure in the air.

Arising from these experiments and other data it was possible to draw a curve representing the value of the constant by which the depression of the wet thermometer must be multiplied in terms of the velocity of the air up to 4 metres per second. The diagram is shown on previous page.

This refers to air pressure of 760 mm. in the formula $p=p_w-K(t-t_w)$.

The formula may be called the convection formula.

The three horizontal lines in the diagram represent Pernter's three values of the constant, *i.e.*,

0·91 for calm air velocity 0 to 0·5 m. per second.
0·61 for light winds ,, 1 to 1·5 ,, ,,
0·50 for strong winds velocity above 2·5 m. per second.

A fuller description of this method is given in the new "Dictionary of Applied Physics."

The convection formula may be applied to explain the "spheroidal" state. A drop of water in a platinum basket in the centre of a flame evaporates rapidly and its temperature will be t_w in our expression. Again, the formula illustrates the extinction of flames by a shower of falling drops. Reference was made to these applications at my lecture before the Royal Institution, April, 1921.

NOTE ON PSYCHROMETRY IN A WIND CHANNEL.

BY

R. A. WATSON WATT, *F.Inst.P.*

ABSTRACT.

Experiments are described showing that the formula used in reduction of data from the ventilated psychrometer applies at air speeds of 60 to 90 miles per hour without appreciable change of constants from those appropriate to a speed of 10 miles per hour.

THE experiments covered by this note were undertaken in order to test for variation, with air speeds of the order of aeroplane flying speeds, of the factor A in the ordinary formula of the ventilated psychrometer.

$$p-p_w=\frac{AP}{1000}(t-t_w)$$

where p is the required vapour pressure.

p_w is the saturated vapour pressure corresponding with the wet bulb temperature.

t and t_w are dry and wet bulb temperatures.

P is the pressure in millibars.

A is a constant for a given air speed.

The work was carried out in the 7 ft. wind channel of the Royal Aircraft establishment. Two wet and dry bulb psychrometers of the type adopted by the Meteorological Office for use in aeroplanes, were mounted on a standard type of wing strut in the centre of the channel. Psychrometer No. 1 had the bulbs 5 ft. above the floor of the channel and was held 1 ft. out from the strut. No. 2 was 3½ ft. above floor and close up to the strut. In both cases the wet bulb was down wind.

Comparison was made with the readings of two Assmann psychrometers outside the channel, No. A 3 ft. above main chamber floor, near channel intake, No. B on a rail 2 ft. above top of channel. The results are set out in Table I.

The vapour pressure has been computed for each psychrometer using the value of the constant A which has been standardised for the Assmann psychrometer (in which the air speed is of the order of 10 miles per hour), and the means for each pair of instruments are tabulated in columns 5 and 7.

The mean difference of the computed vapour pressures for the interior and exterior of the channel is 0·8 per cent., the mean deviation without regard to sign is 1·4 per cent. The variation of computed vapour pressure with the 50 per cent. variation of air speed covered by the experiments is shown by Table II.

These results point to a regular slight decrease of A with air speed, amounting at 90 miles per hour to some 1 per cent. of the 10 miles per hour value.

This variation is, however, distinctly less than the differences of vapour pressures computed from similarly exposed psychrometers, which, as shown in Table I, cols. 6 and 8, amounted in the mean to 2·6 per cent. for the aero-psychrometers, and to 1·4 per cent. for the Assmann instruments.

The conclusion may, however, safely be drawn for the ordinary meteorological

TABLE I.—*Experimental Data and Computed Vapour Pressures.*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		(9)	(10)	
Experiment No.	Date.	Air speed (M.P.H.).	Temperature, °F.	Mean vapour pressure p_a from aero-psychrometers *mb*	Difference between v.ps. from aero-psychrometers Per cent.	Mean v.ps. p_s from Assmann psychrometers *mb*	Difference between v.ps. from Assmann's. Per cent.		Difference between computed vapour pressures *mb* in cols. (5) and (7).	Difference $\frac{p_s - p_a}{p_s} \times 100$ Per cent.	
					+					+	—
1	22/1/20	60	58·0	9·6	2·1	9·6	...		0·0		
		70	59·2	9·75	3·1	9·7	...		—0·05		0·5
		80	60·3	9·8	2·0	9·8	...		0·0		
		90	62·6	9·95	5·0	10·0	+	—	+0·05	0·5	
2	27/1/20	60	61·4	9·2	2·2	9·45	3·2		+0·25	2·6	
		70	62·5	9·7	6·2	9·5		2·1	—0·2		2·1
		80	64·7	10·05	5·0	10·15	1·0		+0·1	1·0	
		90	66·8	10·2	7·8	10·2			0·0		
3	28/1/20	60	60·0	11·0	0·0	11·45	4·4		+0·45	4·4	
		70	61·8	11·35	2·6	11·45	0·9		0·1	0·8	
		80	63·6	11·6	0·9	11·65	0·8		0·05	0·4	
		90	65·5	11·75	3·4	11·85	0·8		+0·1	0·8	
4	3/2/20	60	50·7	7·85	6·4	7·8			—0·05		0·6
		70	51·5	7·9	3·8	7·75		1·3	0·15		1·9
		80	52·9	7·9	2·5	7·85		1·3	0·05		0·6
		90	54·2	8·0	2·5	7·85		1·3	0·15		1·9
5	3/3/20	60	51·9	7·6	0·0	7·45		1·3	—0·15		2·0
		90	55·5	7·7	0·0	7·7			0·0		
6	3/3/20	60	56·3	10·95	0·9	11·65	6·0		—0·30		2·6
		70	57·4	11·5	0·0	12·3	6·5		+0·8	6·5	
		80	58·6	12·0	0·8	12·65	5·5		0·65	5·1	
		90	60·6	12·25	0·8	13·2	3·0		+0·95	7·2	
Means					+2·6%		+1·4%			+0·8%	

TABLE II.—*Percentage Difference between Computed Vapour Pressures at Four Air Speeds.*

Experiment No.	Air Speed.							
	60 M.P.H.		70 M.P.H.		80 M.P.H.		90 M.P.H.	
	+	—	+	—	+	—	+	—
1		0		0·5		0	0·5	
2	2·6			2·1	1·0			0
3	4·4		0 8		0·4		0·8	
4		0·6		1·9		0·6		1·9
5		2·0						0
6		2·6	6·5		5·1		7·2	
Sums	7·0	5·2	7·3	4·5	6·5	0·6	8·5	1·9
	+1·8		+2·8		+5·9		+6·6	
Means	+0·3%		+0·6%		+1·2%		+1·1%	

applications of psychrometry that the value of A in the psychrometric formula applies equally to air speeds of 10 and 90 miles per hour, any error due to its variation being well within the casual error of observations in wet and dry bulb psychrometry.

The range of temperature (51 to 67°F.) and vapour pressure (7·5 to 13·2 *mb*) covered by the experiment is not large; a much fuller series of experiments had been arranged, to cover artificially produced temperatures and humidities of considerable range, but circumstances intervened which prevented the work being carried out.

It seems desirable that work on the wider range should be done, preferably by comparison of the aeroplane psychrometer with a hygrometer also placed in the wind channel.

Thanks are due to the Superintendent, Royal Aircraft Establishment, for making the wind channel available; and to the wind channel staff for their help in the work.

This Note is published by permission of the Director, Meteorological Office, at whose instance the work was undertaken.

THE TILTING HYGROMETER: A NEW FORM OF ABSORPTION HYGROMETER.

BY

The late H. G. MAYO, *M.A.*, *and* PROFESSOR A. M. TYNDALL.

THE following work arose out of a demand for a hygrometer which would be suitable for use in cold stores. While the instrument herein described fulfils this condition, it is also convenient for use at ordinary room temperature, and may appeal to some as instructive as a piece of laboratory apparatus in a university course. It should be mentioned, however, that the form that it takes was determined by the purpose for which it was designed—namely, for cold store work. For use at room temperatures only the instrument can be modified, so that it is more " foolproof."

The method employed is that of abstracting the moisture from a given sample of air with phosphorus pentoxide, and measuring the resultant change of pressure on an oil U-tube gauge.

The following advantages may be claimed for the instrument :—

(1) It is portable.

(2) It is rapid in action, a reading of the water vapour pressure in a cold store being obtainable in less than three minutes, in terms of the length of a column of oil.

(3) Tests made at temperatures ranging from −10°C. to 20°C. show that it is reasonably accurate over this range.

(4) It can be used on board ship or in any other place where a steady support is not procurable.

Its main objection as an instrument for cold stores is that it must be placed in the store sometime before use, and it is necessary for the operator to enter the store for the purpose of taking the readings.

The present form of the instrument is described below under a special heading. But the following historical summary and account of earlier forms which we have tried will serve to bring out the reasons for this particular design.

HISTORICAL SUMMARY OF WORK ON ABSORPTION HYGROMETERS.

Edelmann (Wied. Ann., VI., p. 455, 1879) noted the reduction in pressure brought about by exposing a given volume of air to sulphuric acid, which absorbed the water vapour present.

Van Hasselt (Mbl. Nt. Amsterdam, 1879) brought about the required absorption by breaking a bulb of P_2O_5 inside the vessel. But in neither case can the instrument be regarded as suitable for repetition work in hygrometry.

Sondern and also Matern (Wied. Ann., IX., p. 147) exposed a sample of air to a water surface and measured the change in pressure (or volume) brought about by saturation.

A. N. Shaw (Trans. Royal Soc. Canada, Vol. X., 1916) on similar lines has devised a constant pressure absorption hygrometer, using sulphuric acid as the drying agent. But the influence of the glass walls of the apparatus and the presence of a number of taps, in our opinion, introduce defects into his apparatus which make it both inaccurate and unsuitable for cold store work.

PRELIMINARY EXPERIMENTS.

A theoretical calculation from the coefficient of diffusion of water vapour through air shows that in any practical form of apparatus the method of drying a sample of gas by diffusion leads to a slow rate of working. This was confirmed in an experiment in which, by means of a loosely fitting rubber bung, the upper and lower parts of an ordinary desiccator were virtually cut off from one another. The upper part contained the sample of air, and the lower some phosphorus pentoxide. It was arranged that, without opening the desiccator, the bung could be pulled up by a thread, so that there was a wide free passage opened up, through which diffusion of water vapour could take place.

It was found that ultimately the U-tube gauge connected to the desiccator became steady, indicating that the absorption was practically complete, but that at least half an hour elapsed before this occurred.

In practice, therefore, some arrangement must be adopted in which the air is passed over or through the drying agent instead of simply being left in contact with it.

Accordingly a device was tried in which the sample of air was placed in a tube joining the ends of a glass U-tube half-filled with mercury. Without going into the details of the arrangement, it may be said that it was possible to insert in this sample a rod coated with phosphorus pentoxide, and then to drive the air backwards and forwards over it simply by rocking the U-tube in its own plane.

A capillary U-tube oil gauge communicating with the sample and inserted so that its plane was perpendicular to the plane of the apparatus, responded with a rapidity which showed that the absorption of the moisture in the to-and-fro motion was effectively carried out.

But the heat generated in the P_2O_5 produced a temperature effect which, owing owing to the bad conductivity of the glass walls, died down only slowly, and so greatly decreased the rate of working the instrument.

Moreover an important point was emphasised by this design. If moist air is admitted to a perfectly dry vessel, the humidity of the sample rapidly falls due to adsorption of water vapour by the walls. When the mercury is raised this water film is covered over by or shared with the mercury surface, and not removed by the P_2O_5. In this way the mercury gradually gets wet in a sequence of experiments, and it is then impossible to fill the vessel with a true sample of the external air, because the humidity will change by contact with the walls and mercury before connection between the sample and the outside atmosphere can be cut off. This was shown by the fact that the apparatus gave good results when first set up, but gradually failed.

We believe that this effect vitiates nearly all the early experiments referred to in the historical summary above.

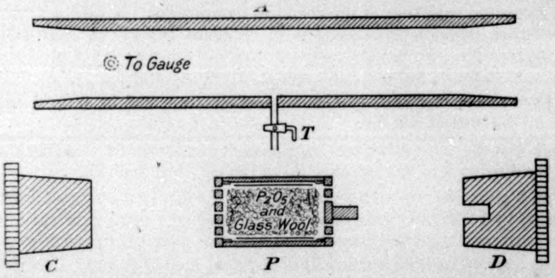
To eliminate these troubles an arrangement is necessary in which the surface area in contact with the sample is practically unchanged throughout and is made moreover of metal instead of glass to reduce both the surface effect and the temperature effect due to heating of the drying agent.

In the first instrument of this type, the air in a brass cylinder was made to surge backwards and forwards through P_2O_5 in a side tube by working a rod and piston in the cylinder. The rod passed through a stuffing box at each end, so that the volume of the vessel did not change with the motion of the piston. Taps were provided so that the cylinder could first be filled with the sample by carrying out a number of strokes without exposing the P_2O_5.

Trials in cold stores revealed troubles due to the hardening of the packing of the stuffing boxes and leakage of the various taps. Moreover, the process of ensuring before starting that the adsorbed film on the inner walls of the cylinder was really in equilibrium with the sample admitted involved a tedious number of strokes of the piston and loss of time. We believe that those difficulties are all met in the final form of apparatus—in so far as it may be said to be final.

FINAL FORM OF APPARATUS.

The rod and piston is replaced by a brass plunger fitting a brass cylinder well but sufficiently loosely to slide from one end to the other without lubrication when the cylinder is tilted through about 30°. The figure shows the details of the arrangement. In the ordinary way the cylinder *A* is left freely open at both ends so that the film on its walls is in equilibrium with the air of the room. When a measurement of humidity is required it is closed at both ends by the insertion of the plunger and two ground-in solid plugs *C* and *D*, the tap *T* being kept open during this operation. A hole in *D* receives at the end of a stroke a pin projecting from the plunger *P*. In this way the plunger may readily be inserted into and removed from the tube before and after an experiment without touching it with the fingers. The plunger *P* is hollow and its ends perforated. It is loosely packed with glass wool dipped previously into P_2O_5 powder. No P_2O_5 is placed on the wool touching the ends of the plunger, and a glass sleeve similarly protects its sides.



Communication with the outside air is cut off by closing the tap *T* and the cylinder is then tilted so that the plunger falls towards the plug *C* ; air passes through the P_2O_5 and dries in the process. Reversing the direction of the tilt reverses the direction of motion of air. Spring clips are fixed to the end of the cylinder to hold the plugs in position and prevent them being loosened by impacts from the plunger.

A few oscillations are sufficient to dry a sample, as may be seen by the motion of the oil gauge which is connected to the cylinder by a side tube. In general, however, about twenty oscillations have been made first and these have been followed by a brief halt and a few more oscillations. In this way the greater part of the effect of evolution of vapour from the walls and of heat from the P_2O_5 is eliminated.

Very consistent results have been obtained with this apparatus both at room temperature and in cold store. Moreover, the possibility of leakage is greatly decreased owing to there being only one tap to look after and no stuffing boxes.

In calculating the humidity from the gauge reading two constants are necessary, the density of the oil and the relative volumes of the wet air in the cylinder and the dry air in the plunger. These factors, once measured, are constants for the instrument if it is made to pattern. The gauge tube is capillary so that the motion of the oil produces no appreciable change of volume in the sample.

The relative internal volumes of cylinder and plunger were measured and it was calculated from these that the observed reduction in pressure must be multiplied by 1·30* to give the true vapour pressure of the wet air before it was passed through the drying chamber. A chemical hydrometer was set up by which the mean humidity of the air over a period of half-an-hour was determined. This reading was compared with the average of a set of readings with the absorption hygrometer, taken at the beginning and end of the same period.

Two typical sets of results are given in Table I., the first obtained at room temperature in the laboratory and the second in cold store. In general six readings were taken with the absorption hygrometer, three just after the chemical hygrometer was started and three just before it was completed. It was assumed that the humidity was changing if at all at a uniform rate during this time, so that the mean reading might be compared with that of the chemical hygrometer. It will be seen that the mean absorption hygrometer reading is in excellent agreement with that of the chemical hygrometer. In fact the results practically always were in good agreement as may be seen from the summarised readings given in Table II.

The hygrometer was tested at room temperature and in several cold store chambers in the Bristol district.

Column 1 of the table gives the temperature of the air in Centigrade degrees, column 2 the difference in level in cms. (h) of the oil gauge column.

Column 3 gives the vapour pressure in mms. of mercury obtained from (h) by multiplying it by 1·30* and by the ratio of the densities of oil and mercury. Column 4 gives the corresponding numbers obtained with the chemical hygrometer.

Column 5 giving relative humidities is included mainly to show that the values are reasonable in amount. The high humidity (97 per cent.) taken at −9·5°C. was obtained by placing the instrument in the duct leading directly from the chamber containing the brine pipes to the store. That of 92 per cent. at −10°C. was obtained in a store recently filled with frozen meat.

The hygrometer therefore when compared with the chemical hygrometer stands the test and is naturally more convenient and rapid for general use.

It should, however, be pointed out that with the instrument in this form the readings are useless in an atmosphere which is rapidly changing in temperature or when the instrument has been brought into a room the temperature of which is different from that of the instrument. For such cases it would be necessary to have a dummy cylinder on the other end of the gauge—as indeed we used in some preliminary earlier experiments—in order that the temperature changes may affect both sides equally. But the simplicity and convenience of the hygrometer is thereby somewhat sacrificed and provided care is taken not to touch the cylinder with bare hands or to breathe upon it, this elaboration is unnecessary in most cases.

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* The four observations marked with an asterisk in Table II. were taken with an instrument for which the constant was 1·19 instead of 1·30.

TABLE I.

Temperature.	Observations.		Calculated vapour pressure in mms. of mercury.
16·3°C.	Absorption Hygrometer—readings in cms....	12·7	
		12·7	
		12·6	
		12·3	
		12·4	
		12·5	
	Mean ...	12·5	8·9
	Chemical Hygrometer—		
	Volume of air	5 litres	
	Change in weight of drying tube ...	0·0454 gm.	8·9
—4·0°C.	Absorption Hygrometer—readings in cms....	3·9	
		3·7	
		3·7	
		3·7	
		3·6	
		3·5	
	Mean ...	3·7	2·9
	Chemical Hygrometer—		
	Volume of air	5 litres	
	Change in weight...	0·0147 gm.	2·8

TABLE II.

Temperature in degrees Centigrade.	Mean gauge reading in cms. of oil.	Vapour Pressure in mms. of Hg.		Relative Humidity.
		Absorption Hygrometer.	Chemical Hygrometer.	
*16·8	14·3	10·2	10·1	72
*16·0	12·93	9·2	9·2	68
*15·4	12·5	8·9	8·9	68·5
13·1	10·2	7·93	8·00	71
12·9	10·6	8·23	8·35	74
12·7	9·8	7·6	7·64	69
12·5	8·9	6·9	7·00	64
—2·5	4·0	3·1	3·20	83
—3·5	3·5	2·7	2·68	79
—4	3·7	2·9	2·80	88
—9·5	2·6 (mean of two readings only)	2·0	2·03	97
*—10	2·1	1·8	—	92

Lastly it may be seen that in special cases by suitable choice of reagent in the plunger the instrument might be adapted for rapid measurement of an impurity in air or another gas within certain limits of percentage of impurity.

A THERMAL HYGROMETER.

BY

PROF. A. M. TYNDALL, *D.Sc.*, *and* PROF. A. P. CHATTOCK, *D.Sc.*

THE primary object of the present experiments was the measurement of the humidity in the egg space under sitting hens, which was required in some work on the physics of hen incubation carried out by one of us (A. P. C.) for the Board of Agriculture.* The results obtained are, however, of more general applicability, and are here described as such.

This instrument is a development of experiments made by Orme Masson (Proc. Roy. Soc., Vols. LXXIV., LXXVIII.) and anticipated by Clayton Beadle (Nature, Vol. XLIX., p. 457; Chem. News, Vol. LXXI., p. 1; Ibid., Vol. LXXIII., p. 180).

The principle of the method is this. Very dry cotton wool exposed to damp air absorbs moisture and warms as a consequence.

The temperature rises rapidly at first; reaches a maximum; and then tails slowly off, until if the temperature of the surroundings is constant, it approaches its original value.

Using about a gramme of wool tied round the bulb of a thermometer, Masson found that the maximum temperature was reached about half an hour after the exposure of the wool to the moist air, but the approach of the temperature to its old value lasted many hours. He obtained an empirical equation to the temperature-time curve and deduced from it that if φ is the maximum temperature attained by the wool and τ the time it takes to acquire it, then φ/τ is directly proportional to the vapour pressure of the air to which it is exposed. He further suggested that this effect might be made use of for hygrometric measurements. The following work was undertaken with a view to obtaining on these lines a practical form of hygrometer suitable either for general work or more particularly for work in confined spaces.

If the temperature of the surroundings varies, the indications of the thermometer will, of course, also vary and the connection between temperature and humidity will be upset.

Masson put his apparatus in a water jacket kept at constant temperature by a thermostat, but this is only feasible under special conditions. It was therefore important to reduce the direct effect of temperature relatively to that of the moisture as much as possible, and one way of doing so was to shorten the period of changing temperature that follows the exposure of the wool.

This we did by using in place of the thermometer a thermo-electric junction of very thin constantan and copper wires, and tying upon it about a centigramme (instead of a gramme) of cotton wool; the result being that when the wires were connected to a dead-beat galvanometer and the wool exposed to damp air after drying, the maximum deflection was reached in about 10 seconds after exposure instead of half an hour. One result of this reduction of the time of exposure is that the total amount of water vapour extracted from the air is relatively very small, an important point when one is requiring the humidity of a small confined space.

It is this maximum deflection that we have used as the measure of humidity.

*** The complete hygrometric results obtained in this work will be included in a Paper on the " Physics of Incubation " now in the course of preparation. (A. P. C.)**

We have used throughout this work a Cambridge Paul Ayrton Mather dead-beat galvanometer of low resistance. In ordinary air of average humidity the deflections were of the order of 20 to 30 cms. In all that follows a resistance of 25 ohms was placed in series with the galvanometer to reduce its sensitivity somewhat. The rises in temperature of the wool which took place were of the order of one-tenth to one degree.

Experiments seemed to indicate that over a wide range of humidity this maximum deflection was directly proportional to the pressure of the water vapour in the air sample provided that other conditions such as temperature, physical state of the cotton surface and geometry of the instrument remained unaltered. That is, at any rate, the conclusion that one would draw from the following results.

The details of the method of exposing the wool to a sample of air of known humidity need not be given here, because the apparatus used in obtaining these results was determined by the circumstances of the main investigation, namely, the measurement of humidity in incubators and under hens. Suffice it to say that a sample of air from a hen incubator, of humidity determined by the chemical hygrometer, was obtained in a small bottle the mouth of which was closed by waxed paper. The thermocouple with its two junctions, one covered with wool and the other bare, was enclosed in a glass tube with a pointed end and a hole in its side opposite the wool. The wool was tied on tightly to the junction in five or six places with sewing-silk; the two junctions were arranged to be fairly close together so that any disturbances other than that set up by absorption of water vapour might affect both equally. It was arranged that the tube could be transferred from a dry atmosphere to the bottle by piercing the waxed paper with the sharp point of the tube, the tear in the paper being sealed immediately afterwards with a rubber pad.

Table I. gives the results obtained, with the time in minutes added during which the cotton wool had been drying, since it was last exposed to moist air.

TABLE I.

Time in minutes of drying of cotton wool.	Grammes of water vapour in 1 litre of incubator air (M).	Maximum galvanometer reading (G).	G/M.
All night.	0·0200*	5·50	275
38	0·0210	5·92	282
20	0·0211	6·22	295
31	0·0211	6·05	286
29	0·0217	6·15	284
15	0·0205*	6·22	303
15	0·0201*	6·18	308
104	0·0202	6·05	299
48	0·0162*	4·98	307
31	0·0142	4·05	285
31	0·0136	4·00	294
30	0·0134	3·73	279
27	0·0132	3·73	282
86	0·0100	2·76	276
49	0·0105	3·10	295

The temperature of the wool was throughout 40°C., the same as that of the incubator.

The figures in the last column of the table show that the galvanometer readings are reasonably proportional to the corresponding vapour pressures in the incubator. Of the four readings that are badly out (shown with asterisks), two were for wool that had only been dried for 15 minutes, one was for wool that had been drying all night ; for the fourth no reason is known for the discrepancy.

Measurements of the time taken by the galvanometer in attaining its maximum deflection show no appreciable variation over the range of humidity used, though its value, 11 seconds, was not large enough to be certain to nearer than 2 or 3 per cent. Under these circumstances proportionality between vapour pressure and maximum deflection is consistent with Masson's results.

The test was sufficient for the purpose in hand when it was made. Later work, however, showed that readings of the temperature rise of cotton wool in air of a given humidity vary from day to day, although the physical conditions appear to be constant. This suggests that similar variations might have been occurring during these readings and that the straight line relationship was, therefore, possibly spurious. However, it was these results that led us to convert the instrument into one of more convenient form which may be termed the " Rotor " form.

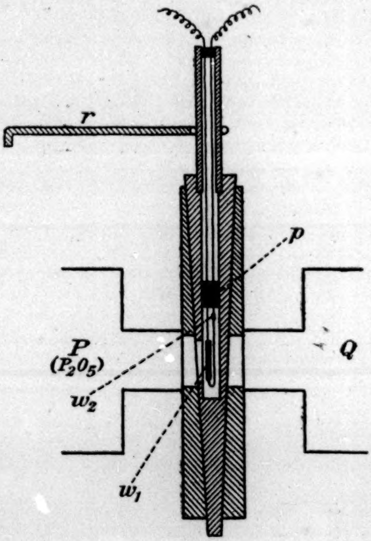


FIG. 1.

In the rotor form the chamber containing the moist air is permanently attached to the tube containing the cotton wool, and it is by rotation of this tube about its axis instead of an end-long motion that the cotton wool is exposed.

Fig. 1 shows a section through the hygrometer as first set up. The centre portion is the rotor. It consists of two brass tubes ground to fit one another with a slight taper. The inner tube contains the junctions w_1 covered with cotton wool and w_2 bare, and is pierced by a single hole opposite the latter. The outer tube is pierced by two holes opening respectively into the two metal chambers P and Q. "*p*" is a sealing-wax plug and "*r*" an arm for rotating the inner tube.

In chamber P a tray of P_2O_5 is placed. Chamber Q is either open to the air or contains a solution, the vapour pressure of which is required.

As shown in Fig. 1, the wool is exposed to P. It is therefore drying. A half turn of the inner R tube will expose it to the moist air of Q.

The procedure was to turn the rotor, note the maximum galvanometer reading, and then turn the rotor back again so that it was left to dry for about half-an-hour before obtaining another galvanometer reading.

We hoped that when the cotton wool had had time to settle down its power of absorbing moisture would become constant, and an occasional single standardisation of the rotor hygrometer with air of known humidity would be sufficient.

This, however, is not the case. The wool reaches a condition of average constancy, but fluctuates up and down. In illustration of this we give in Table **II.** the record of a certain rotor which has been most frequently standardised. Neglecting the very low value of the constant (deflection per mm. pressure) that followed a rest of more than 7 weeks, there are fluctuations of about 8 per cent. above and below the mean value of the constant during the weeks the instrument was in use.

TABLE II.

Date.	Pressure of water vapour in mm. by drying tube (A).	Mean rotor reading in cm. (G).	Temperature of rotor Centigrade.	Constant. G/A.
Aug. 23 ...	13·2	8·09	22·3	0·613
„ 24 ...	11·2	6·47	19·9	0·578
„ 24 ...	16·9	10·26	26·0	0·605
„ 24 ...	15·2	9·74	24·0	0·641
„ 27 ...	20·1	12·00	23·9	0·595
„ 29 ...	16·2	9·74	25·0	0·602
Sept. 1 ...	14·9	9·04	25·1	0·605
„ 1 ...	14·7	8·10	24·8	0·552
„ 4 ...	14·8	8·64	25·4	0·581
„ 4 ...	14·8	8·21	25·3	0·556
„ 9 ...	13·5	7·64	25·1	0·569
„ 9 ...	13·1	7·48	25·3	0·572
„ 12 ...	13·9	8·77	24·4	0·630
„ 12 ...	13·1	7·79	24·4	0·595
„ 24 ...	12·0	6·83	25·1	0·572
„ 24 ...	11·2	6·32	25·3	0·565
„ 25 ...	13·3	7·63	23·9	0·575
	Rotor	not being used, wool	drying.	
Nov. 2 ...	10·5	7·13	22·4	0·680
„ 2 ...	10·1	6·58	22·2	0·650
„ 4 ...	9·65	5·73	20·1	0·595
„ 4 ...	9·25	5·94	19·8	0·646

We have not yet been able to trace these fluctuations to any certain cause; but we feel sure that they are inherent in the method as it is dependent upon the rate of absorption of water vapour by a minute lump of material of complex surface, a phenomenon which is influenced by many factors.

Attempts have been made to find a substitute for cotton wool which, by presenting a more homogeneous surface might act more consistently. Of these cigarette paper was sufficiently sensitive and probably more reliable, if used in the following way.

The fact that both with wool and paper the galvanometer deflection is apparently too small after a long period of drying (for example, the point marked with a cross in the cotton wool curve) suggested that the fluctuations might be due to some inhibiting influence delaying the start of the condensation on a very dry surface. Readings were, therefore, taken of the reverse phenomenon, namely, the cooling of the wool when the rotor was turned from the damp atmosphere to the P_2O_5 chamber.

It was shown that the actual deflection obtained was greater on turning from wet to dry than it was on turning from dry to wet. A typical set of results is given for cigarette paper in Table III., the readings being taken alternately with 10-minute intervals. Actually two hygrometers were used, one for each process. Previous calibration under the same conditions (dry to damp) showed that their constants were sensibly the same.

TABLE III.—*Cigarette Paper.*

Maximum Galvanometer Deflection.	
Dry to damp.	Damp to dry.
(3·99)	5·58
4·07	5·55
4·07	5·56
4·10	5·59
4·13	5·59
4·18	5·60
4·18	5·58
4·13	5·60
4·18	...
4·13 Mean. (Omitting 1st reading.)	5·58 Mean.

$$\frac{\text{Mean damp to dry}}{\text{Mean dry to damp}}=\frac{5{\cdot}58}{4{\cdot}13}=1{\cdot}35.$$

It will be seen that when the rotor is turned from wet to dry, not only is the reading increased 1·35 times, but the variations are certainly less marked. This indicates that when it is possible this is the best way to use the instrument. But some fluctuations in constant did take place from week to week, as may be seen in Table IV.

To what extent the temperature of the surface affects the readings we have no definite information, but in any case the necessity for continual recalibration prevents the instrument from being of practical utility in this particular form except, perhaps, in special cases such as the one for which it was originally designed.

But one point is clearly brought out. Though the fluctuations are marked from day to day, readings taken in rapid succession are extraordinarily consistent with one another, as may be seen from Table III. (damp to dry), taken in ordinary laboratory air on a day when atmospheric conditions were apparently constant.

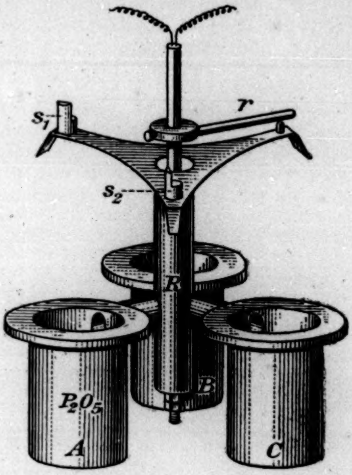


FIG. 2.

TABLE IV.—*Paper Hygrometer* (*Damp to Dry*).

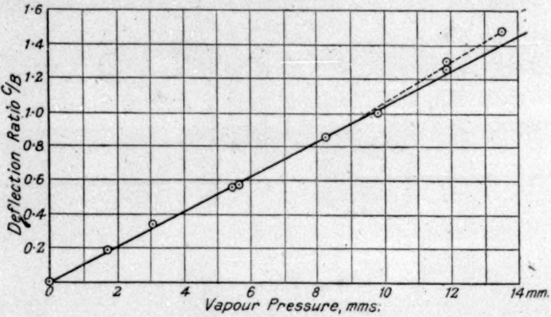
Date.	Temperature of room, in C°.	Change of weight, chemical hygr. C.	Galv. deflection. Paper hygr. P.	P/C.
1920.				
Sept. 9 ...	16·9	0·0252	5·39	214
,, 10 ...	15·5	0·0211	4·52	214
,, 10 ...	15·9	0·0203	4·33	213
,, 12 ...	15·9	0·0228	4·65	204
,, 12 ...	16·2	0·0234	4·65	199
,, 30 ...	14·7	0·0248	5·37	217
Oct. 6 ...	15·3	0·0272	5·91	217
,, 17 ...	15·2	0·0270	5·70	213
,, 23 ...	16·1	0·0222	4·65	212
Nov. 9 ...	16·8	0·0202	3·92	194
,, 10 ...	15·1	0·0179	3·69	206
Dec. 6 ...	13·9	0·0140	2·50	181

Recently, therefore, we have taken the subject up again and converted the instrument into a comparative one by the addition of a third chamber containing a liquid of known vapour pressure, as shown in Fig. 2. "*A*" contains P_2O_5, "*B*" a solution of sulphuric acid of known vapour pressure (*s*)* and "*C*" air the humidity of which is required. The rotor *R* normally open to *A* is exposed by rotating the arm *r* first to *B* and then, after a short drying period, to *C*. The ratio of the two readings obtained gives the vapour pressure in terms of a standard; s_1 and s_2 are stops to ensure as far as possible identity in the degree of exposure in the two cases.

In general the constant of the cigarette paper will not change appreciably in this short time, but in practice one naturally completes the series by repeating the *B* reading and using the mean between them.

A thin strip of paper was used, about 0·3 sq. cm. in area and about 0·5 milligrams weight. It was twisted fairly tightly round the junction and bound to it by a strand of thin copper wire taken from a piece of lighting flex.

Table V. gives some typical results. In experiment 1 the glass lids of *B* and *C* were removed so that the two vessels were both open to the same atmosphere. The equality of the two deflections to within 1 per cent. shows that the geometry of the apparatus was nicely adjusted and the readings, therefore, independent of the direction of turning. In the second, third and fourth the vessel *B* contained an inner glass vessel with a mixture of one part sulphuric acid and four parts water (vapour pressure at 16°=9·9 mm.); the contents of *C* were varied. The last two vertical columns show the comparison of the ratio of the two deflections with the ratio of the vapour pressures of the two solutions obtained from tables.



CURVE I.

These and other results are summarised in Curve I., the mixture in *C* being varied from pure acid to pure water. It will be seen that except near saturation the proportionality between maximum deflection and vapour pressure is quite satisfactory.

It is true that when two solutions of very unequal vapour pressures are compared the agreement with the value from tables might be better. But if the instrument were being used to determine a given vapour pressure one would naturally use, if possible, a standard vapour pressure in *B* of the same order of magnitude as the unknown and not widely different. An illustration of its use in this way is shown in experiment 5 in the Table, in which an acid solution of one part to 11 parts is compared with one of one part to seven parts. The agreement is there good.

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* Ordinary pure sulphuric acid does not appear to be the best standard solution to use, as the sensitivity of paper exposed to it decreases with time. Orme Masson observed the same phenomenon and traced it to the action of oxides of nitrogen. Carefully purified acid did not give rise to it.

TABLE V.

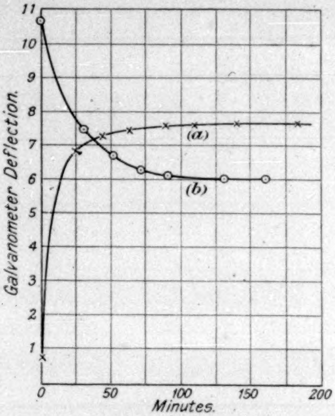
Experiment.	Temperature.	Galvanometer deflections.		Mean C/B.	X/S calculated from tables.
		B.	C.		
		3·70	3·70		
1.	12·5°	3·73	3·62		
		3·70	3·67	0·992	1·000
		3·60	3·60		
		(3·68)	3·65		
	12·7°		(3·65)		
2.	16·8°	7·55			
		7·52	4·17		
		7·42	4·17	0·556	0·56
		7·54	4·17		
	16·8°	7·51	(4·17)		
3.	15·2°	6·44			
		6·51	1·20		
		6·56	1·20	0·185	0·17
		6·57	1·22		
	15·5°	(6·52)	(1·21)		
4.	12·5°		1·95		
		5·83	2·03	0·341	0·32
		5·90	2·02		
	12·6°	(5·87)	(2·00)		
5.	16·2°	11·64	10·80		
		11·60	11·02	0·940	0·93
		(11·62)	10·90		
	16·3°		(10·91)		

For accurate work up to saturation a calibration curve is necessary unless the standard and unknown vapour pressures are nearly equal. But the linear relationship holds up to a value of humidity at least equal to those usually obtaining in ordinary air.

In our opinion, therefore, the instrument in this form is suitable for hygrometric work of various kinds and also for the purpose of measuring fairly rapidly and accurately the vapour pressures of solutions of salts or mixed salts.

A warning note should, however, be given. It is useless to take a reading soon after the vessel containing a solution to be tested has been inserted in the chamber *B* or *C*. In our case the volume of air above the liquid was about 75 c.c., and its area of cross-section 20 sq. cm. and yet it was about 100 minutes before the vapour pressure, as indicated by the deflection attained its equilibrium value.

Two cases are shown in Curve 2 (*a* and *b*); one (*a*) a series of measurements taken every 20 minutes after placing in *C* acid solution (one part to seven parts water), *C* being previously filled with P_2O_5; and the other (*b*) after inserting acid solution one part to three parts in place of water. The deflection-time curves show how with both the liquid surface and the air above it quiescent the vapour pressures only gradually attain their equilibrium value. Unless some means of agitating the contents is adopted this limits the rate at which the vapour pressures of a series of solutions could be examined. Even then in the case of a solution of very low vapour pressure, following one of high, a small change was observed for some hours afterwards. Presumably this was due to slow evolution of vapour from the walls of the metal and the glass.



CURVE II.—(*a*) *and* (*b*).

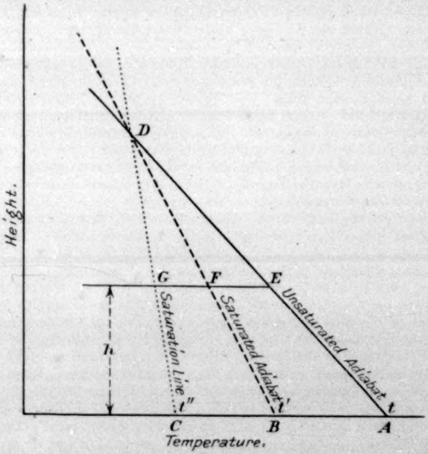
GENERAL DISCUSSION.

Sir Napier Shaw: I find myself in a position of some difficulty in opening this discussion, partly because preceding speakers have viewed the matter mainly from the point of view of the physical laboratory, and partly because the method which seems to me best suited to conditions below freezing point is apparently the one method which has not been tried—I refer to that of the self-recording hair hygrometer. After dealing with the question of hygrometers many years ago at the request of the Meteorological Council, I found that the real need, so far as meteorology was concerned, was for an instrument which should give reasonably reliable results when read by observers, not necessarily highly skilled, scattered throughout the country. Intricate questions of physics relating to sources of small errors are interesting and attractive, but of comparatively little practical importance. Hygrometry is a subject with which meteorologists concern themselves less than is generally supposed, and as hitherto treated it has been really of less service to their science than to certain of the industries. Whenever the study of humidity becomes a vital part of meteorological study the question of instrumental equipment will not present much difficulty. The slide shown illustrates the difference in point of view to which I wish to call your attention. It is a map showing the normal dew-point in July obtained by measurements of humidity by different people all over the Northern hemisphere. The values range from 300 deg. in the Arabian Sea to 275 deg. in Greenland. The question which I wish to put is, supposing that the lines shown on the map are thought to be unreliable for their purpose, which of all the instruments described this evening should be put into the hands of the scattered observers? The temperatures shown do not go below freezing-point, and a real difficulty arises when low temperatures have to be recorded. In this country such temperatures are of comparatively rare occurrence, and we make no adequate provision for them, but in such countries as Norway and Russia they are frequent, and it is the hair hygrometer that is used.

In the study of the upper air, also, very low temperatures occur, and the only instrument used has been the hair hygrometer, I think justly, because the wet-and-dry-bulb hygrometer fails altogether in the circumstances. The recording hair hygrometer seems to be specially convenient also for such conditions as those which obtain in a cold store, because there is not much variation of temperature and observations can be taken through a window more readily than with other types of instrument. I hope I have said enough to indicate that the important matter, from my point of view, is the practical utility of the instrument in service conditions rather than its scientific precision in conditions which are appropriate to a well-equipped laboratory.

Dr. G. C. SIMPSON said that as Sir Napier Shaw had dealt so fully and so well with the meteorological aspect of the case, it was unnecessary for him to say more on that subject, beyond remarking that he agreed with all that Sir Napier had said.

His own contribution to the discussion would be to describe work which had recently been done by Dr. C. W. B. Normand, of the Indian Meteorological Department, as this gave an entirely new importance to wet-bulb temperatures. Dr. Normand considers a mass of air brought into contact with a wet-bulb thermometer. Its temperature is reduced and its humidity increased, but as the bulb maintains a constant temperature there is no transfer of heat; thus the process is adiabatic. With this as a starting point, thermodynamic reasoning is applied and a most important series of conclusions reached. The following are amongst the most important.



(*a*) If t, t', t'' are simultaneous dry-bulb, wet-bulb, and dew-point temperatures respectively, then on a Neuhoff diagram the unsaturated adiabat through t, the saturation adiabat through t', and the dew-point curve through t'' all meet in a point (*D*, Fig. 1).

(*b*) As a mass of air rises adiabatically in the atmosphere its temperature follows an unsaturated adiabat, while the variations of its wet-bulb and dew-point temperature are such that they follow simultaneously a saturation adiabatic curve and a dew-point curve respectively (*F* and *G*, Fig. 1).

(*c*) The wet-bulb potential temperature is an invariant in all adiabatic changes.

(*d*) The wet-bulb potential temperature is a measure of the entropy of atmospheric air.

(*e*) When portions of air at different temperatures, but having the same wet-bulb temperature, are mixed together, the mixture has that wet-bulb temperature.

Dr. Normand also shows how the Neuhoff diagram may be used as a humidity diagram, without any assumptions involving the use of the equations on which psychrometric tables are based.

This work gives an entirely new importance to the physical meaning of the wet-bulb temperature, which is no longer to be considered simply as an interesting instrumental condition from which the humidity of the air can be determined, but, rather, that the wet-bulb temperature represents a definite thermodynamic condition depending on the entropy of the air.

Dr. Normand's Paper is at present in course of publication in the Indian Meteorological Memoirs, and should be available in this country early next year.

Mr. R. Corless: It has already been pointed out that, for meteorological purposes, the only available instruments for the measurement of humidity were the unventilated psychrometer and the hair hygrometer; and that, in the past, little use has been made of humidity observations. A new situation has recently arisen, however, for it was found that observations of humidity made on aeroplanes are of practical utility in weather forecasting, and as temperatures below the freezing-point are relatively frequent in the upper air, it has become important to find a hair hygrometer which can be used under those conditions. Interest in the properties of hair has accordingly been revived, and in view of Dr. Griffiths' remark that no important contribution on hair hygrometry has been made since the time of Regnault, it seems worth while to refer to a Paper published 20 years ago by J. Pircher (Uber den Haarhygrometer: Denkschriften d. k. Akad. d. wiss. Wien., Vol. 73, 1901), which contains a systematic comparison of the hair hygrometer under working conditions with the condensation hygrometer, and the ventilated and unventilated psychrometers. The principal results in this Paper are:—

The coefficient of elasticity of hair is 0·0027, which is greater than its temperature coefficient, and is comparable with its extension coefficient for a 10 per cent. change of relative humidity; hence it is very desirable to keep the load upon the hair constant at all relative humidities. This is not the case in the Richard hygrograph.

Hair hygrometers of the same form, when exposed to the same conditions of humidity, differ from one another by less than 2 per cent.; while the differences of relative humidity indicated by two different kinds of hair hygrometer are less than 5 or 6 per cent.

A hair hygrometer when kept for some days in a closed room at a constant relative humidity gradually reads too high by 4 or 5 per cent.; such a drift of index error does not occur if the instrument is exposed outside to the natural changes of relative humidity in the free air.

The indications of the hair hygrometer are independent of temperature, and differ by not more than 4 or 5 per cent. from the true relative humidity as indicated by Alluard's dew-point hygrometer or the Assmann psychrometer. They are nearly independent of wind velocity.

The sensitivity or the speed with which the instrument adjusts itself to a different value of the relative humidity is greater for the hair hygrometer than for the psychrometer.

In these instruments it is, of course, assumed that both the zero error and the error of scale value of the hair hygrometers used have been reduced to zero or nearly zero.

Mr. W. J. Hall: The question of humidity is of considerable importance in the textile industries. Its commercial aspect may be illustrated by the fact, for instance, that, on the Bradford system, a spinner sells worsted yarn in "18¼ per cent. condition," *i.e.*, the moisture content is 18¼ per cent. of the dry weight of the wool. Obviously with a yarn selling at several shillings per pound, the spinner will be careful not to sell wool instead of water. In certain stages of the manufacture—drawing and spinning by the Continental method for example—it is necessary to humidify the mill artificially. It is true that the limits are fairly wide; but humidification is necessary in order to work the material satisfactorily. If the wool is too dry it becomes strongly electrified during such operations as spinning. The short fibres are then thrown off from the yarn. This causes the yarn to be uneven as well as increasing the "fly waste." In rooms artificially humidified the minimum bulb difference allowed by the Board of Trade is 2°F. It is interesting to observe in a spinning mill the increase in the number of ends breaking down as the day advances, and the atmosphere gets drier.

As regards laboratory work, we have turned to this question only during the last few weeks. We have been using an Assmann Psychrometer, and we find that it is in error by as much as 6 per cent. when compared with the chemical hygrometer. Since reading Dr. Griffiths' Paper we have decided that this is due to insufficient ventilation, for our instrument could not possibly give the necessary minimum air velocity of 3 metres per second. We hope that when this defect is remedied that the instrument will be most valuable, for none will deny its convenience. The chemical method, though perfectly accurate, is tedious and inconvenient for general use.

A suggestion has been made, and is at present under test, to use as a hygrometer a piece of cloth freely exposed to the air. The moisture content of worsted in equilibrium with a moist atmosphere has been investigated by Hartshorne. Preliminary experiments indicate that the lag is rather large. Loose wool might work better, as one would expect it to respond more quickly to changes in humidity than a piece of cloth in which the fibres are twisted together.

In any research in which textile materials are used it is important to investigate and record carefully their history. Otherwise the value of the work might be greatly diminished. As an example the late Prof. Trouton's work on flannel might be mentioned. The term "flannel" covers a range of materials ; flannel may be pure wool, or it may be a mixture of cotton and wool. The results of such experiments must be applied with considerable caution. I am sure that the Director of Research at the British Research Association for the Woollen and Worsted Industries would be only too willing to assist any investigator using textile materials in research work to get samples of known history. Another point to be borne in mind is that woollen materials usually contain about 0·4 per cent. of soap when they leave the mills. This is difficult to remove by water alone. The material can be freed from soap by continued extraction with ether and alcohol.

I should like to ask Dr. Griffiths if he has observed any secular change in the hygroscopic properties of the materials he has used in his researches. Observations taken at the Bradford Conditioning House show that wool kept under test for two years has a distinctly smaller hygroscopic capacity during the second year than it has during the first year.

DR. J. S. ANDERSON: Some years ago, Giraud* suggested that the hygroscopic property of glycerine, combined with the fact that the refractive index of glycerine-water mixtures varied with concentration over a comparatively wide range, might be utilised in determining the relative humidity of the air. The method he employed consisted in placing a piece of cigarette paper, initially soaked with glycerine, on the exposed face of the upper prism of an Abbé refractometer, and measuring the refractive index of the glycerine-water mixture as its concentration changed with varying degrees of humidity. For a temperature of 20°C. Giraud developed the formula—

$$H=100(1-a^x),$$

where $H=$ relative humidity

$$a=\frac{n-1\cdot333}{1\cdot4747-1\cdot333}$$

$$x=n^{2\cdot666}$$

and $n=$ refractive index of mixture, 1·4747 and 1·333 being taken as the refractive indices of glycerine and water at 20°C. respectively. From this formula he calculated tables giving the humidity in terms of the measured refractive index. He also gave the temperature coefficients of refractive index for different ranges of concentration, in order to enable the relative humidity at other temperatures to be calculated. In measurements which he made with this method he obtained for the relative humidity the values 62·89 and 94·20, where the correct values were 63·17 and 94·36 respectively ; no mention is made of what the comparison method was.

Now it seems to me that, although the accuracy which Giraud obtained compares very favourably with that attainable by some of the better known methods, the refractometric method could be made more accurate and more sensitive. For this it would be necessary in the first place to investigate more thoroughly than he appears to have done the equilibrium vapour pressures and the refractive indices of glycerine-water mixtures over a considerable range of temperature. If this were done with the accuracy attainable by sensitive methods which are now available, I think that the reliability of humidity measurements carried out by this method could be made at least to equal that obtained by any other method.

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* F. Giraud, Journ. de Phys., 5th Ser., 3 (1913), 900.

It is, however, with the sensitivity of the method that I wish to deal at present. Even if the laws connecting the relative humidity with variation of index of the resulting glycerine-water mixture are not accurately known, the method, owing to its "openness of scale," is a very sensitive one as it stands, and should prove of considerable use in detecting small variations of humidity or for assisting in the control of constant humidity, especially in cases where the temperature does not vary over a wide range. Its utility would, of course, be greatly widened whenever the above mentioned laws were accurately determined.

As the Abbé refractometer is designed to measure refractive indices from about 1·3 to 1·7, it is necessary to employ for the standard prism a glass having a high refractive index. Thus in one particular instrument $\mu_D=1{\cdot}7336$. Now it occurred to me recently, while repeating some experiments which I carried out last year in connection with differential methods of refractometry,[*] that Giraud's method of measuring humidity could be made more sensitive by using a hollow prism containing pure glycerine instead of a high index glass prism. It would, of course, require to be sealed up in order to prevent any changes in the index of the glycerine other than those due to temperature fluctuations. In cases where the method is applied to a small range of relative humidity it would be possible to increase the sensitivity still further by filling the prism with a glycerine-water mixture having a refractive index just greater than that which corresponds to the lowest degree of humidity in the range.

In order to show the advantage of using such a modification of Giraud's method, I have calculated the angles of emergence corresponding to relative humidities of from 40 to 100 per cent. at 20°C. for different standard refractive indices and for prism angles of 60 deg. and 90 deg. The former angle is that used in the Abbé refractometer, while the 90-deg. angle cell is the arrangement suggested by Hallwachs[†] for measuring small differences in refractive index. We can get a rough idea of the relative sensitivity if we assume that we can with the instrument detect a difference in refractive index of one unit in the fourth decimal place.[‡] This corresponds approximately to a difference in angle of 0·6′. Thus, with Giraud's method one could detect a difference in relative humidity of from about 1/9 to 1/50 per cent., according to the portion of the range under consideration, whereas, if the glass prism were replaced by a hollow prism containing glycerine, the detectable difference would vary from about 1/30 to 1/80 per cent. By restricting the range in special cases, as suggested above, the detectable difference may be made even less than 1/100 per cent.

A further advantage in the suggested modification of Giraud's method is to be found in the fact that the dispersion effects are considerably reduced, since the dispersion of the glycerine-water mixture will not differ very greatly from that of pure glycerine, except perhaps at high humidity values. Thus it would probably be found unnecessary to use a compensating device as in the Abbé refractometer.

In a similar way temperature effects would be reduced, as the temperature coefficient of the mixture will not differ greatly from that of glycerine, except again when one is dealing with high values of humidity. It should be borne in mind, however, that in cases where the temperature is varying rapidly the comparatively large volume of glycerine in the prism would not attain the appropriate steady state so rapidly as the film of the glycerine-water mixture. In such cases it might be advisable to use a fluor crown[§] glass prism as the standard, thereby sacrificing the advantages of similar dispersions and temperature coefficients, but at the same time retaining the advantages accruing from the use of a lower refractive index.

With regard to the relative advantages of the 60 deg. and 90 deg. cells there is very little to choose so far as sensitivity is concerned. The former has the advantage that the refractive index of the standard can be measured directly by removing the cigarette paper and determining the critical angle of emergence. On the other hand, the latter is probably easier to make accurately, and it has the further advantage that the calculations involved are somewhat simpler. For some purposes it might be convenient to use a cell having angles 30 deg., 60 deg. and 90 deg. and to make suitable arrangements for utilising either the 60 deg. or the 90 deg. angle.

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[*] *See* J. S. Anderson, Trans. Opt. Soc., 22 (1920-21), 156.

[†] W. Hallwachs, Wied. Ann., 50 (1893), 577 ; *see* J. S. Anderson, *loc. cit.*

[‡] A good instrument will yield this degree of accuracy, but it is a little doubtful whether one can get a sharp enough edge to set on when using the cigarette paper method, owing to difficulties connected with the illumination. The sensitivity may, therefore, not be quite so great as the deduced values would indicate.

[§] It is possible to obtain a fluor crown glass with approximately the same refractive index as that of pure glycerine.

There are one or two other modifications of Giraud's method which might be found of some use, especially where a high degree of accuracy is not required. For example, instead of setting on the line of division between dark and light portions of the field of view in the observing telescope, it would be possible to use a microscope focussed through the angle of the cell on a portion of the cigarette paper, preferably on an edge.* The microscope could then be rotated about an axis approximately passing through the apparent position of the edge, until the image just disappeared. This position of the microscope would then correspond to the critical angle of emergence. In order to obtain a sharp disappearance of the image it would be necessary to reduce the aperture of the microscope. I have tested this method of observation in a rough way and it seems to show some promise of success. Quite a good approximate setting can even be made without a microscope at all, by observing through two small pin-holes or narrow slits fixed to the two ends of a piece of tubing. Any transverse motion of the image in the field of view, as the angle is altered, could be compensated by introducing a plane parallel piece of glass in the path of the beam and rotating it until the image is brought back to the centre of the field. Such a compensation would not affect the value of the measured angle of emergence, provided the piece of glass is of good quality and has accurately plane parallel sides.

The suggestions which I have briefly outlined may be of some assistance to those who wish to investigate the method more fully. Some of the points dealt with will probably enable those who find the ordinary Abbé refractometer an expensive piece of apparatus to try some of the modifications which involve the use of simpler apparatus already at their disposal.

In the portable form of dew-point apparatus (Fig. 9), described by Dr. Griffiths, there is one minor defect which could easily be remedied. On passing from an observation of the dew-point thimble to a reading of the thermometer scale it is necessary to re-focus the observing telescope. This difficulty could be overcome in a number of ways. For example, the telescope could be permanently focussed on the thimble and a suitable lens introduced between the telescope and the movable mirror so as to bring the image of the scale into the same focal plane. It might be necessary to provide means for altering the position of this auxiliary lens as the mirror is moved into different positions relative to the scale. An alternative method would be to employ a binocular arrangement similar to that used in some types of range-finder. The thimble would then be observed through one of the telescopes and the scale through the other, a focussing adjustment being provided on the latter so as to allow of observations on different parts of the temperature scale.

In connection with the distant reading dew-point apparatus (Fig. 10), I would like to suggest that any disturbing temperature effects due to the heat radiation from the tungsten lamp might be eliminated by utilising the reflection of light instead of heat. The polished surface on which dew forms could be introduced into the path of a beam of light (from which heat radiation had been eliminated by suitable filters) in an optical system which forms a sharp image of a convenient object, such as a fine pair of cross-lines. Then, as soon as dew begins to form on the surface the image will become indistinct. Such a modification would not, of course, readily lend itself to adaptation as continuous record method.

Mr. A. H. Davis, M.Sc.: It is interesting to apply the principle of similitude to the case of the ventilated wet and dry bulb-hygrometer. Radiation being negligible in this case, the temperature of the wet bulb falls until the diffusion of heat to the bulb from the warmer incident air just balances the heat loss by evaporation due to the diffusion of water vapour from the bulb into the same air stream.

The table of variables given contains those involved in the convective transfer of heat by a moving fluid,† together with additional quantities necessary to determine the vapour diffusion effect.

This diffusion of water vapour will depend upon its diffusivity D in air, and upon the difference $(m_1 - m_0)$ between the concentration (m_1) of water vapour in the air in contact with the wet bulb and that (m_0) in the air at a distance. Assuming the air in contact with the wet bulb to become saturated with water vapour, then m_1 represents the mass of water vapour necessary to saturate

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* An alternative would be to use a fine thread of cotton wool or similar substance instead of the cigarette paper.

† *Cf.* Rayleigh. Nature, XCV., p. 66 (1915).

unit volume of air at a temperature t_w. If L is the latent heat of water at temperature t_w the heat equivalent of the transfer of water vapour will depend upon $L\ (m_1-m_0)$. This quantity is represented by β in the table below.

Quantity.	Symbol.	Dimensions.			
		Mass.	Length.	Time.	Temp.
Thermal conductivity of the gas	k	1	1	—3	—1
Capacity for heat of gas per unit volume .	c	1	—1	—2	—1
Kinematical viscosity	ν	...	2	—1	...
Diffusivity of water vapour in air..........	D	...	2	—1	...
(Latent heat of water) × (Excess concentration of water vapour at surface of wet bulb) = $L(m_1-m_0)$ = ...	β	1	—1	—2	...
Temperature excess dry bulb	$t-t_w$	...	...	...	1
Velocity of air stream	v	...	1	—1	...
Linear dimensions of the body	1	...	1	...	...

Then putting $t-t_w=k^p c^q \nu^r D^s \beta^t v^u 1^w$,
we have from dimensions :—

by mass $0=p+q+t$
length $0=p-q+2r-2s-t+w+u$
time $0=-3p-2q-r-s-2t-u$
temperature $1=-p-q$.

Whence $p=-1-p$; $t=1$; $u=w$; $r=-p-s-u$;
so that $t-t_w=(\beta/c)\ \ (k/c\nu)^p\ \ (D/\nu)^s\ \ (v1/\nu)^u$.

By the kinetic theory of gases there is a numerical relation between the diffusivities* of mass (D), momentum (ν) and energy (k/c). Consequently in the above equation the ratios $k/c\nu$ and D/ν are constants, and the equation becomes

$$(t-t_w)=(\beta/c).\ f(v1/\nu)$$

Here, converting to Principal Skinner's notation, $c=S_\rho\rho$ and β, *i.e.*, $L(m_1-m_0)$ is readily proved to be $L(p_w-p)\ \rho\sigma/P$, whence a simple re-arrangement gives

$$p=p_w-\frac{PS_p}{L\sigma}.\ (t-t_w).\ f\left(\frac{v1}{\nu}\right)$$

It appears that with the usual laboratory apparatus the wet bulb reading is independent of the velocity of the air for speeds greater than 4 metres per second, and in these circumstances $f(v1/\nu)$ is a constant. Below this speed, however, the reading depends upon the velocity, which should therefore appear in a complete equation. The form of the result implies that the size of the apparatus may have an influence at speeds less than 4 metres per second.

DR. GUY BARR (communicated remarks) :

Use of Diluted Sulphuric Acid for Standardisation.

The suggestion is frequently made that, since the relative humidity above a dilute sulphuric acid mixture varies only slightly with the temperature, it is unnecessary to take any precautions to keep the temperature constant—*e.g.*, in standardising hair hygrometers. In the absence of efficient stirring the lag in the attainment of equilibrium is, however, so considerable that large errors may be made in this procedure. For example, the air in a closed bell-jar over pure water rarely attains a relative humidity of more than 90 per cent., unless the whole is thermally well insulated. Dr. Griffiths mentions that it is necessary to filter air bubbled through acid mixtures in order to remove spray ; it is, perhaps, not generally known that the air above concentrated sulphuric acid is acid to litmus, even in the absence of spray. Regnault's data, when calculated to relative humidities and plotted against concentration of sulphuric acid, do not lie very well on any smooth curve. In view of the fact that each of his mixtures contained acid and water in fairly simple molar ratios, and of the evidence for the existence of hydrates which is afforded by other physical properties, it appears somewhat arbitrary to use a smoothed curve for interpolation of round values of relative humidity. It is safer, in the absence of equally reliable measurements for intermediate concentrations, to use the same strengths of acid as were employed by Regnault.

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* Maxwell, Ency. Brit., 9th Ed., Article " Diffusion."

Hair Hygrometers.

The effect of exposure to low humidities is noted in Dr. Griffith's paper; it has been the experience of some workers that prolonged exposure to very high humidities causes a similar change in zero. Apropos of Sir Napier Shaw's suggestion that other substances might be used in place of hair, I may mention a Paper by Crockatt and Forster (*J.S.C.*, I., 1919, XXXVIII., 95), in which the behaviour of Chardonnet silk is described. Artificial silks of various kinds have been examined at the N.P.L. by Miss Hadfield, and found to be fairly satisfactory at 40 to 60 per cent. relative humidity. In drier atmospheres the contractions become unduly small for convenient measurement, and above 66 per cent. the stretching is apparently continuous, and requires several days for recovery.

Absorption Hygrometers.

I suggest that, for use at ordinary temperatures, the apparatus designed by Tyndall and Mayo for cold store work would be more convenient if the phosphorus pentoxide were replaced by water; not only is the manipulation simplified, but the heat absorption on evaporation of water is less than the evolution due to combination with P_2O_5, an advantage of some importance when the quantity of moisture in the air is large. In a cold store the plunger might be filled with snow, but it would appear to be essential at these low temperatures (say 50°C. below the body temperature) that a compensation tube should be used, which should be handled in precisely the same manner as the absorption tube; a temperature rise of only 0·7°C. is sufficient to neutralise the whole of the absorption effect from fully saturated air at —10°C. The change in volume due to motion of the oil in the U-tube is not negligible, and should be allowed for in the calculation of the constant.

In this connection, reference may be made to the absorption hygrometer devised by Rideal and Hannah (*Analyst*, 1915, 48), in which the effect of adsorbed moisture on the walls is satisfactorily avoided by measuring the dried gas, not in the original burette over mercury, but over sulphuric acid in the absorption pipette of a simple gas analysis apparatus.

Thermal Effect on Cotton.

In the Paper by Tyndall and Chattock the rises in temperature are stated to be proportional to the vapour pressure. This would be the case so long as the temperature was kept constant, as in the earlier experiments; but when the temperature was varied, as appears to have been the case afterwards, the deflections of the galvanometer would be expected to be proportional to relative humidity, as shown in Dr. Griffith's curve.

Electrical Resistance.

Some results, which I have obtained in endeavouring to use the variation in electrical resistance of cotton as an indication of the hygrometric state, may be of interest. As with other "insulators," it is essential to employ fairly high frequency A.C. in order to avoid polarisation and to keep the current small to avoid warming the cotton. With an ordinary "buzzer" and telephone bridge steady readings were easily obtained. The scoured airship cotton fabric was wrapped on a gilt tube, which formed one electrode, and was covered with a winding of gilt wire, which was the second electrode. The actual value of the resistance depends on the tension used in winding, etc.; but varies to an extraordinary degree with the relative humidity—*e.g.*, in one model it was 850 ohms at 90 per cent. relative humidity, and 780,000 ohms at 33·3 per cent. In the calibration curve the log of resistance is plotted against relative humidity, and gives only a small curvature. Agreement within 1 per cent. can be obtained with rising or falling humidity, but the lag is very considerable, and errors of 5 to 10 per cent. may occur if the humidity is varying with any rapidity. The trouble of lag is experienced in all hygrometers which depend on the attainment of equilibrium between a colloid and the air.

Dew-point Method.

Some precautions in the use of this method may be noted in connection with Dr. Griffiths' experiments. The difference of 0·3°C. between the temperature of the ether and of the silver surface corresponds with an error of 2 per cent. in relative humidity near saturation. The figure refers only to the particular apparatus and particular rate of cooling used in his experiment;

an approximate calculation points to an error of only 0·05°C. for a static difference of 10°C. between the ether and the free air in the case of a silver wall, and of some 0·1°C. for a glass wall 0·1 mm. thick. Most of the difference recorded is due, Dr. Griffiths tells me, to lack of stirring of the ether in these experiments, and is not observed when a slow stream of bubbles is used, as recommended later in the Paper, to keep the ether stirred during the disappearance of the dew. A box is of considerable assistance in avoiding disturbances due to the breath and *hands* of the observer, but it must not be too small. The estimate of the weight of a dew film gives the required data for the size of the box; taking 20 c.c. as an average result for the quantity of air which is saturated by the dew on 1 cm.^{2} at 0°C., it appears that a volume of 2 litres is necessary if the results obtained in a box at 0°C. are to be accurate to 1 per cent.

It is obviously desirable to have the temperatures of appearance and of disappearance of dew as close as possible; if, however, the apparatus is very slowly cooled, the precipitation of dew is not readily visible, since the moisture tends to condense in relatively large drops, and neither the formation nor evaporation of these is sharp. With suitable rates of cooling the interval between appearance and disappearance may be reduced to 0·1°C. in ordinary air. Probably the sharpest determination at low temperatures may be obtained by cooling the polished surface in a very small enclosure and removing the enclosure at various temperatures near the dew-point.

Mr. W. H. **Withey**: The determination of the moisture in a sample of air by passing a known volume through a suitable arrangement of tubes containing powerful dehydrating agents and observing the increase of weight, is a method requiring extreme care in order to secure accurate results, but, with the possible exception of the dew-point method, it is undoubtedly the most accurate method yet available.

In practice, however, it suffers from several objections which prevent its adoption for everyday working conditions, and, especially in those circumstances in which one would most like to employ it. The air in cold storage chambers, for example, contains a very small quantity of water per unit volume and, in order to determine this by an absorption method, it would be necessary to employ very large volumes of air, with the consequent loss of time; and, moreover, the figure so obtained would only represent the amount of water and not its variation over the period in question.

To overcome this objection the method has been modified by taking the sample of air, treating it with a switch drying agent, and noticing the change of volume caused by the removal of the partial pressure due to the water vapour. Such a method was tried by the author, and the experience obtained proved that all such methods, in which a container for the air is used, are liable to very serious errors which render all results obtained by such forms of apparatus open to suspicion.

The apparatus used was a modification of the well-known Haldane apparatus for the rapid estimation of CO_2 in air. The burette was filled with dry mercury and the sample of air collected by lowering the rubber tube. The gas was next passed over in to a cylindrical vessel containing sulphuric acid packed with glass tubes, and the operation repeated until the volume remained constant. It was noted that, after the first transference, the volume rapidly diminished and that, after six, the succeeding decreases became very small.

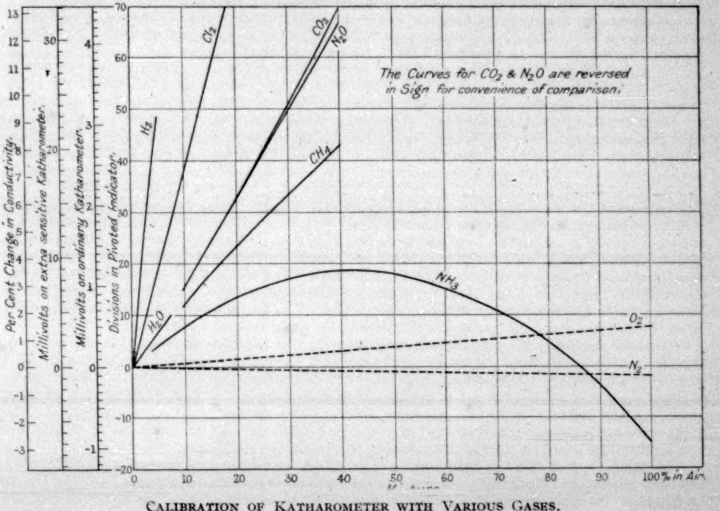
There is little doubt that one to three transferences of the gas would suffice to dry the air, and that the small changes subsequently recorded must be attributed to the moistures given up by the glass bulb, and was not complete for hours. The apparatus is, as regards measuring a change of volume, one of great convenience in speed and accuracy. The graduated portion of the burette contained 600 divisions, each corresponding to 1/2,000 of the whole volume, and it was easily possible to read to 1/10 of each division corresponding to an accuracy of 0·005 per cent., but, owing to this defect of the glass, the apparatus was not suitable. A modification was tried by using water in place of sulphuric acid, and noticing the increase of pressure resulting on complete saturation, but this also proved unsatisfactory, owing to the well-known fact that air in a state of complete saturation is almost impossible to obtain.

It might be of interest to determine whether the substitution of a silica vessel would overcome the difficulty of condensation. If such proved to be the case, the apparatus would be of great service, and would allow of a rapid determination of the moisture.

Dr. G. A. **Shakespear** said that in the brief time at his disposal he would confine his remarks to the form of hygrometer of which he had had the most practical experience during the last two years—viz., the katharometer. As the katharometer had been described at a previous meeting of the Physical Society (Proc. Phys. Soc. Lond., 1921, Vol. XXXIII., p. 163) he needed only to give a short summary of the construction and mode of action of the instrument. When the katharometer was fitted up in the usual form of a Wheatstone bridge, the galvanometer of the bridge could be made to read directly (for a given barometric pressure) the pressure of water vapour in the atmosphere, and by means of a recording galvanometer (such as a thread recorder) a continuous record of the absolute humidity could be obtained. It is important to note that the indications of the instrument are a measure of absolute and not relative humidity. The instrument worked over a wide range of temperatures—he had used it from —11°C. to about 30°C.—and was applicable in a variety of circumstances. Dr. H. A. Daynes would give the meeting some data from which the capabilities of the katharometer might be judged.

Dr. H. A. Daynes: It has been recognised for a long time that the katharometer might be used, if required, as a hygrometer, and a specially sensitive type has been developed for this type of work. It is not until very recently that we have considered the possibility of its use in industry as well as in the laboratory. In particular its use at low temperatures has not received a great deal of attention. We are therefore not in a position to give any considered judgment on its reliability under these more extreme conditions.

Some numerical data are available to show the order of quantities to be dealt with. These are shown in a graph, which gives very roughly the changes of conductivity and E.M.F.'s involved in the measurement of certain gases, including water vapour, when mixed with air.



CALIBRATION OF KATHAROMETER WITH VARIOUS GASES.

If the katharometer is to be useful in cold storage we must consider the case of —10°C. The saturation pressure of water vapour is here about 0·25 per cent. of the total pressure. This changes the conductivity about 1 part in 1,000. If this is to be read to 1 per cent. relative humidity it is necessary to measure the conductivity to 10 parts in a million.

There is no difficulty in making the katharometer sensitive enough, with a suspended pointer galvanometer to show changes of 2 parts in a million. Stability of zero is the limiting factor, and there is reason to believe that this can be made sufficiently good.

Work at higher temperatures is very much easier than at low temperatures, because the katharometer gives an indication of actual percentage of water vapour in the air, not relative humidity.

In cases where CO_2 is present a special form of differential katharometer would be used. Both platinum resistances would be exposed to the gas, but in one case gas would have to diffuse into the sensitive cell through an ante-chamber containing P_2O_5 or water, in which case the indicator would show the absolute humidity or the difference between that quantity and the saturation humidity respectively.

The danger of condensation on the insulation is not as real so it would seem at first sight, as the block is kept some four or five degrees above room temperature. The air in it could never, therefore, be saturated except in a rapid fall of temperature. Moreover, the lag with which the block follows a change in temperature is very much greater than the lag with which the composition of the gas in the sensitive cell follows a change of composition of the gas outside.

Dr. G. W. C. **Kaye**: If any justification were needed for the plea for a more extended study of hygrometry and its measurement, it has surely been provided this evening. Nothing but good can come of the present attempt to bring the subject up to date. Indeed, if this meeting served merely to provide fresh copy for the writers of text-books, it would have justified the efforts of its promoters. At the National Physical Laboratory investigatory work on hygrometry is still going on, and no one has been more surprised than ourselves at the variety of industries which have recently shown a very practical interest in the subject.

It may be recalled that not long ago this question of humidity was of the most vital importance to the State. This was in connection with the artificial seasoning of timber for aircraft purposes during the war—a question to which I have been asked to refer.

It was not long after the commencement of the war that the importance of the new arm of warfare became increasingly manifest and, with the growing appreciation of its utility, the output of aeroplanes was steadily forced up. This is not the place to dwell now on our limited capacity for construction in those early days. That capacity was restricted by several factors, not the least of them being timber. The chief woods used in this country for aeroplanes were walnut and mahogany (for propellers), ash (for fusilages), and spruce (for struts and spars). Such stocks of naturally seasoned highest-grade woods as existed when war broke out soon began to be exhausted, and we were confronted with a heavy problem in attempting to replenish these stocks.

In general, there had always been a prejudice in this country against the artificial seasoning of timber, in spite of its extensive adoption in America for many years. The situation, however, left us no option in the matter, and had to be met vigorously. Enormous orders for timber were placed abroad, and a considerable number of Government-controlled kilns were established throughout the country by the Ministry of Munitions. The kiln-drying was carried out by the Aeronautical Inspection Department. During the latter part of the war it was my privilege to be associated with this work of kiln-drying, and I should like to be allowed to pay tribute to the work of Mr. S. Fitzgerald and his colleagues for the way the gigantic task was tackled.

Roughly speaking, the seasoning of timber consists in reducing the moisture content from, say, 40 or 50 per cent. to a value in equilibrium with the mean atmospheric conditions. This value is of the order of 15 per cent. in this country; in the drier climate of the States the figure is about 8 per cent. The time taken in the natural seasoning of timber is, perhaps, two or three years; in the artificial seasoning, as many weeks. Two classes of kilns were employed, the progressive or Erith kilns, used chiefly for propeller woods, and the compartment (including the Sturtevant) kilns, used chiefly for ash. Spruce requires only conditioning.

In either type of kiln the temperature is gradually raised to a maximum of about 120°F., and then allowed to fall slowly. During this time the air is humidified artificially, being nearly saturated in the early stages and gradually diminished in wetness during the latter half of the process. The reduction in humidity precedes by a few days the reduction in temperature. The whole object is to encourage the timber to give up its excess natural moisture, but to give it up at the right rate. If the conditions are not right for the particular thicknesses of timber in the kiln, or if things are hurried, the timber will develop cracks, or case-harden, or the physical and structural properties will be impaired.

The temperature and humidity of the air in the kilns were indicated by recording long-arm metal-strip hygrometers of the wet and dry bulb type. The thermometer bulbs were mounted in pockets in the walls of the kilns. It is true that they were sheltered from moving air but, on the other hand, they were protected against moving workmen. It may be added that, while the maximum temperature of the dry bulb was about 120°F., that of the wet bulb was about 110°F.

One of the main difficulties of the work was the lack of skilled and intelligent labour to control the kilns so as to ensure that the somewhat complicated conditions of temperature and humidity were followed. The device was adopted (at any rate with the compartment kiln) of providing a series of process charts for the hygrometers. On these charts were two graphs, one for the dry bulb and the other for the wet, and all the operator had to do was to adjust the dampers of the kiln so that the two recording pens did not stray from their appointed paths during the two or three weeks of the operation. The travelling inspector could then tell at a glance whether his instructions had been duly carried out.

There is no doubt a very welcome latitude in timber drying, there is also a certain ambiguity in the readings of self-recording wet and dry bulb thermometers but, thanks to the care and devotion of the staff, the empirical paths of temperature and humidity laid down were religiously followed and, once we got the system established, we rarely had a failure. At the time of the Armistice, many millions of feet of air-craft timber had been kiln-dried successfully, and a popular superstition exploded as to the inherently unsatisfactory nature of artificially dried timber.

It was a source of great satisfaction to us to learn from an International Conference in London, and from a visit I paid to the States in 1918, that in the results of our methods of kiln-drying we need not fear comparison with those obtained by any other country. Had this work continued, we should doubtless have taken steps to increase the precision of the hygrometric measurements, but what I have said will perhaps serve as an illustration of the industrial value of hygrometry, if only of a somewhat rough and ready kind.

One further small suggestion as to humidity measurements. The sparking potential between point electrodes is very sensitive to the humidity of the surrounding gas and there might be more than a passing interest in ascertaining whether an hygrometer could be usefully developed on such a basis.

Mr. F. J. W. Whipple: In quoting the formula of Apjohn and August it has been assumed that it is a correct representation of the facts of the case, *i.e.*, that a certain amount of air loses heat to the wet bulb and is saturated in the process, whilst the rest of the air is not affected until this cooled and saturated air is mixed with it. The theory of Maxwell for the case of a wet bulb in a still atmosphere and the theory of Taylor for the case of a well-ventilated wet bulb both allow for the gradual diffusion of heat inwards, and of water vapour outwards from the wet bulb. In Taylor's theory this diffusion takes place in the thin layer close to the bulb, where there is stream-line motion. On either theory the success of the Apjohn-August formula is to be ascribed to the fact that the thermometric conductivity of air and the diffusivity of aqueous vapour are approximately equal.

In this connection I should like to be allowed to point out that in Principal Skinner's Paper the notations used in dealing with the theories of Maxwell and Taylor do not bring out the relation between the resulting formulæ. In quoting Maxwell's result Principal Skinner inserts a γ in the denominator of the first term in the bracket, writing $K/\gamma D$ in place of Maxwell's K/D. The justification for this γ must depend on the circumstances for which k, the calorimetric conductivity, and K, the thermometric conductivity, are defined. According to Skinner $k=K\rho S_v$, whereas Maxwell used the relation $k=K\rho S_p$. The latter form seems to represent the facts of the case better. For comparison it is perhaps advisable to omit the last stage of Maxwell's argument and quote his formula in the form—

$$p=p_w-\frac{PS_p}{L\sigma}\left(\frac{k}{D\rho S_p}+\frac{AR}{4\pi CS_p\rho D}\right)(t-t_w)$$

whilst Taylor's formula as quoted by Skinner becomes

$$p=p_w-\frac{PS_p}{L\sigma}\cdot\frac{k}{D\rho S_p}\cdot(t-t_w)$$

When radiation is neglected the two formulæ become identical.

I understand that Taylor has given a more elaborate formula in which allowance is made for the finite thickness of the stream-like flow over the wet bulb, and that he has made experiments which show that with gases other than air, and evaporating liquids other than water, the modification of the Apjohn-August formula is fully justified.

Replies to Discussion.

Principal Skinner (communicated reply): Mr. Whipple uses the words "whereas Maxwell used the relation $k=K\rho S_p'$." There is no reason for saying that he did use this relation. At the very end of Maxwell's Heat, in the appendix, he writes "K of air 0·256 square centimetres per second" as the "rate of propagation of thermal effects in still air." I obtained this from the calorimetric conductivity 0·0000558 by the formula $k=K\rho S$. Furthermore Maxwell in his article on Diffusion says the quantity which he writes K/D " is certainly less than unity and probably about 0·77." This would make D equal to 0·33 or something less. Now D has a value from experiment of 0·23 approximately if we allow the temperature to be 15°C. Then the value of $K/\gamma D$ is 0·79, much nearer to Maxwell's approximate value than 0·256/0·23 or 1·1.

On the other hand we may discuss the point from general considerations. If the air is still it appears we must use S_v, since the specific heat S_p contains not only the heat required for raising the temperature, but also the heat required to expand the gas. If a process of expansion were going on it is difficult to conceive the meaning of " still."

Dr. **Ezer Griffiths** (in communicated reply to the enquiry of Mr. W. J. Hall) stated that he had not studied that aspect of the problem. In his method of employing the " thermal effect on cotton hygrometer," changes in the absorptive power would have no influence on the results, as the instrument was calibrated each time it was used.

Dr. A. M. **Tyndall** (communicated) : In reply to Dr. Barr, we only experimented with phosphorus pentoxide in the plunger. It is quite true that in general the effect of the change in volume due to the motion of oil in the capillary tube is not negligible, because it is only a small change in pressure that one is measuring. The readings quoted in the Paper were taken with an instrument in which the error due to its neglect was about 1·5 per cent. Dr. Barr is quite right in pointing out that, in the instrument as made up for demonstration at the Meeting, the correction was considerably greater and must be taken into account ; the text is therefore somewhat misleading on this point. I regret that I was unaware of the publication of the work of Rideal and Hannah in the " Analyst," and have not seen their Paper.

In the experiments in which the temperature varied, there is no disagreement between Dr. Griffiths' results and ours. It is the ratio of two deflections for C and B (both of which are affected by the temperature) that is plotted in Curve I. with the vapour pressure of C. The ratio is unaffected by a change in the " constant " of the Paper, due to temperature or to any other cause, though the constant itself changed considerably during the set of readings detailed in Table V.

LIST OF EXHIBITS.

The meeting was preceded by Demonstrations and an Exhibition of Apparatus in the Astrophysics Laboratory, Imperial College of Science. The exhibits included the following :—

1. Apparatus illustrating the History of Hygrometry. From the Science Museum (by courtesy of Col. H. G. LYONS, F.R.S.).

2. Apparatus used in Hygrometry for Meteorological purposes. From the Meteorological Office (by courtesy of Sir NAPIER SHAW, F.R.S.).
   (1) Alluard Condensation Hygrometer (Symons's Meteorological Magazine, XIII., p. 55).
   (2) Lloyd's Hygrodeik.
   (3) Assmann Psychrometer (M.O. Observer's Handbook, 1921, p. 102).
   (4) Comin Psychrometer.
   (5) Aeroplane Psychrometer.
   (6) Model of U.S. Whirled Psychrometer (U.S. Dept. of Agriculture Weather Bureau. Psychrometric Tables, Part I., Washington, 1918).
   (7) Lambrecht's Polymeter (Hair Hygrometer).
   (8) Klinkerfues Hair Hygrometer.
   (9) Richard Hair Hygrograph.
   (10) Dines' Kite Meteorograph (M.O. 202. "The Free Atmosphere in the Region of the British Isles," p. 23. W. H. Dines).
   (11) Dines' Baloon Meteorograph, *ibid.*, p. 29.
   (12) Marvin Meteorograph (U.S. Dept. of Agriculture, Weather Bureau, Washington, 1921. "Instructions for Aerological Observers," by W. R. Gregg, p. 17).
   (13) Rough Model of a Rope Hygroscope. Designed at the Met. Office, South Kensington.
   (14) Russelvedt Hair Hygrometer as used in Norway.

3. Exhibits illustrating Dr. E. GRIFFITHS' Paper :—
   (A) Apparatus for Calibrating Hygrometers (Section 1 of Paper).
   (B) Ventilated Wet-and-Dry-Bulb Hygrometer (Section 3).
   (C) Dew-point Apparatus (Section 4).
   (D) Wall-fixture Dew-point Apparatus (Section 4c (i)).
   (E) Portable Tube Dew-point Apparatus (Section 4c (ii)).
   (F) Experimental Apparatus illustrating the principle of Distant Reading Dew-point Apparatus (Section 4c (iii)).
   (G) Distant-reading Hair Hygrometer (Section 5h).
   (H) The Refractive index of Glycerine Method.

4. The Thermal Hygrometer described in Paper by Prof. A. M. TYNDALL and Prof. A. P. CHATTOCK.

5. The Absorption Hygrometer described in Paper by Prof. A. M. TYNDALL and the late H. G. MAYO.

6. A Hot-wire Hygrometer : Dr. H. A. DAYNES (Cambridge and Paul Instrument Co., Ltd.).

7. The Edney Recording Hygrometer for Timber Seasoning : Dr. G. W. C. KAYE.

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BY

DR. EZER GRIFFITHS

[This list it is hoped contains most of the important Papers bearing on the present Discussion but it is not intended that it should be regarded as exhaustive.]

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X. *On the Diffusion of Solutions.* *By* T. H. LITTLEWOOD, *M.A., B.Sc., Hon. Research Fellow of University College, Reading.*

RECEIVED JUNE 13, 1921.

ABSTRACT.

An *optical method* is described for finding the *concentration* at various depths in a diffusing solution. The solution is contained in a closed vessel, the top and one side of which are of glass. On the glass side is a vertical scale. This vessel is immersed in another filled with water, which contains a mirror which can be rotated, and the position of which is read on a graduated scale. On a horizontal sliding stand is a telescope, which carries a horizontal wire illuminated by sodium light. The mirror is adjusted so that the image of the wire, after twice passing through the liquid, is seen on the cross wires of the telescope. The corresponding division of the vertical scale is also observed, which can be done to 0·02 mm. These measures enable the concentration to be determined at different depths, with an *accuracy* of about 0·05 gr. per litre.

The second part of the Paper details the *method of calculating the coefficient of diffusion* from a series of measures at different times. It is possible to obtain sufficient data for this in less than a day.

The *results* so far obtained are in good agreement with those obtained by other methods.

SEVERAL years ago the author described to the Physical Society a method of obtaining the concentration at various points in a solution of varying strength at different depths, without disturbing it in any way. (Proc. Phys. Soc., Feb., 1894.)

After a prolonged trial the method proved to be insufficiently sensitive to determine the coefficient of diffusion. Later on a modification of the method was devised which greatly increases the sensitivity, and preliminary results have been obtained with it, which show that it is capable of giving consistent results in good agreement with those obtained by other methods, but in a considerably shorter time.

A method is also described by which the coefficient of diffusion can be calculated from the observations, without making any assumptions as to the previous history of the solution, or as to the nature of the law connecting the observations.

(1) The principle of the method used is as follows. (*See* Fig. 1.) If a solution of a substance is diffusing into water or another solvent, the solution will vary in density at different depths, and we may assume that it consists of a series of horizontal layers, each of the same density at the same depth. Suppose Aa is a narrow pencil of parallel rays of light, incident at a on the upper surface of such a liquid, which is contained in a glass vessel. The face BC of the vessel is approximately vertical. Owing to the total reflection the light will not usually be able to emerge, but on surrounding the vessel by a larger one containing pure water the light will pass through the face. In the larger vessel is a mirror, which can be rotated about a horizontal axis O. The mirror is adjusted until the pencil is reflected back along its previous path. The path will in general be a curved line $Aabcdeh$. Suppose now the pencil is moved parallel to itself into the position $A'a'$. Let the new path be $A'a'b'c'd'e'f'h'$. Then the mirror will have to be turned through an angle hOh', in order to reflect the pencil back on its path again. Theory and experiment show that this angle is a function of the difference between the indices of refraction at e and f, and to a considerable degree of accuracy, ***the angle hOh' is proportional to the difference of the concentrations at e and f.***

This may be proved theoretically as follows :—

Calling the angles of incidence and refraction at the upper surfaces of the layers i_a, r_a; i_b, r_b, &c.; calling the indices of refraction of the layers n_a, n_b, &c., that of air n, and that of water n_0, and the angle of incidence on the upper surface i, we get the equations

$$n \sin i = n^a \sin r_a = n_a \sin i_b = n_b \sin r_b = \text{\&c.} = n_x \sin r_x \quad . \quad . \quad . \quad . \quad (1)$$

This shows that the angle of refraction for the layer x is exactly the same as if all the intermediate layers up to the xth were eliminated, and the ray passed directly from air into that layer. If x is the layer at which the pencil strikes the side of the vessel it will be refracted out into the surrounding water.

If R_x is the angle of emergence, $n_x \cos r_x = n_w \sin R_x$, . . . (2)

From (1) and (2) we obtain $n_x{}^2 = n^2 \sin {}^2 i + n_w{}^2 \sin^2 R_x$.

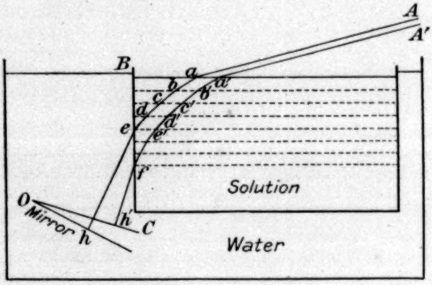


FIG. 1.

If y is the point of exit in another position of the pencil,

$$n_y{}^2 = n^2 \sin {}^2 i + n_w{}^2 \sin^2 R_y.$$

$$\therefore \qquad n_y - n_x = n \sin i \left\{ \sqrt{1 + \frac{n\ ^2}{n^2} \cdot \frac{\sin^2 R_y}{\sin^2 i}} - \sqrt{1 + \frac{n_w{}^2}{n^2} \cdot \frac{\sin^2 R_x}{\sin^2 i}} \right\}$$

which is equivalent to our first statement.

Since n_x and n_y only differ slightly, R_x and R_y are also nearly equal. Hence, denoting by δn the change in n_x, we get

$$\delta n = \tfrac{1}{2}(n^2 \sin {}^2 i + n_w{}^2 \sin^2 R_x)^{-\frac{1}{2}} \times 2 n_w{}^2 \sin R_x \cos R_x \, \delta R_x,$$

where δR_x is the angle through which the mirror is turned.

Experiment shows that the change in the refractive index is very nearly proportional to the change in concentration. Hence the change in concentration is nearly proportional to the angle through which the mirror is turned.

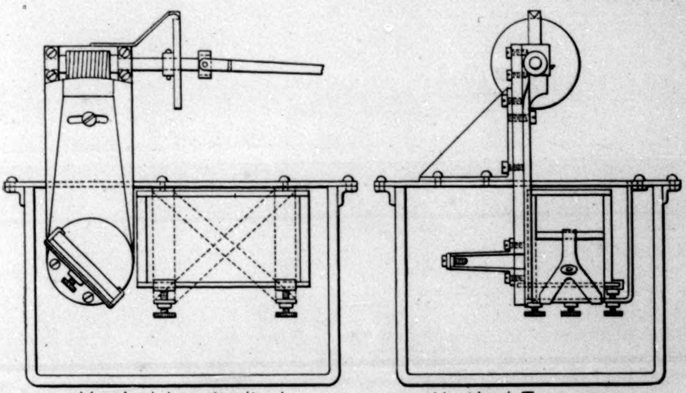
To get the best effect the incident ray should be at a small angle to the surface.

Experimentally it was found that the proportional law could be assumed for considerable differences in concentration, with very fair accuracy.

From the foregoing discussion it is clear that for any particular concentration there is a corresponding position of the mirror. These positions can be determined for several known concentrations, and other points can be calculated by the proportional law.

If we also know the exact points at which the pencil of light passes through the side of the vessel for each position of the mirror, we are able to trace the state of the liquid through the whole extent.

(2) To carry out the principle experimentally, the pencil of light is obtained by illuminating with sodium light a fine wire, which is placed horizontally close to the object glass of a telescope. In an alternative method, the wire may be at the focus of an auto-collimating telescope. The telescope can be moved parallel to itself along an accurately worked slide. A pair of cross wires is placed exactly in the optical axis of the telescope, and the mirror is turned until the image of the wire falls on the cross wires. On the inner face of the front side of the diffusion vessel a scale of millimetres is engraved. By altering the focus of the telescope, it can be focussed on this scale. The telescope can thus be placed so that the central line of the pencil passes through any desired point on the scale, and the corresponding position of the mirror can be determined for this point.



Vertical Longitudinal Section of Tank.

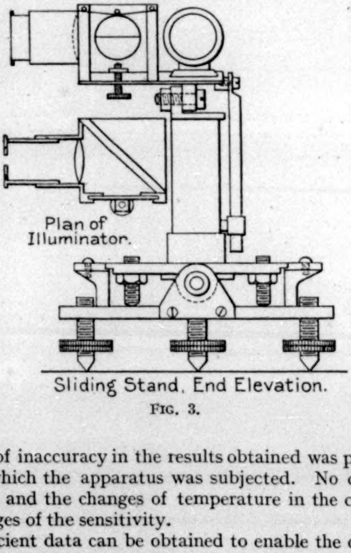
Vertical Transverse Section of Tank.

FIG. 2.

In the apparatus used (Figs. 2 and 3), the mirror was turned by a worm wheel, the screw head of which was divided into 100 parts. Consecutive readings could be obtained which did not differ by more than one or two tenths of a division, corresponding to an angle of a few seconds. The change in position of the mirror from pure water to a solution of potassium chloride, containing 100 gr. per litre, was about 220 div., so that a change in concentration of 0·05 gr. per litre was measurable. In one hour the change in angle at a fixed depth due to diffusion of such a solution was about $1\frac{1}{2}$ divisions. The scale displacement of the telescope could be read by a vernier to 1/50 mm., the corresponding vertical change in the depth of the point of exit of the pencil being about as much.

The glass diffusion vessel was 10×5×5 cm. internal dimensions.

Owing to the curvature of the upper surface, it was necessary to cover it by a plate of glass, in one corner of which was a hole 1 cm. in diameter. The positions of the mirror were first determined for several solutions of known strength. By means of suitably placed stops the vessel could be always replaced exactly in the same position after removal. After calibration the vessel was half-filled with pure water; the solution in use was then poured in through a long funnel with a fine capillary jet, reaching to the bottom of the tank. When full the hole was closed by a rubber stopper to prevent evaporation. After standing about 24 hours sufficient diffusion had as a rule taken place, to detect the change in concentration in the upper part of the vessel.



Sliding Stand, End Elevation.

FIG. 3.

The chief cause of inaccuracy in the results obtained was probably the variation of temperature to which the apparatus was subjected. No constant temperature room was available, and the changes of temperature in the course of a few hours destroy the advantages of the sensitivity.

In one day sufficient data can be obtained to enable the coefficient of diffusion of a solution of KCl to be determined. Also measurements can be made simultaneously over the whole range from pure water to a strong solution.

(3) Having obtained the concentration at various depths, the next step is to deduce from its alteration with time the corresponding coefficient of diffusion.

All we shall assume is that it takes place according to the ordinary Fick's law. Then the concentration at a given point at a depth x being u, the change in concentration in the time δt will be $k \cdot d^2u/dx^2 \cdot \delta t$, where k is the required coefficient.

Suppose that we have measured the values of u at a series of equidistant depths, then by the method of Finite Differences we can calculate the most probable values of d^2u/dx^2 at these points. For convenience we shall take 5 points, a, b, c, d, e, at distances y apart. Then, if u_a, u_b, &c., are the observed concentrations, it is easy to show that (denoting double differentiation by ″)

$$u_b''=\{u_a-5u_b/3+u_c/2+u_d/3-(u_a+u_e)/12\}/y^2$$
$$u_c''=\{4u_b/3-5u_c/2+4u_d/3-(u_a+u_e)/12\}/y^2$$
$$u_d''=\{u_e+u_b/3+u_c/2-5u_d/3-(u_a+u_e)/12\}/y^2$$
$$u_b''+u_c''+u_d''=\{3(u_a+u_e)/4-3u_c/2\}/y^2.$$

Hence the total change in the time t at the three given depths will be

$$\frac{3k}{4y^2}\int_0^t(u_a+u_e-2u_c)dt=\frac{3k}{4y^2}\int_0^t Udt,$$

when U is the value of $(u_n+u_e-2u_c)$ at any instant.

In a similar manner it can be shown that if we have five observed values of U at equal intervals of time z, viz., U_1, U_2, U_3, U_4, U_5, then

$$\int_0^t Udt=z\{14(U_1+U_5)+64(U_2+U_4)+24U_3\}/45.$$

But obviously the sum of the changes in concentration at the three depths is also equal (denoting times by numerical suffixes) to

$$u_{b\cdot5}-u_{b\cdot1}+u_{c\cdot5}-u_{c\cdot1}+u_{d\cdot5}-u_{d\cdot1}.$$

Hence, by dividing this result by $\frac{3}{4y^2}\int_0^t Udt$, we obtain k.

This method assumes nothing as to the real nature of the distribution in the liquid, but only that it has the actual observed values. As for the small range of values which alone are considered in an actual experiment, the proportional law will hold, u_a-u_c, &c., will be equal to change of angle × a constant. In the final division this constant will appear in both numerator and denominator, and therefore will divide out. It follows that we need do no more than use in the calculation the angular positions of the mirror, expressed in any units we like. These considerations considerably simplify the numerical work.

(4) As in the preliminary experiments no thermostatic room was available, the results so far obtained are not very valuable. The following set of observations will illustrate the method.

Sol. of KCl. 100 *grs. per litre diffusing into pure water. Temp.* 16°—17·8°. *Readings of mirror scale (prop. to conc.).*

Depths in cm.	At start.	After 2 hours.	4 hours.	6 hours.	8 hours.
0·7	3·2	7·2	10·1	13·3	16·5
1·0	9·7	13·5	16·4	19·9	23·0
1·3	19·0	22·5	25·6	28·0	31·5
1·6	30·5	33·7	36·4	39·3	42·6
1·9	45·0	48·1	50·0	52·6	54·5
$U=$	10·2	10·3	8·9	9·9	9·0

$y=0\cdot3$ cm. $z=2\times3600$ secs.

Sum of changes of ordinates $=13\cdot3+12\cdot5+12\cdot1=37\cdot9$ cm.

$$\int_0^t U dt = 2\cdot84\times10^5$$

$$K=\frac{37\cdot9\ \times4\times0\cdot09}{2\cdot84\times3\times10^5}=1\cdot601\times10^{-5} \text{ at average temperature } 16\cdot9^\circ.$$

A similar solution at 19·5° gave $k=1\cdot625\times10^{-5}$.

These results do not differ widely from those obtained by other methods.

DISCUSSION.

Mr. B. W. Clack expressed his satisfaction at meeting for the first time another worker in the subject which had engaged his own attention for a number of years. Other experimenters have attempted to measure the angle R^x, but the author goes further and measures the rate of change of this quantity with time. This is only of the order of 1′ per hour, and strikes one as a difficult matter to determine with certainty. The change δR is taken to be proportional to the change δn. Is this assumption justifiable? It would appear to lead to the conclusion that dn/du is also constant, whereas in the speaker's experience variations of 20 per cent. might occur. He expressed the view that the method of the author might be improved by adapting it to the case when the steady state has been attained.

Dr. A. Griffiths hoped the author would continue his experiments so as to accumulate a body of consistent results. He would like to suggest the possibility of Fick's hypothesis being inadequate to account for the facts of diffusion.

Mr. T. Smith considered it very doubtful whether really accurate results were possible with the type of optical method adopted. In a matter of the measurement of seconds of angle very special precautions were required. He thought that an interference method would get over the difficulty. In regard to the theory involved in the calculation it would be an advantage to substitute the method of waves for that of rays, since the latter often leads to wrong conclusions.

Dr. J. S. Anderson asked the minimum difference of refractive index measurable by the author, and whether dispersion caused trouble.

The **Author**, in reply, said that the conclusion that δR was proportional to δn was based on the experimental results. He thought that the method of calculation might be modified so that the assumption of the constancy of k would not be necessary. In regard to Mr. Smith's remarks the greatest care was taken to ensure rigidity of the apparatus; the separate portions were bolted down to a thick marble slab, and thus far consistent results had been obtained. In regard to the theory, he had considered that the shortness of optical path justified the use of the ray method. As to sensitiveness, he had not calculated it out in terms of refractive index, but in terms of concentration he was able to detect a change of 0·05 gr. of salt per litre. No trouble in regard to dispersion was met, using sodium light for the illuminant.

XI. *On a Special Apparatus for the Measurement at Various Temperatures of the Thomson Effect in Wires.*

By H. REDMAYNE NETTLETON, *M.Sc.* (*Lond.*), *Lecturer in Physics at Birkbeck College.*

RECEIVED, SEPTEMBER 29, 1921.

ABSTRACT.

In 1916 (Proc. Phys. Soc., Vol. XXIX., p. 59) the author outlined a *steady state null method* for measuring the Thomson effect in short lengths of wire. The present Paper describes a convenient form of apparatus for carrying out measurements with rapidity and under good theoretical conditions. The short wire under test (S.W.G. 18) passes through *electrical heaters* which may quickly be brought to and maintained at steady temperatures differing by some 50°C. over the *range* 20°C. *to* 250°C. A short coil of the finest double silk-covered copper wire (S.W.G. 44) acts as the Thomson-Joule heat detector. Test experiments were performed with *iron* and *constantan* wires, yielding values of the Thomson effect at various temperatures over the range indicated. Many improvements in the method are described, chief among which is a greatly improved sensibility.

I.—*Introduction.*

In 1916 the author described (Proc. Phys. Soc., Vol. XXIX., p. 59, 1916) a method of measuring the Thomson effect in wires based upon a steady state form of application of the Le Roux-Haga principle. Owing to war conditions and the time given to theoretical study and to the finding of the best conditions of working, progress has until recently been slow; but the writer is now able to describe fully a special apparatus on the development of which he has been occupied during the last three years, and which rapidly and with close conformity to good theoretical conditions enables the Thomson effect to be measured in wires. The use of the apparatus is illustrated by experiments on specimens of constantan and iron wires over the range 50°C.-200°C. Other important improvements in the method, chief among which is a greatly increased sensibility, are also described here.

II.—*Theory of the Method.*

The essential theory of the method is given in the author's previous Paper (*loc. cit.*). A further and more detailed study of the limitations of the Le Roux-Haga principle in general has convinced the writer of the many advantages of still adhering to a rapid but, none the less, steady state form of application of this principle and, more particularly, of always maintaining the thermometric detector with its centre of symmetry in the middle of the portion under gradient of the conductor to be tested. Under such conditions the usual Joulean alternations and the Thomson reversals become merged into one operation; the sensibility is always a maximum; the size of the bolometer coil is no disadvantage; the secondary effects of Joulean heating are of minimum trouble, and, what is most important, the temperature coefficients of resistance and of the Thomson coefficient admit of the most simple interpretation in the formula of the method, given in section 4 below, which may be applied with a higher accuracy than more complex formulæ under other conditions. In particular, the determination of the ratio R/U (of resistance of wire to temperature difference between its ends) is not only practically more simple but theoretically better than that of measuring the usual ratio $w\Big/\frac{d\theta}{dx}$, where w

is the resistance per unit length of the conductor in the neighbourhood of the detector and $d\theta/dx$ the corresponding local temperature gradient.

Accordingly, the apparatus described below is so constructed that both constant temperature sources (and not one only) may be raised in temperature until they differ by some 50°C., and the Thomson coefficient measured is always that corresponding to the mean temperature of the disc-ends of the wire. Since either heater may be made the hotter, the Le Roux longitudinal interchange of the ends of the conductor for the detection and elimination of heterogenity may easily be effected without disturbing the wire and its surroundings.

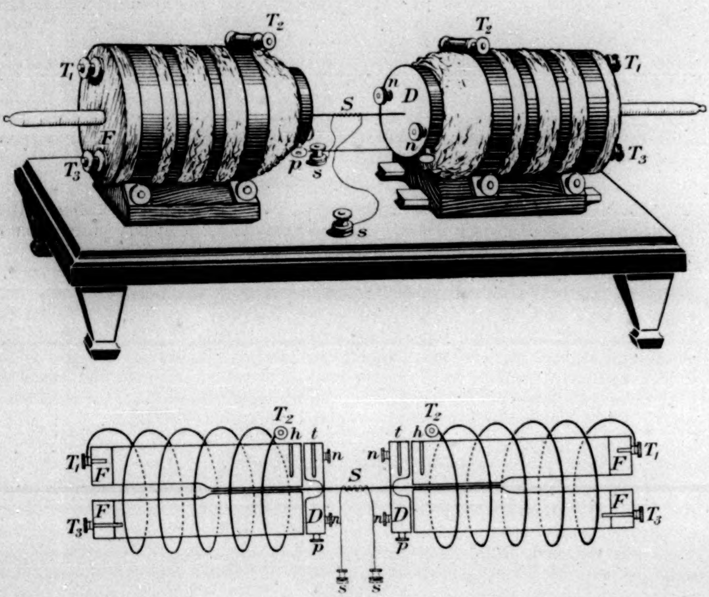


FIG. 1.

III.—*Description of the Apparatus and of the Mounting of the Experimental Wire.*

The general appearance and the essential features of the apparatus will be understood from Fig. 6. Solid copper cylinders of length 7·5 cms. are turned down to a diameter of 4·8 cm. and constitute the electrical heaters when wound with several turns of nichrome wire, S.W.G. 24, separated from them by thin mica. These heating coils are brought on the one hand to the terminals T_1 which are let in to the fibre casings *F*, and on the other hand to the terminals T_2 mounted on fibre rods which are themselves let in to the copper cylinders. Each copper cylinder is drilled centrally for half its length with a hole of just sufficient diameter to receive a thermometer, 0°C.-360°C., with which to ascertain its approximate temperature; for the other half it is drilled with a fine hole just sufficient to permit the entry of the experimental wire of S.W.G. 18. The cylinders, after winding are covered with asbestos strapped on as shown in the figure and are suitably mounted on a base board; the distance between them may be varied at will as the right-hand cylinder may be clamped in any position along its sliding rails. The terminals T_3 pass right through the fibre casing and are thus in conducting communication with the copper. The fine holes at *h* permit the insertion of thermo-junctions.

The copper discs *D* into which the experimental wire is soldered are of thickness 5 mm. and of diameter identical with that of the heaters: they resemble in appearance the ends of the preliminary apparatus (shown in Fig. 3 of the author's earlier Paper) and the cavities leading to the central holes—which just permit the passage of the wire—are carried as deep as possible. Each disc is drilled at *t* with a fine deep vertical hole of diameter 2 mm. to permit the insertion of a thermo-junction, while transverse holes allow the disc to be brought into good surface contact with the corresponding copper heater by slipping the former over screws let into and projecting from the latter and then tightening up with nuts *n*, *n*. Terminals at *p*, *p* serve for potential leads wherewith to measure *in situ* the resistance of the operative part of the wire.

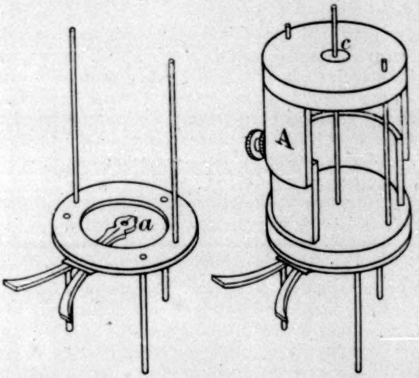


FIG. 2.

The soldering of the experimental wire into its isothermal and equipotential "disc-ends" is greatly facilitated by use of the special soldering stand seen in Fig. 7. For the discs being placed upon it in position and separated to the desired extent by means of the adjustable separator *A*, and the wire being passed through the discs and clamped at *a* when suitably situated, solder is applied at the cavity *c* which is easily accessible to the blowpipe. The discs, wire and separator are then slipped off the soldering stand, inverted as a whole and replaced (the clamp *a* being now pushed to one side) and the soldering of the wire into the other disc is then effected. As the melting point of the solder sets a limit to the temperature of the investigation, solders other than those containing only lead and tin have been experimented with. Aluminium solder (altior brand) when used with a zinc flux has proved very satisfactory for fixing the iron and constantan wires used in this investigation. Since the detecting coil is now wound on the wire after fixing it to the copper discs, it should be possible, if desired, to use a silver solder.

The detecting coil *S*, Fig. 6, is of silk-covered copper wire, S.W.G. 44, the diameters of the bare and covered wire being only 0·07 mm. and 0·14 mm. respectively; it is wound symmetrically as a thread-like covering around the middle of the operative part of the experimental wire and over about one-third of its length. To effect this use is made of the lathe, the one disc being clamped in the main revolving chuck and the inoperative part of the wire protruding some 4 cm. from the back of the other disc resting loosely in a tube held by the small fixed chuck. Lengths of detecting coil wire are left protruding to effect connection with the insulated double terminals *s*, *s*, while the coil itself is lightly varnished to prevent displacement or uncoiling.

The operative part of the experimental wire is now wound with two or three layers of silk yarn (unless temperatures above 250°C. are aimed at, when asbestos thread should be substituted) and then further protected with wool. The wire and discs are now ready for mounting on to the main apparatus, in effecting which the protruding and inoperative portions of the wire are slipped through the narrow central holes in the copper heaters against which the discs are now brought into good surface contact. After screwing up tightly the nuts *n*, *n*, the heaters are pulled and clamped as far apart as possible in order to keep the wire straight. Cotton wool or asbestos wool is now freely supplied to any exposed parts of the discs and heaters in order to adequately allay convection currents.

IV.—*Outline of Experimental Order.*

To perform an experiment the electrical heaters are first raised to the desired approximately constant temperatures which may differ from one another by from 20° C. to 80° C., according to the sensibility of the specimen under investigation. A constant electric current C_1 of some two amperes derived from accumulators is then passed through the experimental wire from hot to cold, being admitted and leaving by the terminals T_3 (Fig. 6). When the steady state is attained this current is reversed and at the same time slightly raised in magnitude (assuming σ is positive) to such a value C_2 found by trial that causes, when in a minute or two the new steady state is attained, no change in the thermometric effect on the resistance of the detecting coil *S*. The small current difference C_2-C_1, referred to hereafter as the balancing current, is measured directly in a shunt circuit on a Cambridge thermo-electric potentiometer, which serves also a moment afterwards to measure the temperatures as recorded by insulated thermo-junctions in the disc holes *t*, *t* and so the difference of temperature *U* between the disc ends of the wire. Lastly, with the electrical heaters still maintained at their respective "hot" and "cold" temperatures the resistance *R* of the wire under gradient is measured *in situ*—again on the same potentiometer now in communication with the disc terminals *p*, *p*. The length of wire having been shown in the previous Paper to be sufficiently short to neglect the effect of emissivity, the Thomson effect σ at the mean temperature of the disc ends is given by

$$\sigma = R/JU\ .\ (C_2-C_1),$$

where *J* is the number of joules equivalent to one calorie.

V.—*The Essential Connections, Operations and Measurements.*

(*a*). The electrical heating.

Both electrical heaters of about 5 ohms resistance are provided with shunts consisting of sliding rheostats of range 0-100 ohms. They may thus be put together in series with appropriate resistances permitting a current of from 0-4 amperes when joined to the 100-volt mains. Usually the current so derived is sufficiently constant during the time of proving a final balance to yield a sensibly steady state as recorded by the detecting coil, though secular changes are, of course, neutralised as far as possible at appropriate moments with the aid of the fine adjustment main rheostat or with the heating coil shunts. Though only giving the approximate temperatures the thermometers in the copper heaters are very useful. The electrical heating on the whole is very satisfactory and it is easy with practice rapidly to reproduce in a heater any desired temperature and to then adjust the current to maintain this temperature constant.

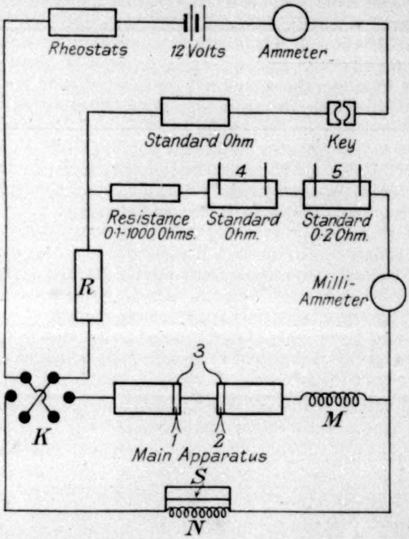


FIG. 3.

(*b*). The main electrical connections.

The electrical connections shown in Fig. 8 are somewhat more convenient than those of the earlier arrangement, for a single key *K* not only effects the commutation of the current, but at the same time produces the necessary change in its magnitude.

The main circuit consists of accumulators (12 volts), an ammeter, rheostats, a standard ohm, the specially designed low-resistance commutator K, the main apparatus and the approximate ohm M and the dummy apparatus, viz., the ohm N shunted by the rheostat S. The shunt circuit includes a milli-ammeter, a resistance box 0·1 to 1,000 ohms, a standard ohm and a standard 0·2 ohm. As the standard ohm is only required in the main circuit during a resistance measurement—in which case the shunt circuit is not used—it is arranged that it may be completely switched over from the one circuit to the other.

By means of the rheostat S it is easy to secure accurate equality of resistance between on the one hand the main apparatus, the ohm M, the operative part of K and the rheostat R and on the other hand the shunted ohm N, the now operative part of K on switching over and again the common rheostat R, so that no matter which of these portions is shunted by the branch circuit the latter is traversed by one and the same current as tested by the equality of potential differences across the standard 0·2 ohm. Under such circumstances, moreover, it is clear that the magnitude of this shunt current is necessarily equal to the difference in magnitude between the larger currents traversing the main apparatus before and after rocking over the commutator K. When now by adjusting the resistance of the shunt circuit a branch current C_2-C_1 has been obtained, which is such as to maintain thermal equilibrium in the detecting coil on operating the commutator K, its magnitude is measured by finding on the potentiometer the milli-volts across the standard ohm or more usually across its known sub-multiple, the standard 0·2 ohm.

Special five-way copper plug keys connected to the Cambridge thermo-electric potentiometer enable the various potential differences which have to be measured during an experiment to be switched quickly on to this instrument.

(*c*). The calibration of the thermo-junctions and the measurement of U, the difference of temperature between the wire disc-ends.

The thermo-junctions, which we may call A and B, are of iron and constantan and are insulated with ohmaline varnish. They dip deep into the disc holes t, t (Fig. 6) containing mercury and are so paired against a similar cold junction C that either of the couples $A-C$ or $B-C$ may at any moment be switched on to the thermo-electric potentiometer. From the millivolts recorded and from the temperature of C together with the data derived from previous calibration, the actual disc-end temperatures are readily ascertained. The junctions of the iron with the copper connecting leads are side by side in glass tubes under water.

The thermo-couples $A-C$ and $B-C$ were most carefully calibrated against a platinum thermometer by immersing C in melting ice and bringing A and B (placed alongside the platinum bulb) to numerous steady temperatures up to 270° C. in a large electrically-heated and mechanically-stirred tank containing oil of high flash point. The maximum difference in the readings for the two thermocouples for the same temperature interval very rarely exceeded 1 part in 2000, and the electromotive force in microvolts for either couple for the interval 0°C.—T°C. was expressed as a result of the calibration by the empirical relation :—$E/T=51{\cdot}85+0{\cdot}0331T-0{\cdot}00007T^2+0{\cdot}000{,}000{,}044T^3$.

(*d*). The measurement of the resistance of the wire.

This is carried out as in the previous Paper with the aid of a constant current of 1·018 amperes save that the wire is maintained under its temperature gradient. Though the millivolts across the terminals p, p now differ considerably with the direction of the standard current, the mean value is independent of the electromotive force of the thermo-couple formed by the wire and the copper potential leads. Between the terminals *p*, *p* and the potentiometer is a plug commutator which must be switched over when the standard current is reversed.

(*e*). The detecting coil connections and the marked improvement in the sensibility.

In a Paper on "Electrical recording thermometers for clinical work" Callendar (Proc. Phys. Soc., Vol. XXII., p. 220, 1910) has investigated the conditions of sensibility in measuring a resistance by the Wheatstone bridge method where, as in the case of electrical thermometry, the limiting condition is imposed by the heating effect of the current on the resistance to be measured. In the light of this investigation a vast improvement in sensibility is attained by interchanging the galvanometer and battery in the Maxwell arrangement shown in Fig. 5 of Part IV., p. 25. For taking these $G=10$ ohms and $S=5$ ohms, so that $X=500$ ohms, we may show that the current c through the galvanometer when the balanced detecting coil, traversed by a heating current C, alters in resistance by an amount dS is given by $c=C.dS/520$, which expresses the Maxwell sensibility. On interchanging galvanometer and battery, however, and raising the voltage so as to secure the same limiting heating current C, we have for the new arrangement a sensibility given by $c=C.dS/25$ which is over 20 times better. Alternatively if we connect the detecting coil to a pyrometer bridge with equal ten-ohm ratio arms we find $c=C.dS/35$, which is nearly 15 times superior to the Maxwell arrangement.

Considerations such as these have led the writer to abandon the early arrangement, and the detecting coil S is now connected directly to a pyrometer bridge with 10-ohm ratio arms which, though not yielding the maximum sensibility attainable, marks the immense improvement already stated, requires no increase of voltage, is convenient and is seen to good advantage as the detecting coil resistance becomes greater at higher temperatures. The source of E.M.F. in the pyrometer bridge is a 2-volt accumulator, and the battery circuit is closed throughout an experiment to maintain constant the Joulean heating effect in the thermometric detecting coil; when S is small, 2 or 3 ohms are put in series with the accumulator. The galvanometer is the low resistance Broca instrument used in the previous research, and similar precautions to those already described are taken when finding and confirming the value of the balancing current.

As a concrete example of the sensibility a full scale deflection of 50 cm. was obtained in 30 seconds on reversing (but not altering in magnitude) a current of 2 amperes in constantan wire with its extremities differing in temperature by 60°C.

VI.—*Illustration of the Method by a Typical Experiment.*

All experiments were performed in a manner similar to that shown for the typical experiment below, for which all essential data are given. The readings on the thermo-electric potentiometer (T.E.P.) are in millivolts :—

AUGUST 8, 1921. IRON WIRE.

5.30 A.M.—Electrical heating commenced.

6.0 A.M.—Heaters reached 84°C. and 134°C. respectively.

6.30 A.M.—Heaters steady at about 84°C. and 134°C. Steady current of about 2 amperes commenced in wire.

7.0 A.M.—Detecting coil resistance stationary at about 9·6 ohms under steady state. Search for balancing current commenced.

7.45 A.M.—Balance confirmed after repeated tests. Balancing current as shown by milliammeter 0·110 amperes. Readings on T.E.P. across

$$0{\cdot}197 \text{ ohm} : \begin{array}{l} \rightarrow 21{\cdot}610 \\ \leftarrow 21{\cdot}590 \end{array}$$

7.47 A.M.—Thermometric observations :—

	T.E.P. reading for thermo-couples.		Thermometer reading.
Hot end	6·440	...	133·8°C.
Cold end	3·610	...	84·4°C.
Cold Junction	—	...	19·2°C.

7.50 A.M.—Current through wire altered to 1·0183 amperes.

8.5 A.M.—Potential differences across disc-ends of wire on T.E.P. $\begin{array}{l} \rightarrow 7{\cdot}290 \\ \leftarrow 8{\cdot}062 \end{array}$

Conclusion of Experiment.

This experiment yields for the balancing current a value $C_2-C_1=0{\cdot}02160/0{\cdot}197 =0{\cdot}109_6$ ampere, and for the resistance of the wire under gradient a value $R=\frac{1}{2}(0{\cdot}007290+0{\cdot}008062)/1{\cdot}0183=0{\cdot}00753_8$ ohm. In order to evaluate the temperatures, 1·007 millivolts—the E.M.F. between 0°C. and 19·2°C. as found from the calibration curve—must be added to the readings 6·440 and 3·610 respectively. From the aggregates and the calibration data we easily obtain the corresponding temperatures, viz. : 135·02°C., and 85·20°C., giving the temperature difference $U=49{\cdot}8_2$°C. Whence taking $J_{20}=4{\cdot}18$ joules per calorie we have :—

$$\sigma_{110}=(0{\cdot}00753_8\times 0{\cdot}109_6)/(4{\cdot}18\times 49{\cdot}8_2)$$
$$=3{\cdot}97\times 10^{-6} \text{ calories per coulomb per } 1^\circ\text{C.}$$

The same standard ohm is used essentially in the measurement of C_2-C_1 as well as in the determination of R ; its absolute value is therefore unimportant.

VII.—*Results of Experiments on Specimens of Constantan and Iron Wire.*

TABLE I.—*Results of Experiments on Constantan Wire.*

Higher temperature.	Lower temperature.	Diff. of temp. U.	Balancing current in amps., C_2-C_1.	Resistance of wire in ohms, R.	Temp. at which σ is measured.	$\sigma\times 10^6$ in calories per coulomb.
80·38	19·80	60·58	0·0738	0·02015	50·1	5·87
80·04	20·19	59·85	0·0728	0·02020	50·2	5·88
110·63	50·07	60·56	0·0753	0·02016	80·4	6·00
140·82	80·39	60·43	0·0762	0·02016	110·6	6·08
171·66	111·81	59·85	0·0760	0·02018	141·7	6·13
201·60	143·44	58·16	0·0749	0·02017	172·5	6·22
232·60	174·40	58·20	0·0750	0·02020	203·5	6·23

Table II.—*Results of Experiments on Iron Wire.*

Higher temperature.	Lower temperature.	Diff. of temp. U.	Balancing current in amps., C_2-C_1.	Resistance of wire in ohms. R.	Temp. at which σ is measured.	$\sigma\times 10^6$ in calories per coulomb.
73·97	22·33	51·64	0·0915	0·006048	48·2	2·56
74·93	24·94	49·99	0·0910	0·006010	49·9	2·62
104·87	54·06	50·81	0·1032	0·006786	79·5	3·30
135·02	85·20	49·82	0·1096	0·007538	110·1	3·97
166·17	114·75	51·42	0·1184	0·008327	140·5	4·59
197·86	147·39	50·47	0·1191	0·009172	172·6	5·18
230·24	179·36	50·98	0·1200	0·010111	204·9	5·69

The results for the constantan over the range of temperature investigated may within the errors of experiment be expressed by the formula :—

$$\sigma_T\times 10^8=564+107T/200-12T^2/10{,}000$$

The results for iron may likewise be summarised in the form :—

$$\sigma_T\times 10^8=138+505T/200-21T^2/20{,}000-5T^3/10^6$$

VII.—*Conclusion.*

The writer must express his gratitude to the Senate of the University of London for a grant of £25 from the Dixon Fund. He is also indebted to the Royal Society for permission to use a supplementary grant of £5 (given originally for his research on the measurement of the Thomson effect by the impressed velocity method) for expenses incurred in this work. He also takes the opportunity of thanking Mr. James Barker for the great pains he took in the construction of the apparatus and Dr. A. Griffiths, Head of the Physical Department at Birkbeck College, for placing at his disposal for so long a time much of the best electrical apparatus in the department.

DISCUSSION.

Mr. C. R. **Darling** congratulated the author on his neat apparatus, and said he looked forward to the accumulation of results which would be of value in settling some of the apparent anomalies in thermo-electricity. The junction of the two materials considered in the Paper furnished a case in point. It is well known that these yield a very closely linear relation between E.M.F. and temperature-difference, leading, on the usually accepted theory, to the inference that the two Thomson effects cancel, whereas they are found in the present experiments to be different in numerical value. Another example was described in a Paper read by the speaker before this Society a few years ago ; in this case both the Peltier and the Thomson effects were (on the usual theory) to be regarded as zero, whereas the combined thermo-E.M.F. actually observed was 13 millivolts. He hoped that the author's experiments would provide data enabling these matters to be tracked to their true causes.

Mr. F. E. **Smith** said it was difficult to understand why the author abstained from the use of the Wheatstone bridge in the determination of his resistance, since this was quite feasible and would undoubtedly lead to greater precision in the determination.

From the **Author** (communicated): In reply to Mr. Darling—the author hopes to measure carefully the Thomson effect in several metals, and to find the thermo-electric power of the same specimens. He has for long been interested in the apparent anomalies mentioned. In nearly all methods of measuring the Thomson effect the error increases with the temperature. The results of the present Paper for the iron and constantan specimens used do not seem seriously out of joint with the electromotive force-temperature curve of the different iron-constantan couples used for the temperature measurements. In reply to Mr. F. E. Smith—it was very convenient to be able to use, in a limited space and with the apparatus available, the Cambridge thermo-electric potentiometer for the measurements of R, U and C_2-C_1. Moreover, by its use in the manner described, the value of the standard ohm became unimportant. The resistance of the wire of the order of 1/100 ohm was measured while under temperature gradient, and the accuracy is probably far more than sufficient in view of the limitations imposed by the sensibility in determining the balancing current. The author is grateful to Mr. F. E. Smith for his remarks, and will consider the Wheatstone bridge suggestion before making further measurements.

XII. *A Defect in the Sprengel Pump: Its Causes and a Remedy.* By John J. Manley, *M.A., Research Fellow, Magdalen College, Oxford.*

Received October 12, 1921.

ABSTRACT.

This communication deals with the limitations and irregularities of the Sprengel pump, resulting from the presence of *air skins* upon the interior surfaces. An elaborated form of the pump is described; also a plan whereby the usual gas skin may be so greatly reduced as to become a negligible quantity. The efficiency of the new pump is markedly superior to that of the older forms, and appears to remain practically constant. Two useful adjuncts to the pump are illustrated and described.

When using a Sprengel pump for producing and maintaining vacua of the order of 0·00004 to 0·00001 mm., I frequently observed that continued pumping caused an increase rather than the expected decrease in pressure. This at first appeared inexplicable, but close observation showed that the growth in the pressure was due to the introduction of air. The pump being in action, minute bubbles of air were seen forming here and there upon the surface of the tube through which the mercury was ascending. Such bubbles originated, or became visible, only at points situated a little below the entrance to the fall-tube; their number and the rapidity with which they formed were greatly increased by tapping the tube. As the bubbles moved upwards, their rather sudden enlargement suggested that the increase in volume was not wholly due to the diminishing pressure of the mercury column, but that it was to some extent caused by the acquisition of other air from the film upon the tube. It is highly probable that the defects here noted are common to all Sprengel pumps.

In the formation of a film upon glass, the weight of the gas absorbed by the growing film will be dependent upon at least four variables—viz., (1) the variety of the glass and the condition of its surface; (2) the nature of the gas; (3) the pressure; and (4) the temperature. In addition, the weight of the film may be affected by the time allowed for its complete formation. When for a given pressure, &c., an air-film is in a state of equilibrium with respect to the free gas, then on reducing the pressure this equilibrium is destroyed, and the tension of the film left superior to the pressure of the residual gas; and, as an inevitable result, there follows a continuous decrease in the mass of the film on the one hand, and a corresponding increase in the pressure of the gas on the other. Preliminary experiments have shown that for the restitution of equilibrium between the film and its parent gas a very considerable period is required. This period has not yet been determined;* but, whatever its duration may be, I find that a marked shortening is effected when the tubes are lightly tapped with a wooden mallet. The beneficial influence of tapping is due to the fact that a vibrating surface throws off catapult-wise some of the film with which it is normally clothed; but as the vibrations cease the film thus attenuated is rebuilt. The correctness of the several statements just made may be verified with the aid of a sensitive McLeod gauge.

Starting with a chemically clean and empty pump, we may suppose that the internal surfaces possess air-films having approximately maximum values. On the introduction of mercury, the films are to a large extent covered and imprisoned and their subsequent removal or escape is slow and difficult ; more especially is this so when, with the pump in action, the mercury flows but feebly, but with a vigorous flow a marked erosive action is obtained ; consequently, the air-film is speedily worn down to the point at which the erosive power of the mercury is balanced by the attractive force of the glass. If, now, the flow of the mercury is reduced to a normal rate, the efficiency of the pump is found to be high ; ultimately, however, air bubbles re-form at the head of the fall-tube, and the effectiveness of the pump again becomes uncertain. The new bubbles are probably formed by air first eroded and then brought upward from the lower films by the stream of mercury. The defect can, of course, be again temporarily and most quickly remedied by the use of a rapid stream of mercury aided by a simultaneous tapping of the tube. I here omit all remarks as to the probable existence of films upon the mercury, and do not consider the possible adsorption of air by the metal ; these omissions do not materially affect the argument and conclusions. I proceed to show how the difficulty was successfully met.

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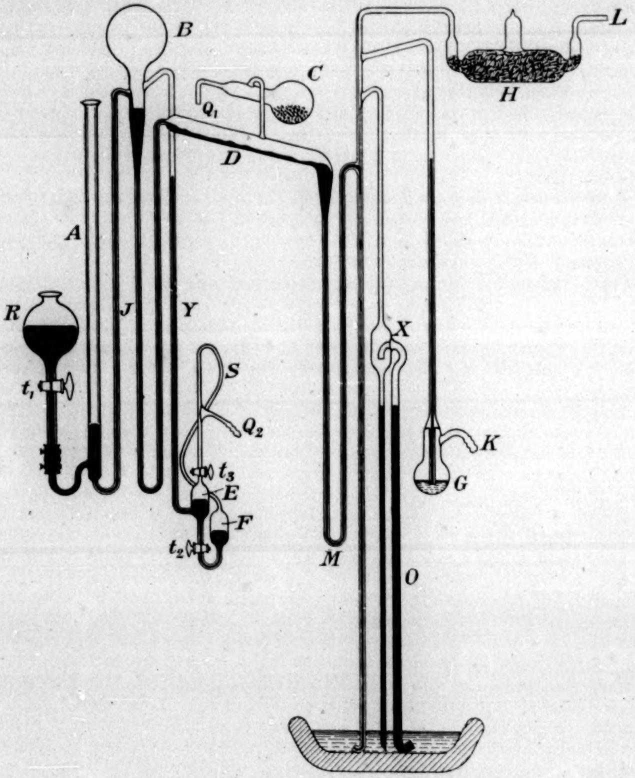
* The actual measurement of the period is at present receiving attention.

First, the new form of the Sprengel pump shown in the figure was designed and built in sections ; these having been chemically cleaned and dried, were subsequently fused together, the completed apparatus conveniently mounted, and the pump was then prepared for action in the following way :—

The tap t_1 and the opening Q_1 were closed, and pure and dry carbon dioxide passed through the apparatus by way of the tube *A* ; meanwhile the whole of the glass, with the exception of the cisterns and traps, was strongly and continuously heated. The pump was then allowed to cool whilst the gas was still passing through it. By this plan skins of carbon dioxide were substituted for those of air upon the interior surfaces of the apparatus. The application of a flame to the taps and their immediately adjacent parts being unsafe, the heating of the main body of the pump was somewhat prolonged in order that the air skins within those portions not directly heated might be removed by the stream of hot carbon dioxide. Referring to the diagram above, and knowing that the density of carbon dioxide is decisively superior to that of air, it is at once evident that the major portion of the gas flowing through the chamber M_1 escapes through the fall tube, and very little by way of *XO* and *K* ; that which passes through the upper limb and into *H* is wholly insignificant, and may be reduced almost to zero by closing *L*. Hence, during these operations, the solid potash in *H* is scarcely affected. The carbon dioxide escaping through Q_2 is regulated by the taps t_2 and t_3. When the apparatus was cold, all five exits were temporarily closed, the plug removed from Q_1 and the bulb *C*, containing solid potash heated to destroy the air skin within. As soon as the air in *C* had been removed by carbon dioxide a Geryk pump was attached to Q_1 and mercury admitted from the reservoir *R* into the tube *A* in quantity sufficient at a later stage nearly, but not quite, to fill the tube *J*. Next, the taps t_1, t_2 and t_3 were closed, and the whole apparatus exhausted as highly as the Geryk pump permitted ; the bulb *C* was then hermetically sealed by the application of a blowpipe flame directed upon the bend above Q_1. During this process all mercury-free tubes were re-heated. At this stage the cistern *E* was half-filled with mercury by partially opening t_2 for a moment ; then t_2, having been closed and t_3 opened, mercury ascended the tube *Y*, and thus ensured the maintenance of the vacuum in the bulb *B*. Finally, the tubes leading into and out of the cylindrical chamber *D* were filled with mercury run into them from *R* ; the pump was then allowed to remain at rest for a few days, after which the tube *L* was opened and the apparatus to be exhausted fused to it. The potash in the bulb *C*, as well as that in the drying chamber H, was re-fused, allowed to solidify, and then broken up just before being introduced into the containing vessels of the pump. Whilst the pump was resting, the residual carbon dioxide in D and C would be absorbed by the potash in C, and the gas pressure reduced to zero, or nearly so. Here it may be remarked that during the removal of the air skins the carbon dioxide on its way through the apparatus would slowly diffuse into the chamber C, and there be absorbed. The large quantity of potash contained in bulb would, however, be but slightly affected.



On admitting mercury from the reservoir R, it passes by way of the tube J into the neck of the bulb B, whence it falls into the tube below and in so doing loses accompanying air. Entering the cylindrical chamber *D* the mercury, in wriggling down the gentle slope, exposes a maximum surface to the high vacuum and is thereby robbed of its last trace of adhering air. As the gas-free mercury flows through the long U-tube *M*, it makes contact with the film of carbon dioxide; and during the action of the pump this film is continuously eroded and so reduced. The detached carbon dioxide is discharged into the fall-tube from whence a little of it is at once carried down and out into the open air the second and by far the greater portion passes backwards towards the vessel that is being exhausted, but on its way thither it is caught and retained by the potash in *H*. Theoretically the efficacy of the pump, when used for the first time, is slightly below its maximum value; but by prolonged action and the consequent erosion of the film upon the walls of *M*, it is raised, step by step, to the highest attainable limit; the greatest power being shown when erosion of the carbon dioxide film ceases and gas is no longer brought into the fall-tube. It will, therefore, be seen that the form and condition of the pump described above secure immunity from certain troubles inseparable from a Sprengel pump of the usual type. In a pump of the ordinary form the air removed by the erosive action of the moving mercury is continuously replaced by other air, and the mass of the film maintained approximately constant. The acquired air appears to be derived partly from that associated with the flowing mercury, but chiefly from the film existing upon the interior of the tube. As to this film the most correct view seems to be that which regards it as a highly attenuated tubular skin; and this when filled with flowing mercury, slides forward but does not terminate; for, although the leading end of the skin on entering the highly exhausted space of the pump is erupted and dispersed, the rearward, having contact with the free atmosphere, is, as it moves on, continuously replenished. Under these conditions even the best pumps of the usual type cannot do more than produce a vacuum in which the pressure is represented by the tension of the advancing and erupting tubular skin. The conclusions here reached are supported by the fact that on using the new pump for the first time, a few gas bubbles appeared at the head of the fall-tube; and that when these had been removed, they were not succeeded by others. Moreover, from that time onward, the efficacy of the pump has, in so far as it has been possible to discover, remained unimpaired.

When, at any time, an appreciable quantity of air has accumulated in *B*, it can be removed by a pump operating at Q_2, provided that the approximate use of the stop-cocks t_2, t_3, is at the same time observed; and this without in the least degree affecting the vacuum in *D* and *C*.

In conclusion I would draw attention to two very helpful bits of apparatus: these are shown with the pump in the figure.

One is a barometric column of Sprengel pump tubing, to which is fused the cistern *G*; the tube *K* serves for the attachment of a Geryk, or other pump, whereby the pressure within a vessel sealed to *L*, can be quickly reduced to a small value. As the barometer tube is always closed by mercury, contamination with air from the Geryk pump is avoided. The quantity of mercury in *G* is such that when the tube is charged, its lower end dips but a little distance beneath the surface. Owing to the small bore of the tube and the wide dimensions of the cistern the level of the mercury is not greatly affected when *G* is exhausted; hence the effectiveness of the manual pump is a maximum.

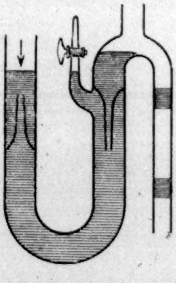
The second piece of apparatus *O*, used for the introduction of gases, consists of a twinned barometric column. The tube shown on the right is, for convenience, 1 cm. wide; the other may be narrower. The lower end of the wider tube is bent upward for the reception of gas delivery tubes; the upper end is fused to the bulb X within which it terminates as a tapering tube directed downwards. When a gas enters a vacuum by way of an ordinary barometric column, mercury is thrown upwards so violently that some is frequently carried into narrow connecting tubes; and these, becoming blocked, are not easily cleared. This inconvenience is avoided by the use of the twinned column; for in this case the mercury that ascends with the gas is, as it enters the bulb X, shot downward into the second tube.

DISCUSSION.

Mr. R. M. **Archer** expressed his interest in the Paper, and thought the use of carbon dioxide, in the manner advocated, a promising departure. He asked the author which kind of glass he recommended for vacuum work, lead glass or soda glass; also whether he could indicate his experience as to the best method of removal of surface films from metallic vacuum vessels.

Mr. F. E. **Smith** remarked that the first person to observe the existence of the surface film on glass was Helmholtz, who demonstrated that a larger quantity of mercury was contained in a tube which had contained paraffin oil than in the same tube when air was previously present, showing that the film of paraffin oil was appreciably less in thickness than that of air. As illustrating the tenacity with which such surface films persisted, he referred to a Paper read before the Society some 15 years ago, in which it was shown that a film of moisture remained on a glass surface after exposure for some considerable time to a temperature of 500°C. He thought that the author had made an interesting contribution towards the removal of the difficulties met in high-vacuum work.

Mr. H. P. **Waran** said: It is a matter of common experience to anyone who has used an ordinary Sprengel pump for the production of high vacua that it is on many grounds entirely unsuitable for the purpose in its old form, as employed by the author in the present Paper. Limits of exhaustion as high as ·00001 mm. are rarely realisable in practice in any reasonable length of time when the system exhausted has any appreciable volume. Generally the pump increasingly ceases to function beyond a pressure of the order of ·001 mm. unless the diameter of the fall tube is specially made less than a millimetre, thus considerably prolonging the exhaustion, already very, very slow. As has already been pointed out by me in an earlier Paper,* at pressures of the order of ·001 mm., when the falling pellets begin to hammer, the thin discs of gas formed by the compression in the fall tube get shattered, and find escape back again into the system much easier than downward progress against the barometric column of mercury.



The film of air coming along the walls of the tube, and possibly also traces absorbed by the mercury, have long been known to be sources of trouble in this form of pump and, after the very earliest types, adequately provided for by the provision of vacuum filters on the circuit of the incoming mercury. In my own experience I find a pair of sharp corners as provided by a pair of internal joints in a short U-tube, as indicated on page 90, a simpler and far more effective safeguard to break and check the forward progress of such films, than any number of long bends and complications of the kind. In the modern automatic types of mercury pumps—largely developed in Germany—acting on the Sprengel principle and worked by a filter pump, the vacuum of a few centimetres at the reservoir end considerably reduces the amount of admixed air carried along by the mercury, and the much faster suction of these pumps renders leakages of this order negligible. Further, when the exhaustion is effected by a process of compression, as in the present case, the provision of a large surface of phosphorus pentoxide, located as near as possible to the head of the fall tube, seems to be very essential.

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* Proceedings of the Indian Association, Vol. VI., 199 (1921).

As a further improvement in the details of the type of automatic mercury pump suggested by me some time back, I have evolved recently a simple type of pump of remarkable speed and efficiency, acting on a modified Sprengel principle, and I hope to give publication to its details at an early date.*

From the AUTHOR (communicated) : In reply to Mr. Archer, I may say that I have confined myself to the use of the best soda glass for constructing pumps and apparatus in general. I have no knowledge of the behaviour of lead glass. Although I have made no experiments with metallic vacuum vessels, I should expect marked improvement to follow the use of carbon dioxide for removing other gas skins. The main quantity of the carbon dioxide could be removed by a Geryk pump, and the last portions absorbed by potash previously re-fused.

The Helmholtz experiment described by Mr. Smith is of great interest. With regard to the retention of moisture by glass at a temperature of 500°C., this is in consonance with some preliminary experiments of my own. These show that the complete removal of gas skins from glass surfaces is extremely difficult if not impossible.

In his initial remarks Mr. Waran would appear to have in mind the more elementary forms of the Sprengel pump. With modern forms, provided the tubes are clean, and the mercury frequently purified, there is no difficulty in reducing the pressure within a closed vessel to 0·0002 mm., and that within a reasonable time.

It is, I suppose, common knowledge that at high exhaustions the air discs in the fall tube are shattered by the descending pellets of mercury. Having made a close study of this matter the conclusion I reached was that the shattered discs rarely if ever re-enter the main apparatus. With a fall tube of the usual length there are in general some two or three pellets descending at the same time, and it is difficult to see how the shattered air disc could successfully dodge those, and so return. Mr. Waran's "sharp corners" are really air-traps ; they afford great assistance, more especially when placed just below the head of the fall tube ; the stopcock at *a* is, however, a source of danger, for a slight imperfection would result in a leakage of air into the pump. A tube terminating in a sealed capillary is quite safe.

May I suggest that as the sole function of the phosphoric oxide is the absorption of aqueous vapour, nothing is gained by placing the drying chamber " as near as possible to the head of the fall tube."

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* See page 120 of the present volume.

XIII. *On the Measurement of the Radium Content of Sealed Metal Tubes.*

By E. A. OWEN, *M.A., D.Sc., and* BERTHA NAYLOR, *B.Sc., The National Physical Laboratory.*

RECEIVED OCTOBER 6, 1921.

ABSTRACT.

Tables have been compiled giving the corrections that have to be applied to the observed radium content of sealed platinum and silver tubes to obtain their true radium content. Two cases have been considered, namely, (1) that in which the active deposit is uniformly distributed throughout the volume of the tube such as in a full tube of salt, and (2) that in which the active deposit is uniformly distributed over the inner wall of the tube, such as in a tube containing a minute quantity of highly concentrated salt.

The results show that when the wall thickness is kept constant, the correction increases with the external diameter of the tube. Also for the same increase of external diameter, the increase of correction is more pronounced for the "empty" than for the full tube.

RADIUM salts are often sealed in platinum or silver tubes for convenience and safety of application in practice. The measurement of the radium content of such tubes presents a difficulty owing to the uncertainty which exists regarding the correction that has to be applied for the absorption of the radiation in the walls of the tubes. An approximate correction is usually made by measuring the absorption of the radiation in flat sheets of the metal and calculating its value for a thickness equal to that of the wall. This, however, gives only an approximate correction even when the internal diameter of the tube is small, but it may be much in error for tubes of larger diameter. The correction to be applied depends not only upon the thickness of the wall and the diameter of the tube, but also upon the distribution of the active deposit in the tube; the active deposit may be distributed throughout the whole volume as in a tube filled with salt, or it may be mainly on the walls as in a tube containing a small quantity of highly concentrated salt which occupies only a small amount of space inside the tube.

There is, in addition, the absorption of the radiation in the salt itself which may be considerable for a tube of large diameter.* The correction due to this cause will be omitted here and attention paid only to the corrections arising from the absorption of the radiation in the walls.

The two metals, platinum and silver, will be considered, as they are the metals most frequently met with in this connection.

Two cases arise which can be completely worked out, namely, (1) the case when the tube is full of finely powdered salt and the active deposit may be assumed to be distributed uniformly throughout the whole volume; and (2) the case when the salt is highly concentrated and the amount of space occupied by it is negligible compared with the internal volume of the tube. In this case the active material will be assumed to be uniformly deposited over the walls of the tube.

In actual practice the majority of cases will be those in which the tubes are only partly filled with salt, so that the correction to be applied to the observed radium content in order to obtain the true content will be intermediate between the results arrived at in the above two cases.

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* *See* Owen and Fage, Proc. Phys. Soc., Vol. XXXIV., Part I., p. 27 (1921).

(1) *Case of the Full Tube.*

It will be assumed that the tube is situated so far from the measuring chamber that an approximation to a beam of parallel rays is obtained. Let O (Fig. 1) be the axis of the tube placed at right angles to the direction OC which passes through the centre of the surface of the measuring chamber, and let I be the measured intensity of the radiation in the direction OC, and I_0 the true intensity if there had been no absorption. Consider an element of area $dx\,dy$ at P. The intensity of the radiation emitted from this element in the direction OC is

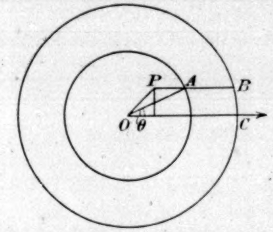

FIG. 1.

$$\frac{I_0}{\pi a^2} \cdot dx\,dy \;.\; e^{-\mu \cdot AB}$$

where
$$AB = \sqrt{b^2-y^2} - \sqrt{a^2-y^2}$$

a and b being the internal and external radius of the tube respectively, and μ the absorption coefficient of the radiation in the metal of the tube. The total intensity of the radiation in the direction OC is, therefore, given by

$$I = \frac{I_0}{\pi a^2}\int_{y=-a}^{y=+a}\int_{x=-\sqrt{a^2-y^2}}^{x=+\sqrt{a^2-y^2}} e^{-\mu\left(\sqrt{b^2-y^2}-\sqrt{a^2-y^2}\right)}\,dy\,.\,dx$$

$$= \frac{2I_0}{\pi a^2}\int_{-a}^{+a}\sqrt{a^2-y^2}\,.\,e^{-\mu\left(\sqrt{b^2-y^2}-\sqrt{a^2-y^2}\right)}\,dy$$

Putting $y = a \sin \varphi$, this becomes

$$\frac{I}{I_0} = \frac{2}{\pi}\int_{-\pi/2}^{+\pi/2}\cos^2\varphi\,.\,e^{-\mu a\left(\sqrt{(b/a)^2-\sin^2\phi}-\cos\phi\right)}\,.\,d\varphi \quad \ldots\ldots \quad (1)$$

which gives the ratio of the apparent to the true radium content of the tube.

(2) *Case of the "Empty" Tube.*

In this case the active deposit which emits the hard γ radiation whose intensity is measured when estimating the radium content of the tube, is assumed to be entirely distributed over the walls. If I and I_0 have the same significance as before, the intensity of the radiation emitted in the direction *OC* from a small element of the surface at *A* is

$$\frac{I_0}{2\pi a} \cdot a\, d\theta \cdot e^{-\mu . AB}$$

where $$AB=a\left(\sqrt{\left(\frac{b}{a}\right)^2-\sin^2\theta}-\cos\theta\right)$$

and θ is the angle which *OA* makes with *OC*.

Then the total intensity of the radiation in the direction *OC* is given by

$$I=\frac{I_0}{2\pi a}\int_0^{\pi} a \cdot e^{-\mu . AB}\, d\theta$$

so that $$\frac{I}{I_0}=\frac{2}{\pi}\int_0^{2/\pi} e^{-\mu a\left(\sqrt{(b/a)^2-\sin^2\theta}-\cos\theta\right)}\, d\theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

The values of the integrals appearing in equations (1) and (2) may be obtained graphically after substituting the proper values for μ.

The measurement of the coefficient of absorption (μ) was carried out on the same apparatus that is employed to standardise samples of radium preparations. This consists of a combined ionisation chamber and electroscope,* the radiation entering the chamber through a lead sheet 5 mm. thick. The source of radiation in the present instance was placed as far as was convenient from the chamber so as to obtain an approximation to a beam of parallel rays. Also the sheets of platinum and silver employed for measuring the absorption coefficient were small—a little larger than that necessary to shield the whole tube from the chamber, the axis of the tube being placed at right angles to the surface of the chamber. In this way the secondary radiation emitted by the absorbing screens was reduced to a minimum. Large sheets would tend to give a value of the absorption coefficient which would be too low on account of the increased amount of scattered radiation superposed upon the primary radiation transmitted by the screen.

The values of the absorption coefficient showed little variation with thickness of screen over the ranges of thickness employed, namely, up to about 0·5 mm. for platinum, and 2·0 mm. for silver. The following mean values were obtained :—

	μ mm.$^{-1}$
Platinum...	0·124
Silver	$0{\cdot}036_6$

It should be borne in mind that these values apply to radiation emitted by radium in radioactive equilibrium which has passed through 5 mm. of lead before entering the measuring chamber.

The above values of μ were substituted in the exponentials appearing in equations (1) and (2), and the expressions under the integrals evaluated for different values of a and b/a. For each combination of values of these two quantities, a series of values for each expression was obtained for different values of angle, and curves were drawn on a large scale, whose areas were measured by means of a planimeter capable of giving the results to about 1 in 1,000. A summary of the results obtained in this way are tabulated in Tables I. and II. They give the percentage correction which has to be added to the observed radium content in order to obtain the true radium content of tubes of various external diameters and thickness of wall. For the empty tube no further correction is needed, but for the full tube an additional correction has to be made for the absorption of the radiation in the salt itself.

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* Owen and Fage, *loc. cit.*

TABLE I.—*Platinum Tube.*

External diameter in millimetres.	Full tube.			Empty tube.		
	Thickness of wall in millimetres.			Thickness of wall in millimetres.		
	0·3	0·4	0·5	0·3	0·4	0·5
1·5	4·3	5·3	6·6	4·9	5·9	7·0
2·0	4·4	5·5	6·9	5·3	6·4	7·6
3·0	4·5	$5{\cdot}7_5$	$7{\cdot}1_5$	5·7	7·1	8·3
4·0	$4{\cdot}5_5$	5·9	7·3	6·0	7·4	8·8
5·0	4·6	$5{\cdot}9_5$	7·4	$6{\cdot}2_5$	7·7	9·1
6·0	$4{\cdot}6_5$	6·0	$7{\cdot}4_5$	$6{\cdot}4_5$	7·9	9·4
8·0	4·7	$6{\cdot}0_5$	7·5	$6{\cdot}7_5$	8·3	9·9
10·0	$4{\cdot}7_5$	6·1	7·6	$7{\cdot}0_5$	8·6	10·3

TABLE II.—*Silver Tube.*

External diameter in millimetres.	Full tube.			Empty tube.		
	Thickness of wall in millimetres.			Thickness of wall in millimetres.		
	0·5	1·0	1·5	0·5	1·0	1·5
1·5	$2{\cdot}0_5$	...	...	$2{\cdot}2_5$	...	...
2·0	$2{\cdot}1_5$	...	...	2·5	...	...
3·0	2·2	3·9	...	$2{\cdot}7_5$	$4{\cdot}1_5$	...
4·0	2·2	$4{\cdot}0_5$	5·8	2·9	4·5	6·1
5·0	$2{\cdot}2_5$	4·1	5·0	3·0	4·7	6·5
6·0	2·3	4·2	6·1	3·1	4·9	6·8
8·0	2·3	4·3	6·2	3·2	5·2	$7{\cdot}2_5$
10·0	2·4	$4{\cdot}3_5$	6·3	$3{\cdot}3_5$	5·5	7·6

The tables show that when the wall thickness is kept constant, the correction increases with the external diameter of the tube. Also for the same increase of external diameter, the increase of correction is more pronounced for the empty than for the full tube. This is to be expected in view of the nature of the distribution of the active deposit in the two cases.

When the internal diameter approaches zero, it will be noted that equations (1) and (2) both become

$$\frac{I}{I_0}=e^{-\mu b}, \quad \ldots \ldots \ldots \ldots \quad (3)$$

so that the correction tends towards the value obtained for a flat sheet for both empty and full tubes as the internal diameter diminishes. The values of the corrections when the internal diameter is nearly zero are given in Table III.—

TABLE III.

	Platinum tube.			Silver tube.		
	Thickness of wall in millimetres.			Thickness of wall in millimetres.		
	0·3	0·4	0·5	0·3	1·0	1·5
Percentage correction.	$3{\cdot}6_5$	$4{\cdot}8_4$	$6{\cdot}0_1$	$1{\cdot}8_1$	$3{\cdot}6_0$	$5{\cdot}3_5$

It was of interest to test the above calculations on a few practical cases. A glass tube full of radium salt was measured, and found to contain 269·0 milligrammes of radium element. This tube was afterwards divided up between six platinum tubes of 3·0 mm. external diameter and 0·5 mm. thickness of wall. Each of these tubes was filled with the salt and hermetically sealed. After the state of radioactive equilibrium was reached the tubes were measured, and the total apparent content was found to be 252·5 milligrammes radium element. The correction to be applied to this to obtain the true content, according to Table I. above, is + 7·1 per cent. of the observed content—*i.e.*, 18·0 milligrammes—so that the true radium content becomes 270·5 milligrammes radium element. This figure agrees within about 0·6 per cent. with the radium content of the original tube before it was divided up.

In another case a glass tube contained 121·6 milligrammes radium element before being divided up between eleven platinum tubes as follows: Two tubes of 3 mm. external diameter and 0·5 mm. wall; three tubes of 2·5 mm. external diameter and 0·5 mm. wall; and six tubes of 1·6 mm. external diameter and 0·3 mm. wall. The sum total of the contents of these tubes after the appropriate correction had been added in each case, was found to be 122·5 milligrammes of radium element, which agrees within about 0·7 per cent. with the radium content of the original tube.

This agreement is good in view of the possibility of slight loss during sub-division and the accumulation of small experimental errors in the measurements of the tubes.

The correction for the absorption of the radiation in the salt itself which would be greater for the original tube than for all the small tubes added together, would tend to bring the two figures in each case into still closer agreement.

The above two cases suffice to show that the figures given in the above tables are sufficiently accurate for most practical purposes.

Note added 7th March, 1922 :—

Table IV. contains the results calculated for tubes of soda glass. The value of the absorption coefficient (μ) for soda glass was found to be 0·010 mm.$^{-1}$. The figures given in the table, as in the foregoing tables, are the percentage corrections which have to be added to the observed radium content to obtain the true radium content of the tubes.

TABLE IV.—SODA GLASS TUBE.

External diameter in millimetres.	Full Tube.			Empty Tube.		
	Thickness of wall in millimetres.			Thickness of wall in millimetres.		
	0·5	1·0	2·0	0·5	1·0	2·0
3	0·5%	...	...	0·5%	...	...
5	0·6	1·2%	2·2%	0·6	1·2%	2·2%
10	0·7	1·2	2·4	0·7	$1{\cdot}3_5$	2·5
15	0·7	1·3	2·5	0·8	1·5	2·7
20	...	1·3	2·6	...	1·6	2·9
25	...	1·3	2·6	...	1·7	3·1

DISCUSSION.

Dr. J. S. ANDERSON inquired whether the necessity for a correcting factor might not be avoided by using tubes of rectangular cross-section.

Dr. A. RUSSELL suggested that instead of resorting to a graphical method of integration, use might be made of Weddle's Rule.

Dr. E. A. OWEN, in reply, stated that cylindrical tubes are in common use by the medical profession, as they are easy to handle. Flat applicators are also used, however.

XIV. *The Crystal Structure of Ice. By* PROF. SIR W. H. BRAGG, *K.B.E., M.A., F.R.S.*

RECEIVED JANUARY 7, 1922.

ABSTRACT.

The methods of X-ray analysis have been applied to ice by Ancel St. John and by D. M. Dennison. The former refers the structure to a lattice composed of right triangular prisms of side 4·74 A.U. and height 6·65 A.U.; the latter to a similar lattice of dimensions 4·52 and 7·32 respectively. The actual arrangement of the atoms is not found in either case.

On certain suppositions, the arrangement can be found independently of direct X-ray analysis in the following way. Let it be assumed that each positive ion is surrounded symmetrically by negative ions, and vice versa; and in view of the low density of ice, let the number of neighbours be in each case as small as possible. The crystal is to be hexagonal, and is to have the right density. The result is that each oxygen atom is at the centre of gravity of four neighbouring oxygens, from each of which it is separated by a hydrogen atom. The dimensions of the structure agree with Dennison's figures.

The conclusion is supported by a comparison between the calculated intensities of reflection and the observed intensities as recorded by Dennison.

THE methods of X-ray analysis have been applied to ice by Ancel St. John* and by D. M. Dennison.† The two investigators agree in referring the structure to a lattice composed of right triangular prisms of side a and height c. St. John gives a and c the values 4·74 A.U. and 6·65 A.U. respectively; Dennison gives the values 4·52 and 7·32. The complete structure is composed of four such lattices, interpenetrating, and the relative positions of the lattices may be expressed in the following way:—

Let axes of x and y be taken, making an angle of 120° with each other, and the axis of z at right angles to both. The prism $x=o,\ x=a:\ y=o,\ y=a:\ z=o,\ z=c:$ contains two of the triangular lattices described above; each corner of the prism is a point representing a molecule and, therefore, the prism contains the mass of one molecule. According to St. John the three other prisms are derived from the first by parallel shifts, defined by moving the prism corner from 000 to $\frac{a}{3}\cdot\frac{2a}{3}\cdot\frac{c}{n};\ \frac{a}{3}\cdot\frac{2a}{3}\cdot\frac{c}{2};$ and $0\,.\,0\,.\,c\left(\frac{1}{2}+\frac{1}{n}\right)$ in turn. The quantity n is undetermined. Dennison's suggestion is more general. Only two lattices are to be imagined; and the corners are to be $0\,.\,0\,.\,0,\ \frac{a}{3}\cdot\frac{2a}{3}\cdot\frac{c}{2}$. *Two* molecules are, however, to be associated with each corner of the prism; which amounts to saying that the positions of two more lattices are to be derived in the same way by shifting (together) the two that are already defined. St John's definition is, therefore, more particular than Dennison's, but is otherwise in agreement with it. Neither gives a definite value for n.

The differences in the values assigned to the constants by the two investigators are rather more than might be expected, especially in view of the care which both seem to have taken.

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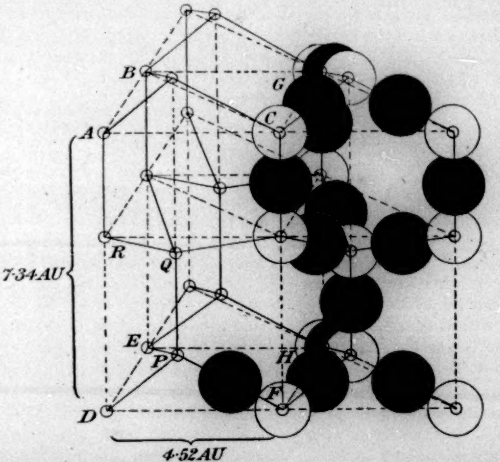
* Proc. Nat. Acad. Sci., p. 193, July (1918).

† Science, Sept. 24 (1920); Phys. Rev., Jan. (1921).

The values can be shown to give in both cases, as they ought to do, the correct value for the density of ice.

It is interesting to make an attempt to arrive at the structure of ice by an independent method, which, though depending on general information gained by X-ray analysis, does not require special application of the analysis to ice itself.

We assume, as it seems reasonable to do, that ice belongs to the class of crystals in which the molecules are broken into positive and negative ions ; and the structural arrangement is such that each positive surrounds itself with negatives, and each negative with positives. In these crystals the individual molecule is lost. Thus, for example, in sodium chloride or calcium carbonate, the positive (metal) ion is surrounded by six negatives and vice versa. In the case of fluorspar, each calcium atom is surrounded by eight fluorine atoms, each fluorine by four calciums.



Black Circles Represent Hydrogen Atoms ; White Circles Represent Oxygen Atoms.

In the case of ice we assume that the hydrogens have given up their valency electrons to the oxygens ; and the structure must be such that there are twice as many hydrogens round each oxygen as oxygens round each hydrogen. But the structure must be much more open that that of fluorspar ; a simple calculation shows that if we replace calcium by oxygen and fluorine by hydrogen the distance from the centre of an oxygen to a hydrogen in contact with it must be 2·2 A.U. This is too great : the radius of an oxygen atom in combination with others in crystal structure is in all known cases nearly 0·65 ; we cannot suppose the radius of *H* to be 1·55 A.U.

The structure must be such that each ion of one sign is closer to its neighbours of the other sign than the fluorspar arrangement allows. The number of atoms in a given volume is limited by consideration of density : if an atom is to be closer to its neighbours it must have fewer of them. The density of ice is so low, viz., 0·9165, as to suggest that the most economical spacing should be tried. In diamond each atom of carbon has only four neighbours ; the number could not be fewer, if each atom is to be similar in position to all the rest. Diamond is cubic, but there is an alternative structure in which each atom, as in diamond, is at the centre of gravity of its four neighbours, but the crystal is hexagonal. One structure can be obtained from the other by parallel shifts of the (111) planes, without finally altering the value of the universal distance between two neighbours. Zinc oxide is of the latter variety, zinc sulphide of the former.

Let us replace each carbon atom in the hexagonal lattice by oxygen, and insert a hydrogen between each pair of oxygens. Each oxygen is now surrounded by four hydrogens and each hydrogen by two oxygens ; and clearly the number of neighbours has been reduced to a minimum.

The dimensions of the new structure can readily be obtained by comparison with diamond. Each point of the lattice now represents a molecular weight 18 instead of 12, and yet the specific gravity has been lowered from 3·52, the density of diamond or of its alternative arrangement to 0·9165, the density of ice. Hence, if the linear dimensions are increased in the ratio $1 : p$ we have

$$p^3 = \frac{18 \times 3{\cdot}52}{12 \times 0{\cdot}9165}$$

whence $$p = 1{\cdot}79$$

The distance between the centres of two carbon atoms in diamond is equal to 1·54. Hence the distance between the centres of two oxygen atoms is $1{\cdot}54 \times 1{\cdot}79 = 2{\cdot}76$. The distance between consecutive basal planes is in diamond 2·05 and in the new structure $2{\cdot}05 \times 1{\cdot}79 = 3{\cdot}67$; and the distance between two atoms in the same plane is in diamond 2·52 and in the new structure $2{\cdot}52 \times 1{\cdot}79 = 4{\cdot}52$. These figures agree exactly with those of Dennison, because the triangular lattice in terms of which he describes his suggested structure has its base points on three neighbouring atoms in a basal plane, and its height is twice the distance between two (111) planes.

The details of the geometry are shown in the figure. $ABCDEF$ is the right triangular prism the dimensions of which, according to Dennison, are : $AC = 4{\cdot}52$, $AD = 7{\cdot}32$. The prism $x = o$ or a, $y = o$ or a, $z = o$ or c, is the prism $ABCGDEFH$. The four lattices mentioned by St. John are derived from this by shifting A to P, Q, and R in turn. The value of n is 8.

The structure is extremely empty. It is easy to imagine that a loose arrangement of H_2O molecules would occupy less space, and that pressure would tend to melt the ice. The hydrogen atom has apparently a larger diameter than the oxygen, if we give to oxygen the value 1·30 ;* but it must be remembered that we can only say—assuming the correctness of the structure—that the sum of the diameters of O and H is 2·76 A.U.

The derivation from the hexagonal structure which is the alternative of diamond amounts to assuming that every oxygen atom is at the same distance from each of its four neighbouring oxygen atoms, and that their orientation is regular. The result agrees exactly with the X-ray measurements made by Dennison. In order to get the figures found by St. John it would be necessary to spoil this regularity somewhat; the substance must be flattened down in the direction of the z-axis, and either PQ is no longer equal to PD, PE and PF, or the four lines are not regularly oriented about P.

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* W. L. Bragg, Phil. Mag., Aug. (1920).

Dennison gives in his Paper an estimate of the relative intensities of the different lines based on his photographic results. It is possible to calculate, for the structure given above, the effect—

1. Of the phase differences of the various atoms.
2. Of the relative number of co-operating planes.

The two factors are set out singly and combined in the following table; and Dennison's estimates are also set out for comparison. In making the calculations it has been assumed that the always expected falling off of intensity with increase of glancing angle (θ) is proportional to the square of the sine of the latter; this may be only true approximately. The effects of the hydrogen atoms have not been included.

Plane.	Phase factor.	Number of co-operating planes.	Spacing observed.	Factor proportional to $F.N/\sin^2\theta$.	Observed intensities (Dennison).
	F.	N.			
0001	0	1	...	0	0
0002	8	1	3·67	100	100
0003	0	1	...	0	0
0004	0	1	...	0	0
$10\bar{1}0$	4	3	3·92	177	10
$20\bar{2}0$	4	3	(1·96)	44	0
$30\bar{3}0$	16	3	1·30	78	2·5
$10\bar{1}1$	1·76	6	3·44	136	20
$10\bar{1}2$	2·00	6	2·68	83	15
$10\bar{1}3$	10·24	6	2·065	243	50
$10\bar{1}4$	0	6	...	0	0
$10\bar{1}5$	10·24	6	1·368	110	20
$11\bar{2}0$	16·0	3	2·26	236	10
$11\bar{2}1$	0	6	...	0	0
$11\bar{2}2$	8·0	6	1·92	170	10
1123	0	6	...	0	0
$11\bar{2}4$	0	6	...	0	0
$20\bar{2}3$	10·24	6	1·53	143	15
$20\bar{2}5$	10·24	6	1·167	80·0	5·0

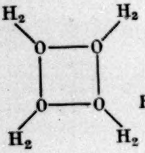
The most striking disagreement is in the large value which Dennison finds for the reflection from the principal plane. Dennison placed a small quantity of distilled water in a thin walled capillary tube of lime glass, which was then plunged quickly into liquid air. Freezing was so rapid that only very minute crystals were formed. The tube was kept at liquid air temperature in a specially-constructed Dewar flask, and rotated continuously during its 10-hour exposure to X-rays. The large (0002) reflection might be due to a tendency to crystallise so that the crystal axis was perpendicular to the tube axis. There appears to be a general tendency throughout the table for the planes more nearly at right angles to the (0001) plane to suffer in comparison with the others; which would point to the same original cause. Dennison records no reflection by $20\bar{2}0$; but he records a strong reflection for $11\bar{2}2$, which may really be a compound of the two since the spacings are so nearly the same.

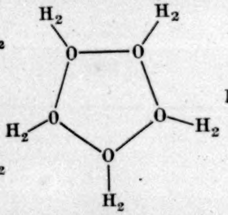
Apart from this, the agreement is really very good. There is, of course, no trace of (0001) itself, nor any (0003) as must be the case for any structure satisfying Dennison's conditions. But, also, there is no (0004), which agrees with the value here given to *n*, viz. 8, and the (0004) should be clear in the experimental results if it really existed because the (0002) is so strong.

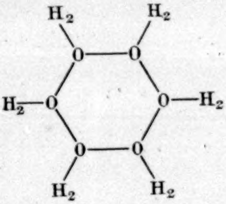
I am much indebted to Mr. Shearer for making the calculations set out in the table.

DISCUSSION.

Prof. H. E. **Armstrong**: The Paper to which we have listened interests me particularly because, as a chemist, I have speculated as to the molecular composition of water, and concluded that a variety of forms are possible. The formula OH_2 (hydrone) represents, not water, but dry steam, for in saturated steam such agglomerates as $H_2O\,.\,OH_2$ may also be present. No doubt water is saturated with hydrone in a proportion dependent on the temperature, but in addition there may be closed systems containting 4, 5 or 6 oxygens, thus:







Now the author's model shows that frozen water contains the potentiality of hexagonal rings of the benzene type. It is remarkable that many substances known to chemists do contain precisely six H_2O units which come and go together. I have here a model constructed of unit spheres to represent a derivative of benzene, the carbon system in which may be taken to represent a slab of water. I assume, however, that the hydrogen atoms occupy half as much space as the oxygen atoms, and that the structure is close-packed. I should like to ask the author whether he could not re-arrange his model in such a way as not to interrupt the continuity of the oxygen chains. At present all the oxygen atoms in it are separated from one another by hydrogen atoms as large as themselves.

It would be interesting if he could also give us some positive information about water in the liquid state, and as to whether a change in structure of the electronic system may not take place at the freezing point. A similar problem arises with regard to various anomalies in the behaviour of metals—for instance, with regard to the atomic heats of metals at very low temperatures. Modern electronic theory may be able to throw some light on these questions.

Dr. A. O. **Rankine**: An important feature of the Paper lies in the fact that the space occupied by the hydrogen atom in a crystal has been measured for the first time. It is surprising, in view of the small atomic number of this atom, that it occupies roughly the same space as that of oxygen. It is unfortunate that the X-ray analysis does not indicate the hydrogen planes, but only those of oxygen, because it may be that the layers of hydrogen are not exactly half-way between those of oxygen. My reason for suggesting this is that a hydrogen atom has only one electron to dispose of. If this electron be lent to the oxygen atom on one side of it, there is nothing left for the oxygen atom on the other side. Consequently the binding forces would be different on the two sides, and one would expect uneven spacing.

Dr. F. L. **Hopwood**: It would be interesting to know whether any crystals containing water of crystallisation have been submitted to X-ray analysis. If so, possibly a similar structure may be found in such crystals.

Dr. H. **Borns**: With regard to the question raised by the last speaker: Rinne, by the aid of optical tests and Laue radiographs, found that the crystal lattice was not disturbed when the water was expelled by heat from brucite—$Mg(OH)_2$—and in some other cases.

Dr. J. L. **Haughton**: The anomalous density of water in the neighbourhood of the freezing-point could no doubt be explained if we knew what structural change occurs when water freezes. It is curious that many substances which crystallise in the hexagonal system exhibit such anomalies. Bismuth, for instance, expands on solidifying, and the electrical properties of both bismuth and antimony are correspondingly peculiar at the melting point. Zinc crystallises in the same system, and on solution in acid it gives off less heat if it has been worked than if it has been annealed.

Sir **William Bragg**, replying to the discussion: The points raised this evening would furnish material for a great deal of research! The suggestions made by Prof. **Armstrong** as to the hexagonal ring in water are intensely interesting. The method tells us nothing about the liquid, and nothing therefore about the structural change at the freezing-point, but it is quite clear that the structure is *not* close-packed, and that the unit in the crystal is the atom, not the molecule. It cannot be said that a given hydrogen atom belongs to one oxygen atom more than another. The oxygens are entirely separated by hydrogens: it is a striking feature, so far, of the models of polar crystals that the positives touch negatives only and *vice versa*; exceptions to this rule occur, but they seem to be accidental and due to some special cause. For instance, the *Al* atoms in ruby touch each other, but they are driven into contact.

On the question raised by Dr. Rankine as to whether the hydrogen atoms are located half-way between the oxygen atoms, we have no guide except symmetry. If I move a given hydrogen atom nearer to a given oxygen atom, I shall be obliged to move a corresponding hydrogen atom in the opposite direction, and there will be a loss of symmetry.

In reply to Dr. **Hopwood**, some alums, which contain water of crystallisation, have been investigated.

XV. *A Method of Exciting Vibrations in Plates, Membranes, etc., Based on the Bernoulli Principle.*

By PROF. KERR GRANT, *University of Adelaide.*

RECEIVED JANUARY 1, 1922.

(COMMUNICATED BY SIR W. H. BRAGG, F.R.S.)

ABSTRACT.

It is known that a plate placed close to a flanged orifice from which a stream of air or liquid is issuing is attracted towards the orifice. If the plate be mounted as a diaphragm it can be excited to strong vibration by a suitable blast, and a loud sound is produced with high efficiency.

THE apparent attraction which is exerted between two closely juxtaposed surfaces when a stream of air flows between them is well known. Witness, for example, the experiment of lifting a light object, such as a matchbox, by blowing through a pipe, provided with a flange placed in loose contact with the surface of the object.

It occurred to the writer in 1918 that this effect might be applied to maintain vibrations in a plate or diaphragm, since the vibratory motion of the plate or diaphragm could operate as a valve to stop or permit the flow of air, and thus give the variations of pressure necessary to maintain the vibrations. (For brevity any sound-producing appliance based on this principle will be termed a hooter.)

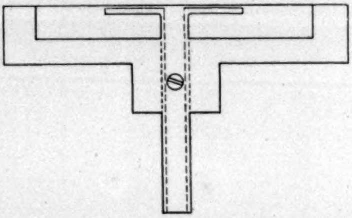


FIG. 1.

To test this idea a piece of parchment was mounted on a circular wooden ring about 5 in. in diameter and a three-eighths inch brass pipe provided with a flange of brass-plate about 2 in. in diameter, mounted in such a way that the surface of the flange could be adjusted parallel with and in the closest proximity to the parchment diaphragm (Fig. 1). When the pipe was supplied with air from a compressor the parchment was excited to vigorous vibration with the production of a very intense sound, the pitch of which varied with the pressure of the air supplied and the degree of approximation of the flange to the parchment. In general overtones were produced, but by operating at lower pressures and by careful adjustment the diaphragm could be made to vibrate in its fundamental mode.

A pressure of a few pounds to the square inch was found to be sufficient to maintain vibration. In fact, a powerful sound can be obtained by using this appliance as a trumpet.

When a plate of thin sheet-metal was substituted for the parchment somewhat higher pressures were found to be necessary for excitation, the pressure requisite depending, as might be expected, on the stiffness of the plate.

With diaphragms of sheet-metal which are so thick as to function as plates rather than as membranes and yet not thick enough to permit of being accurately machined there is some difficulty in putting on the diaphragm in such a way as to secure a good approximation to a plane. A fairly satisfactory method of doing this is to leave a slight rim on the inner periphery of the supporting ring and then screw down the holding-down ring while the diaphragm is kept hot by a hot metal plate laid upon it. The plate then approximates very nearly to a true plane except for a short distance inwards from its periphery. If it is not expanded during the process of screwing down, the surface takes a hemispherical form. Fig. 2 illustrates the construction of a sheet-metal hooter on the above lines.

A metal hooter was found to operate successfully when the water in the supply main, at a pressure of about 80 lbs. per square inch, was substituted for air as the driving agent. The hooter operated in this case whether it were held in the air or immersed in water.

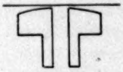


FIG. 3*a*.

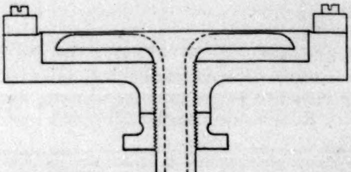


FIG. 2.

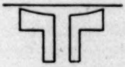


FIG. 3*b*.

While the author was engaged during the summer of 1919 at the Research Laboratory of the General Electric Co., Schenectady, N.Y., several hooters were, by the courtesy of the Director, constructed, the largest having a plate of sheet-iron about 1 ft. in diameter and one-eighth inch in thickness. This operated very well on a compressed air supply at a pressure of 80 lbs. per square inch, and could probably be heard under favourable conditions at a distance of one mile. There seems no reason why hooters of very much larger dimensions should not function equally well or better, in which case a sound production of intensity comparable with those of the most powerful sirens or whistles might be expected.

A rough comparison as regards efficiency of sound-production was made between this large hooter and the standard type of compressed-air whistle used on the Schenectady Electric Cars. The sound of this whistle on full air-consumption was appreciably louder than that of the hooter, but since it required for this a 1-inch supply-pipe as against the three-eighths inch on the hooter, it would appear that the latter is very much more efficient.

The efficiency of the hooter as a sound-producer depends in a high degree upon the details of its construction. One condition for good operation is that the mass of the flanged pipe, together with that of the frame in which it is rigidly supported, should be large compared to that of the vibrating plate or membrane. If this is not so the flange will also vibrate and not only waste energy but interfere with the vibration of the diaphragm.

The amount of air used depends also very largely on the closeness with which the flange can be approximated to the plate—actual contact does not prevent operation—and on the nearness with which exact parallelism of the two surfaces is attained.

The more massive the vibrating plate the better is the adjustment necessary in these respects for satisfactory operation, and for large hooters it is likely that both exact geometrical form and polishing of the juxtaposed surfaces would be desirable.

It is an advantage to vary the surface of the flange from a true plane to a form slightly convex towards the plate and to expand the end of the pipe, as shown in Fig. 3*a*. Probably the space between vibrating plate and flange should vary in sectional area according to the principle of the Venturi-nozzle. If, on the other, the departure from a true plane is in the direction of concavity towards plate or diaphragm (Fig. 3*b*), the operation of the hooter is adversely affected even to the point of failure.

No absolute measurements of the efficiency of the hooter as a sound-producer have yet been made. The comparison with the whistle alluded to above, and the very striking reduction in the quantity of water issuing between flange and plate, which takes place as soon as the latter commences to vibrate, would indicate that this is of a fairly high order.

No attempt at exact mathematical treatment of the energy and force relations involved in the action of this hooter will be made in this paper. Qualitatively the theory of operation is no doubt somewhat as follows: Let the plate be initially at rest with no fluid flowing. The static pressure of the fluid in the supply pipe produces a displacement of the plate or membrane calling into play an opposing force due to elastic resistance or tension, as it may be. Fluid now flows between plate and flange reducing the pressure and permitting return of the plate or membrane up to and beyond its equilibrium position. The flow is thus nearly or completely stopped with consequent rise of pressure up to and beyond the static pressure in the supply system; the plate is then driven back by this pressure and the cycle repeated.

During the first stage work is done by the plate upon the fluid in the supply pipe; during the second, positive work is done by the fluid upon the plate. Since the average pressure is clearly higher during the second stage, the total work done by the fluid on the plate in a cycle is positive.

The production of the sound is due no doubt mainly to the vibrations of the plate or membrane itself, but also in some degree upon the siren-effect of the intermittent outflow of air. This is put in evidence by the fact that the sound is emitted most strongly, at least for some hooters that have been tried, on the side of the nozzle. It might, therefore, be an improvement to modify the construction in such a way as to cause the escaping air to issue in front instead of, as at present, behind the plate.

The frequency of the note depends, of course, mainly on the natural period of the plate or membrane; but this is probably modified in the case of a membrane very considerably, in the first place, by the pressure of the driving fluid, which will, of course, cause variations of tension in the membrane, and, secondly, by the very heavy damping due to the air-cushion between flange and plate. The presence of this air-cushion probably accounts for the fact that a hooter will vibrate freely even when the flange is actually in contact with the plate or membrane before vibration commences. The damping is sufficient for all hooters yet experimented with to cause instant cessation of the sound when the supply is cut off. An easy method of varying the frequency of a given plate is to load it centrally with different masses tightly screwed or soldered on.

It is probable that frequencies well above the limit of audibility could be obtained, though no hooter has yet been made to test this point. An application to the purposes of submarine signalling suggests itself. For this steel plates comparable with those used in the Fessenden oscillator would be necessary. An auxiliary high pressure water pump with a suitable rapid action control valve would be the only accessories required. The radiation of high frequency waves from an area large compared to the wave length is, as is well known, highly directive in character. In fact, the addition of a horn to the large hooter described above, the frequency of which was certainly below 1,000 d.v. per sec., made very little increase in the amount of sound radiated along the axis of the plate.

I have to thank Mr. A. L. Rogers, mechanic to the Physics Department, for giving the benefit of his skill and experience to the construction of the hooters made and tested here.

DISCUSSION.

Dr. F. L. Hopwood remarked that the ingenious device which had been described afforded an instance of the solution of a new problem by the inversion of an old one. It might be advantageous to provide the instrument with a resonator.

XVI. *The Number of Radio-active Transformations as Determined by Analysis of the Observations.* *By* H. LEVY, *M.A., D.Sc.* (*Asst. Professor of Mathematics, Imperial College of Science*).

RECEIVED DECEMBER 24, 1921.

(COMMUNICATED BY DR. J. S. ANDERSON.)

ABSTRACT.

It is known that the expression for the nth product of a series of radio-active transformations is properly represented as the sum of n terms of the type $a_n e^{-\lambda_n t}$. When n is known, the coefficients a_n and λ_n are determinable with comparative ease. The present Paper finds a criterion for determining n, the number of transformations, when this is not known from *a priori* considerations. This consists in the successive evaluation of a system of simple determinants easily constructed from the observations. The value of n is at once found from the order of the particular member of the system that vanishes.

IN a Paper "On the Resolution of a Curve into a Number of Exponential Components,"* Mr. J. W. T. Walsh has developed a method for the determination of the coefficients a_r and λ_r in the expression for the amount of the nth product $N(t)$ in a series of successive transformations, where

$$N(t)=a_1e^{-\lambda_1 t}+a_2e^{-\lambda_2 t}+\ \dots\ +a_ne^{-\lambda_n t} \quad \dots\dots \quad (1)$$

In the subsequent discussion on the above Paper it was urged that a crucial difficulty in the application of the analysis to any particular case was the lack of knowledge with regard to the actual number n of the transformations which occurred, unless this deficiency could be made up from a priori considerations. Unless the appropriate number of terms were adopted in the expression for $N(t)$, it seemed obvious that wrong values would be found for the coefficients a and λ. As stated in the Paper referred to, the method there adopted was analogous to the Lagrange-Dale method of analysis of periodic observations.

In actual fact, a still closer analogy with the latter method enables the number n of transformations to be easily determined, and indicates the effect that would be produced on the coefficients if a wrong number were selected. The present Paper deals principally with these two points.

Let $N_0, N_1, \dots N_r \dots$ be a series of observations of the products of radio-active transformations corresponding to times $t=0, 1, \dots r \dots$, or a series of values taken from a smoothed curve of such observations for equidistant values of the time interval if the observations were not actually recorded at these times. The unit of time adopted may for the moment be supposed so selected as to provide from the experimental curve any desired number of values N_0, N_1, &c. When there are n successive radio-active transformations, I assume N can be expressed in the form (1). In effect, the problem is to determine n and the $2n$ constants $a_1 \dots a_n, \lambda_1 \dots \lambda_n$.

Following the method adopted in the Paper referred to, except that we do not restrict ourselves only to $2n$ observations, let

$$x_1=e^{-\lambda_1},\ x_2=e^{-\lambda_2},\ \dots\ x_n=e^{-\lambda_n}$$

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* Proc. Phys. Soc., Vol. XXXII., p. 26.

Then the λ's will be determined when the x's are obtained. Expression (1) now becomes

$$N_t = a_1x_1^t + a_2x_2^t + \dots + a_nx_n^t \quad \dots\dots\dots\dots \quad (2)$$

The observations N_0, N_1, etc., corresponding to the successive times $t=0, 1, \dots r \dots$ must accordingly satisfy the following relations

$$\left.\begin{array}{lllll} N_0 = a_1 & +a_2 & + \dots & & +a_n \\ N_1 = a_1x_1 & +a_2x_2 & + \dots & & +a_nx_n \\ \dots & \dots & \dots & \dots & \dots \\ N_r = a_1x_1^r & +a_2x_2^r & + \dots & & +a_nx_n^r \\ \dots & \dots & \dots & \dots & \dots \end{array}\right\} \quad \dots\dots\dots \quad (3)$$

The system (3) will consist of as many equations as we care, by taking the same number of values of N from the experimental curve. If we choose $2n$ such values, we shall have just sufficient equations to determine the $2n$ constants a_1, a_2, etc., and x_1, $x_2 \dots$ etc. If we choose more than $2n$ values of N, the system (3) will only be consistent if certain relations hold among the known values of N. These latter relations are in effect the conditions that must be satisfied if the additional values of N are to lie on the curve (1), as determined by the first $2n$ observations. These conditions, it will shortly be seen, enable us quite simply to determine n, the number of terms in (1).

Let the n quantities $x_1, x_2 \dots x_n$ be roots of the equation

$$x^n + p_1x^{n-1} + \dots + p_{n-1}x + p_n = 0 \quad \dots\dots\dots\dots \quad (4)$$

Consider the expression

$$S(n,r) = N_{n+r} + p_1N_{n+r-1} + p_2N_{n+r-2} \dots + p_nN_r \quad \dots\dots \quad (5)$$

where r is any positive integer. Inserting the expressions for the N's from (3)

$$\begin{aligned} S(n,r) &= \sum_{s=1}^{s=n} a_sx_s^{n+r} + p_1\sum_{s=1}^{s=n} a_sx_s^{n+r-1} + \dots + p_n\sum_{s=1}^{s=n} a_sx_s^r \\ &= \sum_{s=1}^{s=n} a_sx_s^r[x_s^n + p_1x_s^{n-1} + \dots + p_n] \\ &= 0 \end{aligned}$$

in virtue of the fact that $x_s (s=1, \dots n)$ is a root of (4). Accordingly, we can have as many expressions of the type (5) equated to zero as our observations will allow.

Corresponding to successive values of $r=0, 1, \dots$ the following system of relations between the observations must be satisfied

$$\left.\begin{array}{lllll} N_n & +p_1N_{n-1} & +p_2N_{n-2} & + \dots & +p_nN_0 = 0 \\ N_{n+1} & +p_1N_n & +p_2N_{n-1} & + \dots & +p_nN_1 = 0 \\ \dots & \dots & \dots & \dots & \dots \\ N_{n+r} & +p_1N_{n+r-1} & +p_2N_{n+r-2} & + \dots & +p_nN_r = 0 \\ \dots & \dots & \dots & \dots & \dots \end{array}\right\} \quad \dots \quad (6)$$

By selecting any arbitrary block of $(n+1)$ of these equations we are enabled to eliminate the unknown coefficients $p_1 \dots p_n$, deriving a whole system of determinants, each of which must vanish, and restricted in number only by the number of observations available. These determinantal equations are simply the condition that the observed values of N shall consistently correspond to points which fall on the curve defined by (1). The constants in (1) are, moreover, determined by associating with (4) any n equations of (6), and again eliminating $p_1 \ldots p_n$ when a series of determinantal equations, of which the following is a simple type, will be obtained :—

$$\begin{vmatrix} x^n, & x^{n-1}, & \ldots & x, & 1 \\ N_n, & & \ldots & N_1, & N_0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ N_{n-1}, & & \ldots & N_n, & N_{n-1} \end{vmatrix} = 0 \quad \ldots \quad (7)$$

This is the equation obtained by Mr. Walsh for the various values of x, but, of course, it is only one of a whole series of such equations. If the observations were exact, and if n be correctly chosen these equations must all be identical, but in practice they will differ. As in the Lagrange-Dale method for periodic analysis, the equations of type (7) enable us to replace them by one of similar form to (7), embodying all the observations. The single observations are there replaced by the sum of groups of observations. I do not propose to develop this here, however.

The vanishing of the determinants formed by eliminating $p_1 \ldots p_n$ from any $n+1$ equations of the system (6) obviously provides us with the criterion for the determination of n, for the observations are merely set down in a series of symmetrical determinants of $n+1$ rows and columns of the form

$$\begin{vmatrix} N_n, & N_{n-1}, & \ldots & N_0 \\ N_{n+1}, & & \ldots & N_1 \\ \ldots & & \ldots & \ldots \\ N_{2n-1}, & & \ldots & N_{n-1} \\ N_{2n}, & & \ldots & N_n \end{vmatrix} \quad \ldots \quad (8)$$

for successive values of $n=1, 2, 3, \ldots$, until for some value of n the determinant in question vanishes. It cannot moreover vanish for n less than the correct value, for otherwise this would imply that the points corresponding to the observations could lie on a curve for which n is less than the correct value. Every such determinant formed by taking n greater than the correct value will also vanish since all the minors are zero. The query was raised in the discussion on Mr. Walsh's Paper, whether, if too many terms are assumed in (1), say, by the addition of a term $a_{n+1}e^{-\lambda_{n+1}t}$ when there have been only n radio-active transformations, all the constants will readjust themselves in value. In the event of such an error, however, since all the determinants of which (8) is a simple type must vanish, all the minors of the new determinantal equation (7)—an equation now of degree $n+1$ in x—will consequently vanish and the coefficients of the various powers of x in (7) will all be zero ; or at any rate, in practice their values will be due to observational errors.

We have thus established a complete criterion for the determination of the number of radio-active transformations. In any particular case since the values of N are subject to experimental errors the determinant (8) will not vanish exactly and it becomes necessary to compare the magnitudes of the successive determinants. For a legitimate comparison, therefore, it is advisable to reduce the observations taking, say, N_0 as unity, *i.e.*, by dividing each observation by N_0, otherwise the effect of the errors may increase in absolute amount with increase in order of determinants.

While for a determinant of any particular order it is a comparatively simple matter to find an expression for the effect on the value of the determinant of small observational errors in the quantities N_t, I have been unable to find a simple expression from which the effect of such errors in general may be directly estimated. Where there is a systematic proportional error, that is, where instead of the accurate value N_t, the erroneous value $N_t(1+a)$ occurs where a is a small constant, it is evident that the process of dividing each observation by $N_0(1+a)$ before constructing the determinant will completely eliminate any error from the value of the determinant. If again the observational error increases with the value of t so that, say, instead of the accurate value N the erroneous value $N\ (1+ta)$, or $N_t(1+a)^t$ is observed, where a is a small constant, the vanishing of the determinant is likewise not affected; for determinant (8) becomes—writing down merely the diagonal terms:

$$| N_0, N_2(1+a)^2, N_4(1+a)^4, \ldots\ldots N_{2n}(1+a)^{2n} | = (1+a)^{n(n+1)} | N_0, N_2, \ldots N_{2n} |$$

It follows that the error in the estimation of a determinant of $n+1$ rows and columns is approximately $n(n+1)a$ times the value of the determinant, and, therefore, vanishes with the determinant. These two cases seem to suggest that no serious difficulty is to be anticipated on these grounds.

As an illustration we may take the example given in Mr. Walsh's Paper, where the author assumes or knows from *a priori* considerations that only two terms of the type (1) are present. Plotting the given observations on a smooth curve I derive the following data, by interpolation where necessary:—

$t =$	0	1	2	3	4	5	6 . . . .
$N =$	278	142	107	84	70	59	50 . . . .
$N_t/N_0 = N_t' =$	1	0·512	0·385	0·302	0·252	0·212	0·180 . . . .

$$\therefore \begin{vmatrix} N'_1, & N'_0 \\ N'_2, & N'_1 \end{vmatrix} = \begin{vmatrix} \cdot 512, & 1 \\ \cdot 385, & \cdot 512 \end{vmatrix} = -0{\cdot}123$$

$$\therefore \begin{vmatrix} N'_2, N'_1, N'_0 \\ N'_3, N'_2, N'_1 \\ N'_4, N'_3, N'_2 \end{vmatrix} = \begin{vmatrix} \cdot 385, \cdot 512, \ 1 \\ \cdot 302, \cdot 385, \cdot 512 \\ \cdot 252, \cdot 302, \cdot 385 \end{vmatrix} = -0{\cdot}002$$

$$\therefore \begin{vmatrix} N'_3, N'_2, N'_1, N'_0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{vmatrix} = \&c. \ \ldots\ldots = 0{\cdot}000007$$

Since the data are scarcely correct to the third figure and the above calculations have been carried through on a simple slide-rule, we may conclude that the determinants of third and higher orders vanish; it follows that there must be two terms in the appropriate expression (1).

DISCUSSION.

Mr. T. Smith said that the method of finding the number of terms described by Prof. Levy placed the analysis of observations of this type on a satisfactory basis. He wished to suggest that the absolute magnitude of the determinants used as an illustration by the author did not provide the key to the number of terms which should be considered, but rather the relative magnitude of successive determinants should be taken. In the preliminary examination to find n, it would be convenient to adopt as the unit for the N's the probable error due to experiment and graphical interpolation combined. The successive determinants will then at first rise, come to a stationary value, and afterwards fall. The number of determinants on the rising portion of the series would indicate the number of terms justified by the observations. For instance, if in the original example the total error is not likely to exceed ± 1 in any observation, the values of successive determinants arranged with the extreme readings on the main diagonal are

$$278, \ 9582, \ 36721, \ -2344,$$

showing that the observations require the presence of three terms. If on the other hand the individual errors may amount to ± 4, the values of the same determinants may be written approximately as

$$4 \times 70, \quad 4^2 \times 600, \quad 4^3 \times 570,$$

so that the evidence for the inclusion of a third term is inconclusive. The observations themselves show that, if the assumed mathematical form is applicable to them, the accuracy in this case is not as great as ± 4, since the solution when three terms are assumed to exist involves a harmonic component in one of them, and the character of the phenomena observed may be assumed to exclude the reality of such a factor.

A preliminary analysis by means of differences of increasing orders appears to be valuable when the observations are thought to be consistent with laws of this class. In the present example from the series

$$278, \ 142, \ 107, \ 84, \ 70, \ 59, \ 50, \ \ldots$$

the leading differences are

$$278, \ 136, \ 101, \ 89, \ 86, \ 89, \ 97 \ \ldots .$$

and the tendency to increase without changes of sign after a certain point indicates the presence of a term alternating in sign in successive observations. Corresponding to the original observations

$$\begin{aligned} N_0 &= a_1 + a_2 + a_3 + \ \ldots \\ N_1 &= a_1 x_1 + a_2 x_2 + a_3 x_3 + \ \ldots \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ N_r &= a_1 x_1^r + a_2 x_2^r + a_3 x_3^r + \ \ldots \end{aligned}$$

we have the successive differences

$$\begin{aligned} D_0 &= a_1 + a_2 + a_3 + \ \ldots \\ D_1 &= a_1 y_1 + a_2 y_2 + a_3 y_3 + \ \ldots \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ D_r &= a_1 y_1^r + a_2 y_2^r + a_3 y_3^r + \ \ldots \end{aligned}$$

where $x_1 + y_1 = x_2 + y_2 = x_3 + y_3 = 1,$

showing that the differences form a series of the same type as the original series. This formula indicates that if after some point the D's increase in magnitude without change of sign at least one of the y's is greater than unity—that is, at least one of the x's is negative. The analysis can evidently be carried out on the differences just as well as on the original figures. In complicated cases it appears desirable to determine short period terms from the original figures, and after these are removed to calculate long period effects from the differences for the remainder.

Mr. J. W. T. Walsh (communicated remarks): The criterion which Dr. Levy has found for determining the number of the terms in a composite exponential curve will be of great value to those who have to analyse such curves without any *a priori* guide to the number of terms present.

The principal difficulty in the practical application of the method is, of course, the presence of the unavoidable errors of observation, and the magnitude of these may be such as to make the verdict of the criterion somewhat uncertain, especially if two of the components really present happen to have values for the exponent which only differ by a comparatively small quantity.

It seems difficult to imagine a method which will avoid this difficulty, and probably the only course to adopt in such a case is to determine the constants for n components, where n is the smallest value indicated by the application of the criterion. If the differences between the observed values and the values thus calculated for n components do not in any case much exceed the estimated experimental error, then n must presumably be adopted as correctly representing the observations to experimental accuracy. If, however, the differences somewhat exceed the experimental error, then a further component may justifiably be included.

The AUTHOR, in reply, stated that Mr. Walsh's analysis had been developed from a method given by Dr. Dale, and applicable to periodic functions (indices of exponential terms imaginary). The extension given by Mr. T. Smith was analogous to a further procedure indicated by Dr. Dale, and, like the latter, had the advantage that the experimental errors of a plurality of terms were averaged, whereas without such procedure undue effect would be given to the errors of individual observations. Dr. Dale had shown that sums and differences of blocks of observations could be employed in the place of individual observations. Undoubtedly, the method indicated by Mr. Smith was the correct one for determining the required constants.

XVII. *On a Graphical Method of Treating Fresnel's Formulæ for Reflection in Transparent Media.* *By* CHARLES H. LEES, *D.Sc., F.R.S.*

RECEIVED FEBRUARY 7, 1922.

ABSTRACT.

It is shown that Fresnel's Formulæ for Reflection in Transparent Media may be treated by a simple graphical method which allows the effects of variations of the angle of incidence and of the refractive indices of the media to be traced readily.

(1.)

IN tracing the variations of the amplitude of the reflected wave R', when an electromagnetic wave I is incident at different angles on a surface separating two media transparent to the wave, I have found that a graphical method appeals more readily to students than an analytical treatment of the two well-known Fresnel expressions

$$\frac{R'}{I}=\frac{\tan(r-i)}{\tan(r+i)}$$

when the electric vector is in the plane of incidence, and

$$\frac{R'}{I}=\frac{\sin(r-i)}{\sin(r+i)}$$

when it is normal to that plane, i and r being the angles of incidence and of refraction respectively.

As it is possible that others have had a similar experience, I reproduce here the graphical method I have found most convenient, in order that its suitability may be tested by a wider circle.

(2.)

In the figures which follow, IP is the incident, PR' the reflected and PR_1 the refracted ray,[*] μ the index of the first and μ_1 that of the second medium. The incident IP and reflected PR' rays are produced beyond the surface if necessary. The distance PI is set off along the incident ray equal on some convenient scale to the component, either in or perpendicular to the plane of incidence, of the amplitude of the electric oscillation in the incident ray.

Through I, IQ' is drawn parallel to the refracted ray PR_1, cutting the reflected ray PR' or the ray produced in Q'. When PI represents the component perpendicular to the plane of incidence of the amplitude of the electric oscillation in the incident ray, PQ' represents on the same scale the amplitude of the electric oscillation perpendicular to the plane of incidence in the reflected ray, since the angle PIQ' is $i \backsim r$ and the angle $PQ'I$ is $i+r$, and the sides of the triangle IPQ' are as the sines of the opposite angles.

Through Q', $Q'Q$ is drawn perpendicular to IQ', to cut the incident ray IP or the ray produced in Q. When PI represents the component in the plane of incidence of the amplitude of the electric oscillation in the incident ray, PQ represents on the same scale the amplitude of the electric oscillation in the plane of incidence in the reflected ray, since if a perpendicular PP' were drawn from P on to IQ',

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[*] In Figs. 1 and 4 the refracted ray is not shown in order to avoid confusion.

$$QP/PI = Q'P'/P'I = \cot(i+r)/\cot(i\sim r).$$

As it is a convenience to represent the amplitude of the electric oscillation in a given ray by a length measured along that ray, QS' is drawn through Q parallel to the reflecting surface at the point of incidence, cutting the reflected ray PR' or the ray produced in S'. Thus, PS' represents the amplitude of the electric oscillation in the plane of incidence in the reflected ray.

In order to distinguish in the figures the two directions of oscillation, dots have been placed alongside the line representing the amplitude of an oscillation perpendicular to the plane of incidence, and short lines alongside and at right angles to the line representing the amplitude of an oscillation in the plane of incidence.

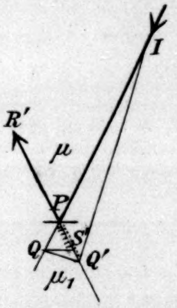

FIG. 1.—INCIDENT AMPLITUDE *PI*, REFLECTED *PQ'* AND *PS'*.

When the amplitude is shown as a line measured from the point of incidence P along the reflected ray PR' it indicates that there is no reversal of the oscillation, or, what is equivalent, no loss of half a wave length, on reflection. When the amplitude in the reflected ray is shown as a line measured from P along the reflected ray produced backwards, it indicates reversal of the oscillation or loss of half a wave-length on reflection.

If the electric oscillation is neither in nor perpendicular to the plane of incidence, it is resolved into its components in these two directions, the reflected oscillations PQ' and PS' due to them are determined by the construction and the resultant of the two is then $\sqrt{PQ'^2+PS'^2}$.

If, as in the case of ordinary light, the electric oscillation is equally in all directions perpendicular to the incident ray, the mean reflected amplitude is still $\sqrt{PQ'^2+PS'^2}$, if PI is the mean incident amplitude.

(3.)

The most important facts connected with the reflection of electric waves from the surfaces of transparent media are readily given by these constructions. Thus, if the index μ of the first medium is less than μ_1 of the second, and the incidence is not far from normal, we see from Fig. 1 that the amplitudes both for oscillations in and perpendicular to the plane of incidence are measured backwards from *P*, and that in each case the oscillation on reflection loses half a wave-length or is reversed.

As the angle of incidence decreases the angle of refraction decreases, and at nearly normal incidence the tangents and sines become equal to the angles $r \sim i$, $r+i$ and sice $\rho/r = \rho/i$, $R'I = (\rho - \rho_1)/(\rho + \rho_1)$ for both directions of oscillation.

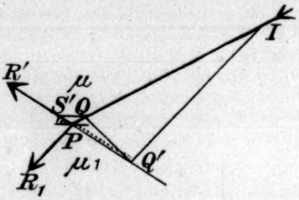

FIG. 2.—INCIDENT AMPLITUDE *PI*, REFLECTED *PQ′* AND *PS′*.

As the angle of incidence *i* is increased, *Q* and *S′* approach *P*, and the amplitude of the reflected wave when the oscillation is in the plane of incidence decreases. When *Q* and *S′* coincides with *P*, the reflected and refracted rays are at right angles, the amplitude is zero, and the reflected ray contains no oscillation in the plane of incidence. We thus get Brewster's law of complete polarisation of the reflected ray for an incident ray of any kind.

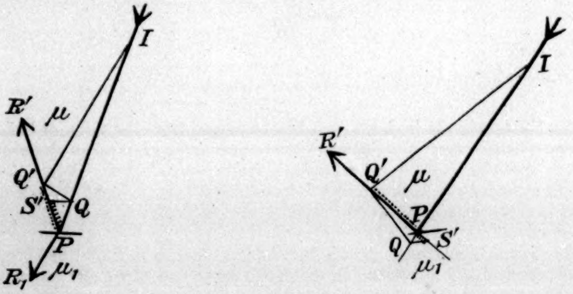

FIG. 3. FIG. 4.

INCIDENT AMPLITUDE *PI*, REFLECTED *PQ′* AND *PS′*.

When the angle of incidence is increased beyond the polarising angle, as in Fig. 2, the construction gives *Q′* still behind *P*, and the reflected ray losing a half wave-length if the oscillation is perpendicular to the plane of incidence; but *S′* is now between *P* and *R′*, and there is, therefore, no loss of half a wave-length for reflection of an oscillation in the plane of incidence.

As the angle of incidence approaches 90° PS' increases rapidly, and at grazing incidence both PQ' and PS' are equal to PI, and the whole of the incident energy is reflected.

(4.)

When the index μ of the first exceeds μ_1 of the second medium, the construction (Fig. 3) shows that there is no loss of half a wave-length on reflection in either case, since Q' and S' lie between P and R'. As the angle of incidence increases, S' approaches P, and the amplitude of the oscillation in the plane of incidence diminishes, becoming zero when S' coincides with P, that is at the Brewsterian angle of incidence, and the reflected ray oscillates in all cases perpendicular to the plane of incidence. For larger angles, S' lies behind P, showing that there is a loss of half a wave-length on reflection for oscillations in the plane of incidence, although there is no loss for oscillations perpendicular to the plane of incidence (Fig. 4).

When the angle of incidence is further increased, the refracted ray becomes nearly parallel to the reflecting surface (Fig. 5), PQ' and PS' becoming nearly equal to PI, and when the refracted ray grazes the surface $PQ'=PS'=PI$, and the whole of the incident energy goes into the reflected ray.

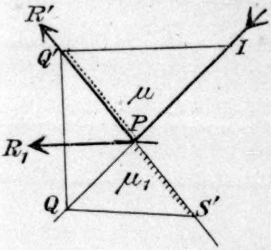

FIG. 5.—INCIDENT AMPLITUDE PI, REFLECTED PQ' AND PS'.

For angles greater than the critical angle the construction becomes impossible, since there is no refracted ray.

(5.)

When used in conjunction with any one of the graphical methods of determining the direction of the reflected and refracted rays,* the two constructions solve completely the problem of reflection so far as it is covered by the Fresnel formulæ.

DISCUSSION.

The PRESIDENT congratulated the author on his elegant construction, which would be welcome both to teachers and pupils.

From Prof. R. Ll. JONES (communicated): I have used the following method of tracing the variations in the amplitude of the reflected wave given by Fresnel's expressions. Describe a circle AEA', centre O, and on the diameter $A'A$ take a point C, so that $OC=\mu.OA$, μ being the ratio of the refractive indices of the two media. Through C draw $BC\,B'$ making, with OC, an angle r equal to the angle of refraction. Then $OB\,B'=i$, the angle of incidence, and $BOC=i-r$. This construction gives Fig. 1 when $\mu>1$, and Fig. 2 when $\mu<1$; the construction is due, I believe, to Tait.

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* The method in which two circles or radii proportional to μ and μ_1 are used, reproduced as Fig. 43 in the Intermediate Practical Physics of Schuster and Lees, is one of the simplest.

From the two triangles OBC, $OB'C$ with the side OC and the angle r in common we get

$$\frac{\sin(i\sim r)}{\sin(i+r)}=\frac{CB}{CB'}=\frac{CB^2}{CE^2}=\frac{1}{(\mu^2\sim 1)OA^2}\,.\,CB^2 \quad \dots\dots\dots\dots \quad (1)$$

where CE is tangent to circle in Fig. 1, and at right angles to AA' in Fig. 2.

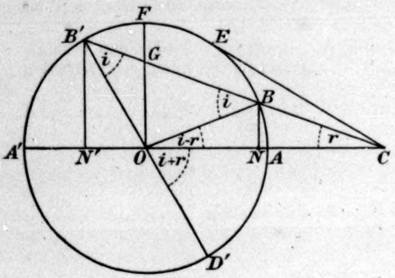

FIG. 1.

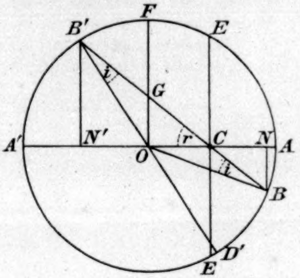

FIG. 2.

At normal incidence $CB=CA$ and

$$\frac{\sin(i\sim r)}{\sin(i+r)}=\frac{(\mu\sim 1)^2 OA^2}{(\mu^2\sim 1)OA^2}=\frac{\mu\sim 1}{\mu+1}.$$

At grazing incidence in the first case and at the critical angle in the second case, $CB=CE$ and the expression$=1$.

For light polarised at right angles to the plane of incidence Fresnel's expression for the amplitude of the reflected vibration is

$$\frac{\tan(i\sim r)}{\tan(i+r)}$$

From B, B' draw BN and $B'N'$ at right angles to AA' and let OF drawn at right angles to AA' meet CB' in G. Then we get

$$\frac{\tan(i\sim r)}{\tan(i+r)}=\frac{BN}{ON}\,.\,\frac{ON'}{BN'}=\frac{ON'}{ON}\,.\,\frac{B\,N}{B'N'}=\frac{B'G}{GB}\,.\,\frac{CB}{CB'}=\frac{1}{CE^2}\left[\frac{B'G}{BG}\,.\,CB^2\right] \quad \dots\dots \quad (2)$$

At normal incidence $B'G=BG$ and $CB=CA$, and the expression becomes $\dfrac{\mu\sim 1}{\mu+1}$.

When B' coincides with F the reflected and refracted rays are at right angles and the value of the expression is zero.

When B' passes through F, B and B' are on the same side of OF, indicating a change of sign and change of phase of half a period in the reflected ray.

The method given above follows naturally from the construction which Tait* uses to prove the proposition that in any refraction the deviation increases as the angle of incidence increases. The expression for the tangent formula in (2) is rather clumsy as it involves several varying quantities. In the important case of normal or nearly normal incidence (1) and (2) show the value of the amplitude of the reflected vibration in a perfectly clear and unmistakeable form.

Prof. LEES (communicated reply): It has always seemed to me that the graphical method of finding the refracted ray which depends on two circles with radii equal to the refractive indices was superior to the one Prof. Jones gives in his Fig. 1 and ascribes to Tait. Simple constructions for the Fresnel ratio of sines are easily found. The ratio of tangents gives more trouble and I should endorse Prof. Jones's statement that his expression is "rather clumsy."

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* Tait's "Light."

XVIII. *A New Form of High Vacuum Automatic Mercury Pump.* By H. P. Waran, *M.A., Ph.D. (Government of India Research Scholar of the University of Madras).*

Received February 9, 1922.

ABSTRACT.

The pump is based on a modified Sprengel action. It works automatically, the mercury being removed from the lower to the upper reservoir mixed with a current of dry air which is sucked through a side tube by a filter pump. The defects of design of former types of pump are discussed, their inefficiency at low pressures explained, and suitable simple remedies suggested. The introduction of an intermediate reservoir in the middle of the fall tube, kept automatically exhausted by the Sprengel action in the lower fall, allows the upper half to exert a positive exhaustion for every pellet of mercury falling down, even at the lowest obtainable pressure. The absence of compression in the first fall enables the maximum bore to be used for the fall tube; and hence speeds and efficiencies of exhaustion comparable to those of a Gaede pump are obtainable, though less than a pound of mercury is required to operate the pump.

Introductory.

The rapid production of high vacua is increasingly becoming necessary in modern work, and the difficulties that confront experimenters in situations where none of the modern high-class mechanical or mercury pumps are available for the purpose are only too well known. Any arrangement improvised locally to meet the situation must satisfy by the nature of the case the principal requirements of simplicity, low cost, quickness of action, as well as automatic working. A Töpler pump of the Antropoff or inclined barrel form is convenient enough, though it requires a lot of mercury as well as constant attention. But a mercury pump that operates on the old Sprengel principle is evidently the most convenient, since, by the introduction of the well-known arrangement for the automatic circulation of the mercury, not only is the pump kept in operation without attention for long periods, but the amount of mercury required for its working is rendered surprisingly small, if the parts are properly designed. Further, considering the modern practice of maintaining a steady suction at one end of an apparatus and balancing it by a steady gas feed at the other end, such a form of pump has its special advantages for the purpose.

Various forms of automatic mercury pump, of the design of Boltwood, Khalbaum, and Zehnder, were in use before the advent of the Gæde pump. But these have not come into general use, since, on account of their defective design, they were rather slow in action and ineffective at very low pressures. This can easily be seen by a brief consideration of the working of such a pump using the simple Sprengel action. When the mercury begins to hammer in the fall tube the situation is as represented in Fig. 1, and efficient exhaustion then practically ceases. The mercury stands up to nearly barometric height in the fall tube, and a pellet of mercury falling down this tube traps the low-pressure gas between A and B, compressing it to an inappreciably thin disc of gas against the head of the mercury column. Finding downward progress against the atmospheric pressure difficult, this disc gets shattered into bubbles which stick to the glass walls, and find an easy escape upwards. This escape back of the gas is facilitated in the automatic types of pump by the oscillatory motion of the mercury column. The pressure at which this process sets in is determined solely by the width and height of the effective fall above the barometric height,

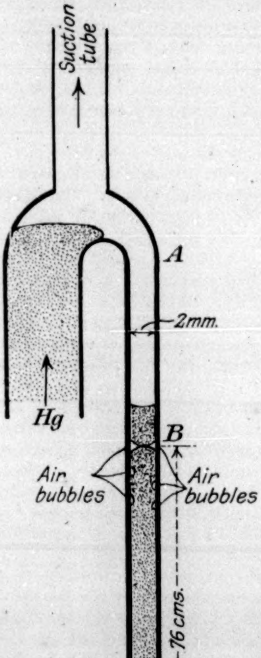
Suction tube
A
2mm.
Hg
B
Air bubbles
Air bubbles
76 cms.

FIG. 1.

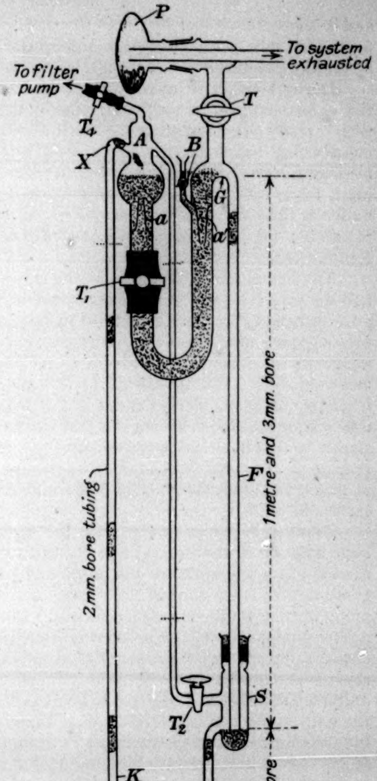
P
To system exhausted
To filter pump
T
T_4
X
A
B
G
a
a
T_1
2 mm. bore tubing
F
1 metre and 3mm. bore
S
T_2
K
H
50cms. and 2mm. bore
R
T_3
D

FIG. 2.

P_1 = Phosphorus pentoxide drying chamber.
D = Drying bottle of concentrated sulphuric acid.
T_1, T_3, T_4 = Rubber connections with screw clamps.
T, T_2, B = Vacuum glass stop cocks.

and it is easy to see that by the employment of a greater length and narrower section for the fall tube, a higher vacuum is obtainable (though correspondingly more slowly) before the pump ceases to act on this account.

During some experiments with pumps of this kind it occurred to the writer that this objection could be easily overcome by the following simple device, which carries with it many other advantages. The fall-tube is divided into two sections, the upper one F being longer than the lower one H; and between them is mounted an intermediate reservoir S, as shown in Fig. 2. Tube F is connected to S through a rubber joint, made easily by slipping on a short length of thick-walled rubber tube that makes a tight fit round the outside of the fall tube, as well as inside the tubular reservoir; the joint being rendered air-tight by a pool of mercury over the rubber forming a mercury seal.

The lower half H of the fall tube is connected to the side of the reservoir S, and the projecting end of F inside S so adjusted as to be just below the surface of the pool of mercury in S, which is filled to the verge of overflowing into H. The fall of mercury now takes place in two stages: the first one in F, whereby the pellet of mercury pushes down in front of it the column of rarefied gas into S, which is kept automatically exhausted to nearly the same pressure; and a second one in H, whereby this accumulation of gas in S is pumped away whenever it is sufficient in amount for the simple Sprengel action to come into play. In fact the action is very similar to that in a Gæde pump—hence its high efficiency and speed even at very low pressures. The introduction of such a device in the fall tube enables the exhaustion to proceed to limits previously unattainable. (Reference may be made here to an earlier Paper.*)

Another principal trouble with the earlier forms of the pump arises from the large amount of air carried along by the mercury into the vacuum system, partly carried along the walls of the tube, and partly adsorbed. Elaborate devices to guard against the fouling of the vacuum due to this cause are in use in every type of pump working on the Sprengel principle. In the modern automatic types, worked by a filter pump, the vacuum of a few centimetres of mercury at the main reservoir end considerably reduces the amount of such admixed air. I find a pair of traps in a short U-tube in the circuit of the incoming mercury, as indicated at *a* and *a′* in Fig. 2, a simple and effective safeguard to break and check the progress of air working along the walls of the tube; the traces of admixed air being simultaneously eliminated by the filtering operation of the traps. Further, as with a Gæde pump when the rate of exhaustion is sufficiently high, residual minor leakages become quite negligible.

On such lines I have modified my original design of the pump, and have arrived at a compact and simple design illustrated in Fig. 2, the performance of which under ordinary circumstances compares favourably with that of a small size Gæde pump.

The Working of the Pump.

Keeping T_1 open (Fig. 2) the pump is charged with mercury to the levels indicated by pouring it through B, which is then closed. On starting the filter pump, with T, T_2 and T_4 open and T_1 and T_3 closed, it sucks away the system down to a centimetre of mercury in a few minutes through the circuit $PTFST_2AT_4$. After closing T_2, if T_3 is slightly opened a current of dry air is sucked through DT_3RKAT_4, and the back pressure of this makes the level of mercury in H rise about 15 centimetres higher than that in R, and this pressure tends to drive the mercury from A into F on opening T_1 a little. The mercury from A passes through a and a', and getting rid of the admixed and creeping air in these filters falls down F in pellets which trap columns of air of the volume of F, and push them into S. If T_2 is again opened a little, the gas accumulating in S escapes readily into the filter pump, while the mercury flows down H, raising the level in R past the opening from T_3. The current of air coming in through T_3 pushes this mercury into K, and aided by the suction of the filter pump at the other end it goes rapidly up the tube K, and on reaching the bend at X the mercury gets shot down into A, while the air is sucked away by the filter pump. This action of the pump is automatically maintained; and in a few minutes, when the pressure has gone down to the order of 0·01 mm. (or earlier if the volume exhausted is small) the tap T_2 is closed again. The second fall H now begins to function, rapidly exhausting S and maintaining it at about 0·001 mm. The flow of mercury may now be speeded up to the maximum rate it is possible to use with advantage, allowing for the slow flow of gas through various tubes at such low pressures, and the definite time of fall down F. The exhaustion in the present apparatus now proceeds rapidly at the very stage when the usual types cease to act; for every pellet falling down F produces a positive exhausting effect in P even at the lowest pressures obtainable.

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* Proc. Indian Association for the Cultivation of Science, Vol. VI., p. 199 (1921).

For the most effective and rapid working the relative size of openings of T_1 and T_3 have to be so adjusted by trial that, with the minimum opening of T_3 and the maximum of T_1, all the mercury coming down F is transferred back to A from R at the quickest rate, without, however, causing back pressure of mercury in H of more than 30 centimetres. An interesting property of the pump is its capacity of self-adjustment within wide limits in regard to this requirement. For example, if an excess of mercury is coming down F it gets transferred up K rather slowly; and in the meantime the filter pump, producing a higher vacuum at A, reduces the back pressure driving the mercury into F, and so the mercury flow becomes automatically reduced.

To stop the pump the taps T, T_1, T_3, and T_4 are closed in the order named, before shutting off the filter pump and then its water supply.

Though the pump is easily improvised according to the above design by anyone acquainted with the elements of glass blowing, for its efficient performance a number of minor points have to be attended to, and the few complications introduced into the design have for their object the improvement of the efficiency and quickness of action of the pump. It would take too long to go into the details and considerations that govern the design of each part; the principal points requiring special attention are, however, briefly outlined below.

The side tube from S, containing tap T_2, may be dispensed with in the simpler models intended for exhausting small volumes. The object of this side connection is solely to expedite the suction in the preliminary stages. For by opening T_2 the filter pump can straight away exhaust the system (up to its limit of about a centimetre of mercury) through $PTFST_2A$, instead of through $PTFSHRA$, in which case it would have to suck against the head of a few centimetres of mercury at R. Further, if the system to be exhausted is of any considerable volume, because of the accumulation of gas in S in the preliminary stages at a rate at which the second fall cannot pump it away, the pump cannot be speeded up at once to its full capacity. With the provision of the side connection this accumulation is readily removed by the filter pump, through T_2 kept partially open for the purpose; this tap being completely closed for the final stage of high exhaustion.

If the side connection with tap T_2 is provided, it is preferable to have it of narrow bore capillary tubing, so that the fluctuations of pressure in A during the working may not produce any effect at S when T_2 is kept partially open in the preliminary stages.

To ensure the rapid automatic circulation of the mercury in the pump the dead capacity on the filter-pump side, *i.e.*, the volume of the air circuit $RKAT_4$ up to the filter-pump, must be made as small as possible.

Though T, B and T_2 must be good vacuum taps, the others may with advantage be good rubber joints with screw clamps over them, thus providing greater flexibility of adjustment.

The rounded shape of the bend at G is also important, since upon this depends the proper formation of a single large drop of mercury which shall on falling into F form at once an effective plunger.

Concluding Remarks.

A great advantage secured by the adoption of the modified Sprengel action (above described) is that it enables a high vacuum to be attained at a high speed, because of the large bore and length of fall-tube which it permits. Using a 3 mm. bore fall-tube (in place of the usual one of 1 mm.) and an effective length of 100 cm. (instead of the usually available one of about 50 cm.), and bearing in mind the rate of exhaustion varies directly as the volume of the effective fall, it is readily seen that this form of pump exhausts about twenty-five times as rapidly as the ordinary form of Sprengel pump.

Since the gas is not compressed appreciably in the pumping out process in the first fall, the apparatus is effective against traces of vapours, and thus the limit of exhaustion is as good as, if not better than, that given by a Gæde pump. It requires less than a pound of mercury to run it, while for a Gæde pump about 50 lbs. of mercury must be used. Again, the extra source of power required to work a Gæde pump is dispensed with.

The present design of pump has also the advantage that it can be made or bought in five ready-made units that require only to be mounted rigidly on a stand, and connected up by rubber joints (preferably mercury seated) in positions indicated by the dotted lines in Fig. 2, the use of rubber joints being permissible since they lie on the low vacuum circuit only. (The complete pump, or its component parts ready for mounting, can now be had from Messrs. Baird and Tatlock, Hatton Garden, London.)

A few months ago I had occasion to make and use pumps of this type at the Cavendish Laboratory, Cambridge, for running a 20-ft. hydrogen discharge tube (Prof. R. W. Wood's pattern) of large volume, with a steady gas feed at one end, and its performance was quite satisfactory throughout the few months it was in almost daily use. Connected to other apparatus a degree of exhaustion represented by an alternative gap of about six inches has been obtained without difficulty.

DISCUSSION.

Mr. R. M. ARCHER: I should like to put three questions to the author. (1) Does he not find that the bottom of the tube S occasionally gets knocked out by the falling mercury? (2) How long would the pump take to evacuate a space of, say, 1 litre to a pressure of 0·0001 mm.? If it can do this in a reasonable time it has a great advantage over the Gaede pump, which is apt to get out of order in consequence of leakage of lubricant, etc., and is difficult to put right. Further, the present pump has the advantage of being transparent, and it can be easily kept clean. (3) Can it be used to extract residual gases for analysis?

Mr. J. Guild: Would the pump become too complicated if it were provided with several fall-tubes in parallel? In an ordinary Sprengel pump it is common to use half a dozen of these, thus compensating for the disadvantage of a narrow bore. The present apparatus seems to me to demand considerable skill for its construction, whereas a Sprengel pump is generally used by persons who wish to erect a cheap and simple apparatus for themselves. Is not the apparatus very liable to breakages?

The Author: In reply to Mr. Archer—The tube *S* does not get broken by the falling mercury—in fact, the bottom of this tube may be of rather thin glass. I could not say offhand how long the pump would take in the case he gives, but in the case of a space of 500 c.c. it produced a vacuum corresponding to a 4-inch spark-gap in about an hour. In reply to Mr. Guild—My idea has been to construct a simple pump which anyone can make. The additional fall-tubes would certainly add to its complexity, and they would also take up more mercury. I do not think any one possessed of moderate skill in glass-blowing should have much difficulty over the construction. As regards accidents, I have used four of these pumps and nothing worse has happened than the cutting off of the water supply by some unauthorised person! The apparatus should not be left idle unless the stop-cocks are turned off, otherwise mercury may get forced into the tube *P*. It will be noticed that a trap is provided against this contingency—a device which has proved an adequate safeguard.

DEMONSTRATION of an Electrostatic Voltmeter, and of Apparatus for Weighing and Density Determinations, by the Research Staff of the General Electric Co., London. (Shown by Mr. E. M. EDEN and Dr. F. S. GOUCHER.)

(*a*) *A Rapid-Weighing Balance* for quantities up to 40 mg. (scale-divisions 0·1 mg.). This instrument is more particularly used for weighing filaments, which are hung on the lever of a torsion balance, thereby tightening a fine vertical wire. To the middle point of the latter is attached one end of a horizontal wire, the other end of which is fixed. The middle portion of the horizontal wire is optically projected on to a scale.

(*b*) *Electrostatic Voltmeter.*—This comprises a fixed and a movable disk, the distance apart being adjustable. The movable disk when attracted tightens a catenary of fine wire, the middle portion of which is optically projected on to a scale.

(*c*) *Apparatus for the Measurement of Density of Fine Wires.*—The wire to be tested is arranged as a cantilever and the sag of its end is measured in air and in a liquid of known density.

DISCUSSION ON THE ELECTROSTATIC VOLTMETER.

Prof. C. H. **LEES** commented on the high damping of the instrument, which contrasts favourably with that of many electrostatic voltmeters.

Mr. C. C. **PATERSON** remarked that the design in some respects resembles that of Abraham, which also employs sliding electrodes. The chief novelty of the present instrument lies in the direct measurement of the sag of the catenary.

Mr. K. **EDGCUMBE** said he should be interested to know how the instrument was calibrated.

Mr. B. S. **GOSSLING**, in reply to the previous question, stated that the electrometer was standardised by comparison with an electro-dynamometer. It might be of interest, he added, to give some further details. A theoretical calculation shows that for very small voltages the scale is somewhat close, but for greater voltages it is more open, and is nearly uniform, closing again as the wire becomes taut. The calculated scale agrees very well with that obtained by the calibration, and the scales corresponding to different adjustments of the disks agree equally well with one another. The catenary is so designed that the movement of the small attracted disk is very small compared with its distance from the fixed disk.

XIX. *On the Electro-magnetic Screening of a Triode Oscillator.* *By* R. L. SMITH-ROSE, *B.Sc., D.I.C., A.R.C.S., A.M.I.E.E.* (*Communication from the National Physical Laboratory.*)

RECEIVED FEBRUARY 16, 1922.

ABSTRACT.

The results of the many experiments on screening described in the accompanying Paper show that the most complete method of screening a valve set is to enclose it in a box made of metal of suitable thickness for the frequency used, and to seal hermetically all joints in the box and its lid. The smallest crack or hole is sufficient to allow of the escape of a detectable amount of the high-frequency energy, and it is shown that iron is far more effective than copper of the same thickness in preventing direct penetration of radio-frequency magnetic fields through the metal.

While a practicable means of using an oscillator inside an hermetically sealed screening box is described for work of the highest precision, a much more convenient arrangement for the use of continuous-wave wireless direction-finding is described, with some results which have been obtained with an experimental sample in actual use for this work.

The experiments herein described were carried out at the National Physical Laboratory in connection with investigations undertaken for the Radio Research Board.

1.—*Introductory.*

THE problem of screening a body or piece of apparatus from a high-frequency electro-magnetic field is known to be very much more difficult than the protection from steady electric or magnetic fields. Electrostatic screening may be effectively carried out by completely surrounding the body with a metal covering (a Faraday cage), the thickness of which is immaterial. In cases where transparency of the screen is desirable to permit of inspection of the interior, the screen need not be absolutely continuous, *e.g.*, metal gauze or perforated sheet is effectively used to screen instruments from stray electric fields, or from capacity to external objects. Magnetic shielding of a body may be carried out by surrounding it with a heavy iron screen, which, however, only partially screens by deflecting the magnetic field through the space of greater permeability.

With an alternating magnetic field, however, the shielding effect of the iron is much more complete, and in this case, also, the shielding may be produced by other metals than iron, due to the demagnetisation by the eddy currents induced in the metal by the initial magnetic fields. Simple experiments with oscillation generators give the impression that when the frequency of the alternating magnetic field is very high, the difficulty of the screening is greatly increased. This is, however, a false conception and arises from the fact that the induced E.M.F. in any circuit placed in an alternating magnetic field is directly proportional to the frequency, and thus becomes very large at frequencies of the order employed in radio work. It is essentially this fact, together with the very high magnifications produced by multi-stage triode amplifiers, that leads to some rather surprising results when attempts are made to shield instruments from radio-frequency alternating magnetic fields. That the principles underlying the effective screening of a radio-frequency field have been somewhat misunderstood in the past is borne out by the fact that several experimenters write of employing a Faraday cage for this purpose, the cage often being constructed of wire gauze. The term "Faraday cage" can only strictly be employed in connection with electrostatic screening, and its effect is altered immediately the field is fluctuating or alternating.

2.—*Object of Present Experiments.*

The present investigation arose from the necessity of a screened local oscillation generator for use in continuous-wave reception on a system of wireless direction-finder. The local oscillations are most conveniently generated by a simple form of triode oscillator operating entirely from a 6-volt battery, and the problem is to protect the receiving coil system from induced local oscillations, which were to be led to the required point in the circuit by a purely conductive connection. The final form of screened triode oscillator may, however, be applied to other purposes than the above. It is, for example, a very necessary appliance for certain methods of measuring the strength of received radio signals.

The transfer of electrical energy from one oscillating circuit to another may take place by either pure mutual induction or by radiation, or by both these factors. In the case under consideration, in which the closed oscillating circuit of the generator is placed comparatively near the receiving circuit, the transfer is almost entirely by mutual inductance, since the radiation will be negligible by comparison. The problem may, therefore, be considered correctly as the limiting of the high-frequency magnetic field around the oscillator by induced eddy currents in the screening body.

3.—*Preliminary Experiments.*

Some early miscellaneous experiments serve to indicate the difficulty of constructing a perfectly screened oscillation generator. These experiments were carried out with a compact form triode oscillator, using a single R-valve operating entirely from a 6-volt battery, and having a wave-length range of from 1,000 to 10,000 metres. With this instrument and a large-frame coil receiver and a seven-valve amplifier in use at Teddington, the more powerful and near C.W. transmitting stations, such as Horsea and Chelmsford, give a heterodyne note which can be heard several yards away from the telephones, even when the oscillator is placed at the far end of the wireless hut, about 100 ft. away. This indicates how very small is the oscillating current required to give a beat note with incoming signals.

When the oscillator and battery were placed inside a thin copper box, with a loose-fitting copper lid, the induced oscillations in the receiving coil were considerably reduced and were only effective in producing a beat note when the oscillator was within 20 ft. of the coil.

When the copper box was replaced by a galvanised-iron bin, a similar reduction in the induced oscillations was obtained of about the same order. In both these cases it was observed that the placing of the lid on the box had a considerable effect in the reduction; but the position of the otherwise open end of the box relative to the receiving frame appeared to be immaterial to the strength of the induced signals. A piece of fine brass wire gauze was not so effective as a lid as a solid sheet of material, whether copper or iron.

To obtain much greater sensitiveness in the later experiments and also for more convenience when no suitable incoming signals were received, a second valve oscillator was used, placed inside or near the large receiving coil which was also tuned to the oscillator frequency, thus producing a constant beat note in the telephone receivers when any appreciable oscillation was induced by the generator under experiment. The large receiving coil with tuning condenser and seven-valve amplifier thus acted as an extremely sensitive search-coil method of detecting the presence of any radiated or induced radio-frequency field.

The former oscillator and its battery was next placed inside the copper box, and this in turn was supported on paraffin blocks inside the iron box and with the respective lids in position. In this condition the beat note was only appreciable when these boxes were placed within about 12 ft. of the coil. By using a small coil in the amplifier circuit as a search coil, it was found that a considerable increase in the strength of the note was obtained with this coil placed near the aperture between box and lid, indicating that some, at least, of the energy escaped in this direction. Making metallic connection between the copper and iron box had no effect on the strength of the received signal.

Further experiments showed that the copper box provided with a well-fitting lid giving about 3 in. overlap was much more efficient in screening than with the lid formerly employed, only ½ in. deep.

4.—*Experiments on Boxes with Multiple Copper Linings.*

As the combined copper and iron boxes seemed to give hopeful results, as above mentioned, further experiments were carried out with boxes with two and three linings.

One such box was made up as shown in Fig. 1, consisting of a wooden box *A* lined inside and outside with thin sheet copper, and supplied with a completely enveloping lid *B*, similarly double lined.

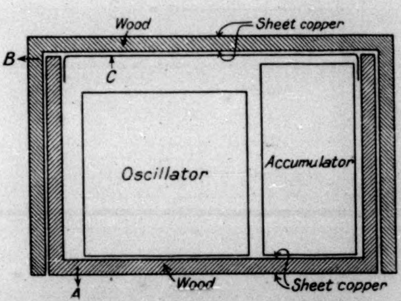

FIG. 1.—SIMPLE SCREENED OSCILLATOR.
A = Double-Copper-Lined Wooden Containing Box.
B = ,, ,, ,, ,, Lid.
C = Inner Copper Lid.

With this box it was found that putting on the lid made a considerable reduction in the escaping energy from the oscillator as indicated by the diminution in intensity of the note heard in the telephones. This box appeared, in fact, to be about as good as the combined copper and iron boxes previously tried, but was much more convenient from the point of view that the double-lined lid can be removed at one operation. The diminution effect was continuous as the lid was placed over the box, being a minimum when the box was completely enveloped.

It was subsequently found that the fitting of an auxiliary inner lid, as shown at *C*, effected a still further reduction in the emitted energy.

Another arrangement consisted of two copper boxes, complete with close-fitting lids, separated from each other by 1 in. wooden strips. This arrangement was found to screen most efficiently when the inner box was inverted, as shown in Fig. 2, when it was about the same as the last arrangement used. Again, no difference was experienced on making contact between the two boxes.

At this and subsequent stages of the experiments the need was felt for some simple and fairly accurate means of obtaining a measure of the emitted energy from such a type of partially-screened oscillator. What is essentially required is to measure the comparative strengths of the induced energy with the unscreened oscillator, and with the latter enclosed by the various screening arrangements which were used during the present investigation. The shunted telephone method might possibly be used, but the intensity of the note heard in the telephones does not necessarily indicate the amount of energy intercepted by the frame search coil, particularly when the oscillator is unscreened and the induced E.M.F. very large. It was often noticed, for instance, that even when the oscillator was partially screened, the intensity of the note in the telephones increased to a maximum before diminishing, when the oscillator was moved away from the coil. Also the interpretation of the results in terms of the distance of the oscillator from the frame is not simple.

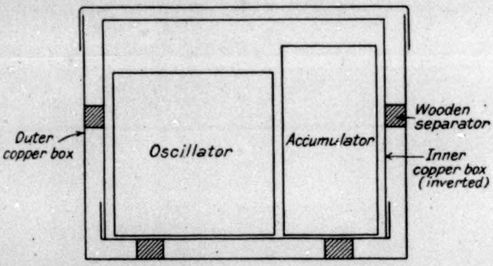

FIG. 2.—TRIODE OSCILLATOR, SCREENED WITH TWO COMPLETE COPPER BOXES.

It is possible that a somewhat elaborate arrangement would meet the requirements, involving a very constant amplifier and a means for calibrating it, but this was not considered to be within the scope of the present investigation.

5.—*Effect of Bringing Leads out from Screening Box.*

With a type of double-screened box as used above, the external field, therefore, could be rendered inappreciable when it was at a greater distance than 12 ft. from the search coil. In this condition a small coupling coil of a few turns was introduced into the inner copper box, and a flexible conductor brought outside both boxes. So long as this conductor remained entirely within the outer iron box, the beat note was undetectable, but immediately a few inches of it were exposed outside the iron box, even with a closely-fitting lid, good beat note signals were immediately obtained, giving evidence of energy thrown off from these leads. Replacing the flexible wires by a lead-sheathed twin conductor overcame this trouble, which, however, recurred on connecting a small coil to the end of the screened leads, with a view to coupling on to the amplifier. By varying the coupling to the oscillator, this latter " radiation " could be reduced to an extent inappreciable compared with the oscillating energy which could be conveyed through the outer coupling to the amplifier.

6.—*First Experimental Screened Oscillator.*

Embodying the essential results of the experiments carried out as above, a first experimental oscillator was made up to the author's design by Messrs. H. W. Sullivan. This was in principle, although not in detail, similar to the arrangement shown in Fig. 3 below, consisting of a wooden box double lined with sheet copper 1/64 in. thick, fitted with an inner copper lid and with a second enveloping lid similarly constructed of wood and double lined with sheet copper. Ebonite connecting pieces were used between the control handles on the top of the lid and the variable condenser and switches for filament current and inductance tappings within the box. The metal parts of the control handles were arranged to make spring contact with the outer coating of copper.

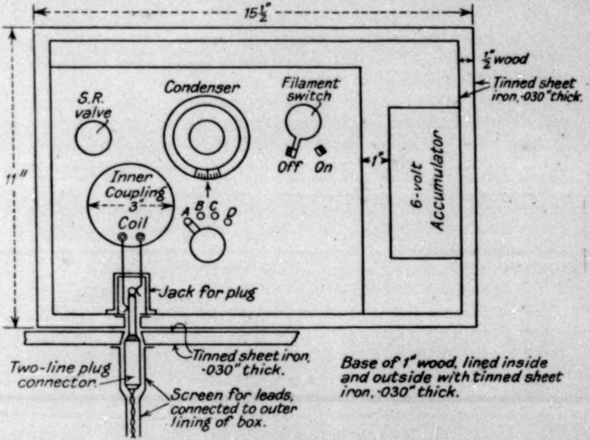
15½"
½" wood
Tinned sheet iron, ·030" thick.
S.R. valve
Condenser
Filament switch
Off
On
1"
6-volt Accumulator
11"
Inner Coupling 3" Coil
A B C D
Jack for plug
Tinned sheet iron. ·030" thick.
Two-line plug connector.
Screen for leads, connected to outer lining of box.
Base of 1" wood, lined inside and outside with tinned sheet iron, ·030" thick.

FIG. 3.—PLAN OF INTERIOR OF SCREENED OSCILLATOR.

A separate coil was used within the box to couple to the oscillator inductances, and leads from this coil were brought out through the side of the box by a simple telephone jack and plug connection. In the present arrangement, the jack is mounted inside the box and the plug is pushed through holes in lid and box which are only in alignment when the lid is in its final position over the box. The plug is screened by a metal coating in contact with the outer lining of the box and the connections are taken through a metal-sheathed twin flexible conductor to a double-D-shaped coil, forming half of the outer coupling. Leads from the other pair of D-shaped coils are taken to the output terminals mounted on ebonite at the side of the box.

The following points may be emphasised in regard to this screened oscillator arrangement :—

1. The entire control of the oscillator is obtained from outside, and it need never be opened except for inspection, replacement of valve, or recharging of accumulator. A valve (type SR) consuming only 0·1 ampere filament current is used, which with the accumulator fitted will give continuous operation for about 120 hours.

2. The opening up of the oscillator, when necessary, is a simple operation. It entails the removal of the connection plug, control handles and the two screening lids.

3. Fairly close metallic contact is obtained at all points in the paths by which the radio frequency energy may escape from the box, other than by direct penetration through the sides.

4. No part of the primary oscillator circuit is outside the inner metal lining. The secondary circuit is brought out by a screened conductor to an astatic coupling coil arrangement. The other half of this astatic coupling forms the tertiary circuit containing leads from the direction-finding coil to the amplifier. This coupling forms a convenient means of varying the strength of the local oscillation led to the receiver, varying from a maximum down to zero.

There is thus no part of any tuned circuit common to the inside and outside of the box, and all leads from the inside are carefully screened to reduce to a minimum any energy thrown off by these. With this oscillator in operation, but disconnected from the D.F. receiving circuit, beat-note action could just be detected on the latter, when the oscillator was about 5 ft. from the centre of the D.F. coil. In the case of the reception of a C.W. station, this heterodyne action was negligible, compared with the note intensity that could be obtained by connecting up the oscillator to the receiver in the proper manner.

As the pick-up effect from the oscillator by the large frame coil (5 ft. by 3 ft. 6 in.) is so small, it is reasonable to assume that that picked up by the amplifier will be negligible. The oscillator may, therefore, be placed close up to the amplifier to make the connections between the two as short as possible. As it is essential in the system to have the amplifier at least 5 ft. from the D.F. coil, the oscillator can conveniently be placed below or behind the amplifier and so be in a position in which its inductive or radiative action on the frame is quite negligible.

7.—*Use of the Experimental Screened Oscillator for Continuous Wave Direction-Finding Observations.*

With this first screened oscillator, used under the conditions outlined above, a number of observations were taken in duplicate on the few C.W. stations that can be received at Teddington with and without heterodyne. The results, all of which were made in the daytime, are summarised in the following Table :—

Station observed.	Wave-length km.	Without Heterodyne.		With Heterodyne.		Difference. A—B
		No. of observations.	Mean bearings. (A)	No. of observations.	Mean bearings. (B)	
Le Vallois ...	2·2	20	145°·0	18	145°·0	0°·0
Chelmsford	3·8	22	56°·3	24	56°·5	—0°·2
Horsea.........	2·5	22	220·5	22	220·8	—0°·3
,, ...	4·0	44	219·8	50	220·0	—0°·2
,, ...	8·0	Signals too weak		18	219·8	—

These observations show that on the stations observed the difference in the mean readings taken, with and without heterodyne, is usually negligible and never greater than the ordinary errors of D.F. observation. The normal position of the oscillator in the above results was 5 ft. west of the coil.

A number of observations were also carried out on Chelmsford, with the position of the oscillator varied, and these showed that no serious error is introduced until the oscillator is within 4 feet from the centre of the coil, in which position it is only about 1 foot from the side of the coil during part of its swing.

With the oscillator on the floor immediately underneath the D.F. coil, the error introduced was not more than 0·3°, while with the oscillator placed inside the frame, the error ranged up to 2° in either direction, according to the orientation of the oscillator relative to the frame.

No consistent difference in effect was obtained when the tertiary circuit of the oscillator was connected in either the grid or filament lead of the amplifier, but it may possibly help to balance up the "antenna effect" of the system as a whole if it is introduced into the grid lead.

As a typical example of the effect of "unscreening" the oscillator when connected up in its normal position the following result may be quoted: With the oscillator fully screened, the error introduced by it was about 0·1°; when the outer lid was removed, this error increased to 4·3°, and with the inner lid also removed, it became 8·3°.

Where it is possible to make the necessary connections, it would be preferable to connect the tertiary circuit of the oscillator in the grid circuit of the rectifying valve of the amplifier, to reduce the possibility of reaction from the high-frequency transformers.

8.—*Further Experiments on Screening.*

Using the above screened oscillator in conjunction with the coil receiving set as a very sensitive detector of a high-frequency magnetic field, a number of further experiments were carried out on the screening of the oscillator originally employed :—

(*a*) *Hermetically-sealed Copper Box.*

One of the original screening boxes was used, size 12 in. square by 8 in. high, made of sheet copper 25-30 mils thick. All joints at sides and bottom were sweated together with about ½ in. overlap, the box being liquid-tight and probably gas-tight. A well-fitting lid was made for this of similar copper sheet, giving about ½ in. overlap. This lid was actually used as the base with oscillator and battery placed on it, and the box over all. In such condition the external field was easily detectable by the receiving set when the box was within 20 ft. of it. A wave-length of 2,400 metres was used, as being fairly free from interference by external signals, and also giving greater possibilities of screening than a longer wave-length.

Without altering any of the adjustments the box and lid were solidly soldered up, using a good thickness of solder to cover up all holes and cracks. In this condition the effect of the syntoniser could still be detected when within 10 ft. of the frame. When the box was placed inside the frame the resultant beat note in the telephones could be heard 100 ft. away. Also by tilting the box inside the frame a position giving practically dead silence could be obtained in much the same way as is obtained with the oscillator without the screening box. No effect could be obtained by removing the box from the frame and placing it near the amplifier.

The effect of hermetically sealing up this copper box is, therefore, to reduce some of the emitted energy which presumably came from the cracks round the lid, while still leaving a very appreciable amount, apparently passing through the metal, the external field retaining the characteristics of that from the unscreened oscillator.

(*b*) *Hermetically-sealed Iron Box.*

A similar experiment to the above was carried out with a box of the same dimensions constructed of tinned iron sheet 28 mils in thickness.

With the oscillator placed inside the box, with a well-fitting lid giving ⅜ in. overlap, the resulting effects of beat note were about the same as for the copper box. Placing another iron covering over the sides of the box made no appreciable difference, but when the lid as well was covered the emission was considerably reduced. This seemed to indicate that the major portion of the energy from the inner box was coming from under the lid.

Using the same wave-length of 2,400 metres, the box and lid were solidly soldered up as before, after which no beat note was audible in the telephones. The fact that the oscillator was still in operation undisturbed was proved by making a slit in the box, when a beat note was easily heard in the telephones. A subsequent repetition of this experiment showed that enough energy escaped through a slit ½ in. long to be easily detected. Further experiments indicated the absolute necessity of stopping all holes and cracks and of getting good metallic contact across all the joints, as a " dry soldered " joint may easily be the cause of an external magnetic field.

These experiments indicate therefore that it is only possible to screen a valve oscillator completely, as far as the sensitivity of the above apparatus goes, by placing it inside an hermetically-sealed box of tinned sheet iron of sufficient thickness to prevent the direct penetration of the high-frequency magnetic field through it. In the latter respect iron would appear to be far superior to copper, which is a result in complete accordance with the theory of the penetration of alternating currents into conductors. (Skin effect.) This theory indicates that iron should be equivalent to about four to six times its thickness in copper. It is, therefore, probable that if experiment (*a*) above were repeated using a box of ⅛ in. copper sheet, complete screening would also have been obtained.

(*c*) *Mercury-sealed Boxes.*

The process of soldering and reopening metal boxes containing a valve oscillator is very laborious and considerably limits experiments on these lines. It was, therefore, thought that a mercury seal, if of sufficient thickness, would be as efficacious as solder in stopping up holes and cracks.

A suitable tray 2½ in. deep was therefore made on which either the copper or the iron box could be placed giving an all-round clearance of about ½ in. Placing oscillator and battery in this tray with the iron box over it, the external field was easily detected when within 20 ft. of the frame. Filling up the tray with ½ in. of mercury to close all cracks entirely stopped all detectable field, even when the box was placed inside the frame. On raising the box at one side the smallest crack was immediately detected by the note in the telephones.

Using the copper box in the above manner, however, a beat note was still obtained when the mouth of the box was entirely below nearly 1 in. of mercury. This comparison experiment is very striking. With the copper box placed over, the beat note steadily diminishes, and a just detectable drop in intensity is obtained when contact is made with the mercury and the last crack closed up. With the iron box, however, the drop in note intensity to entire inaudibility when contact is made with the mercury is very sudden and a most marked effect.

These experiments were repeated for wave-lengths of 5,000 and 9,000 metres, and the residual effect with the copper box was slightly more noticeable on the latter wave-length, although the initial note-intensity was less due to the inefficiency of the amplifier at this wave-length. While these results indicate the conditions for effective screening over the range 2,000—10,000 metres, the problem is probably much simpler for shorter waves, but somewhat more difficult for longer waves. A slight increase in thickness of the box would, however, probably be effective up to the greatest wave-lengths at present in use. These results thus confirm fully those previously obtained and indicate that the most complete method of screening a valve oscillator is to enclose it in an iron box of sufficient thickness for the frequency used, and to seal hermetically all joints either with solder or mercury. Where very complete screening is required, therefore, this may be obtained on the above lines, using a plain iron box standing over an iron tray with welded joints, since mercury readily amalgamates with tin and solder. No external controls may be permitted, owing to the aperture necessary for these, but it would not be very difficult where an approach to ideal conditions is very essential, to remove the box when adjustments are required to be made. One serious disadvantage to this, however, is that the frequency generated by the valve set alters considerably when the box is placed in position over the oscillator. In all the above experiments, this fact always necessitated the continual readjustment of the auxiliary oscillator to retain the beat note within the audible limits.

9.—*Design of Screened Heterodyne for D.F. Sets.*

The general result of this investigation is that for applying auxiliary oscillations for C.W. reception on the D.F. directional wireless receiver in use, *an almost completely* screened oscillator, similar to the experimental sample used above, is suitable, with certain modifications. These consist in a reduction in the size and weight of the instrument, with the substitution of tinned-iron plate for the copper sheet previously used.

The details of the design of the instrument are shown in the accompanying Figs. 3, 4 and 5, which indicate the essential parts of the oscillator and screening box, to the scale of the modified design.

Fig. 3 is a plan view of the interior of the box, showing the positions of the oscillator and battery, with the various switches; also the position of the inner coupling coil and the plug connection. Fig. 4 shows the positions of the screening lids with the method of obtaining external control. The exterior of the top of the box is illustrated in Fig. 5, showing the position of the D-shaped coupling coils.

10.—*Variation of the Screening Effect with Frequency.*

When a sheet of metal is placed in an alternating magnetic field, eddy currents are set up in the conductor, the field from which will tend to counteract the original magnetic field. The penetration of these eddy currents into the conductor will be governed by the ordinary well-known " skin-effect " formulæ, and hence the latter will determine the fraction of the original magnetic field which penetrates to any depth in the conductor. The decay factor governing the amplitude of the high-frequency current at a depth "x" below the surface of the metal is $e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}$,

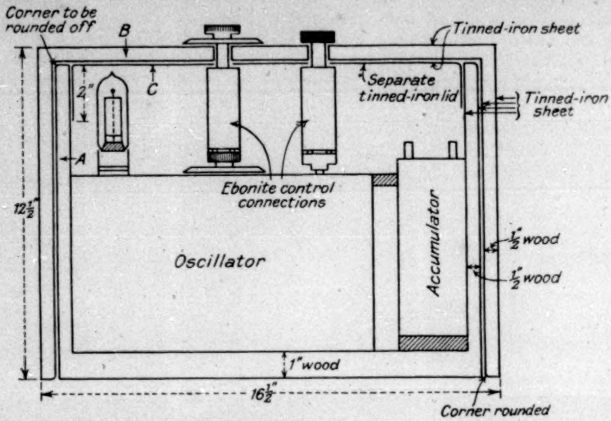
Corner to be rounded off
B
Tinned-iron sheet
2"
C
Separate tinned-iron lid
Tinned-iron sheet
A
12½"
Ebonite control connections
Oscillator
Accumulator
½" wood
½" wood
1" wood
16½"
Corner rounded

FIG. 4.—SECTIONAL ELEVATION, SHOWING TINNED-IRON SHEET LININGS, AND CONTROL HANDLES.

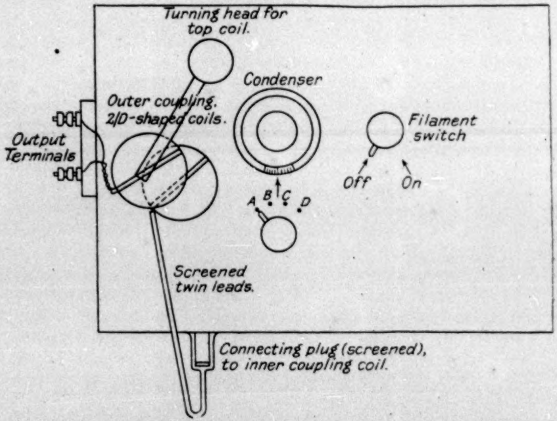
Turning head for top coil.
Condenser
Outer coupling, 2/D-shaped coils.
Output Terminals
Filament switch
Off
On
A
B
C
D
Screened twin leads.
Connecting plug (screened), to inner coupling coil.

FIG. 5.—PLAN OF EXTERIOR OF SCREENED OSCILLATOR.

where "μ" and "ρ" are the permeability and resistivity respectively of the metal and "f" is the frequency of the alternations.

In the present case, this indicates that the field at a depth "*x*" is reduced to the fraction $a=e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}$ of its original value at the surface of the metal. Hence, if the strength of the original magnetic field and the limiting value to which this must be reduced be known, the thickness of the metal required for the screen can be calculated.

In the absence of knowledge of the absolute magnitude of the "screened" field the comparative dimensions of the screens for different metals and frequencies may be obtained. For example, for a given frequency the thickness of iron and copper to produce the same values of "*a*" above may be obtained.

For copper	$\mu=1$	$\rho=1{,}600$
,, iron	$\mu=300$	$\rho=16{,}000$ approximately.

Hence

$$\frac{x_{\text{copper}}}{x_{\text{iron}}}=\sqrt{\frac{1{,}600}{1}\times\frac{300}{16{,}000}}$$

$$=\sqrt{30}$$

$$=\ 5{\cdot}5$$

i.e., it requires 5·5 times the thickness of copper to screen as effectively as a given thickness of iron.

The above expression also shows that the thickness of metal required to produce a given value of "*a*" varies as $\sqrt{\frac{1}{f}}$. In screening a circuit from a stray magnetic field, however, the usual condition is that the induced E.M.F. in the circuit shall be reduced below a certain limiting value. Now the E.M.F. induced in a coil by an alternating field of strength $H \propto f\times H$. If the circuit is now shielded by a metal sheet, the induced E.M.F. becomes

$$E=f\times e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}\times H$$

which is the true equation giving the variation of the screening effect with frequency.

Thus, suppose it is required to determine the comparative thicknesses (x_1, x_2) of a metal necessary to give the same screening effect at frequencies of 100 and 1,000,000 cycles per second. In this case we have

$$10^2\times e^{-2\pi\sqrt{\mu 10^2/\rho}\,.\,x_1}=10^6\times e^{-2\pi\sqrt{\mu 10^6/\rho}\,.\,x_2}$$

$$e^{-2\pi\sqrt{\mu/\rho}\,.\,x_1}=10^4 e^{-2\pi\sqrt{\mu/\rho}\,.\,10^3 x_2}$$

$$-20\pi\sqrt{\frac{\mu}{\rho}}x_1=\log_e 10^4-2000\pi\sqrt{\frac{\mu}{\rho}}\,x_2$$

For copper, $\mu=1$ and $\rho=1{,}600$; hence

$$\frac{\pi}{2}x_1=50\pi x_2-2{\cdot}3\times 4$$

$$x_1=100x_2-\frac{18{\cdot}4}{\pi}$$

$$x_1=100x_2-5{\cdot}86.$$

Thus, if a thickness of copper $x_2=\frac{1}{8}''=0{\cdot}32$ cm. is required to give effective screening at 10^6 frequency, then at a frequency=100 cycles per second the thickness required to give the same screening effect is

$$\begin{aligned} x_1 &= 100 \times 0{\cdot}32 - 5{\cdot}86 \\ &= 32 - 5{\cdot}86 \\ &= 26{\cdot}14 \text{ cms.} \end{aligned}$$

This example would appear to demonstrate that the difficulties of screening are greatly enhanced at frequencies well below the radio limit—*e.g.*, within the range of audible frequencies, as used in many forms of inductance and capacity bridge measurements.

Since this Paper was written I have learnt that some experimental work on screening has been carried out at the R.A.F. Instrument Design Establishment, Biggin Hill, and at the Army Signals Experimental Establishment, Woolwich ; but no details of this have yet been published. The results of the work at Biggin Hill are already being used, however, in the design of wireless instruments.

DISCUSSION.

Mr. Morton: On behalf of the staff at Biggin Hill, I should like to express our appreciation of the author's work, and of the reference to our own with which his Paper concludes. The errors in wireless direction finding which are produced by adventitious inductive coupling with the local oscillator were fully recognised during the war. They were counteracted to some extent by fixing the oscillator to the rotating coil, and adjusting its position in such a way as to minimise the evil. Experiments on screening were carried out in 1918, but this plan was not adopted in practice till February, 1921, when devices somewhat similar to those described by the author were brought into use.

Dr. J. H. Vincent: The author has called our attention to the surprisingly large amount of energy that can escape through a crack in the screen surrounding an oscillator. A similar fact was proved in the case of sound by Lord Rayleigh ; and in experiments on the screening of sound carried out some years ago, I found that at a pitch such as middle C a quite considerable amount of energy can get through a crack measuring, say, ⅛ in. by 1 in. in a solid screen. It might be interesting to design an oscillator calculated to give out a minimum of energy by induction or radiation. By constructing the inductances and condensers in a suitable manner, it might be possible to confine the bulk of the emitted energy to the purely conductive path.

Mr. F. E. Smith: I might remark that the author's research was undertaken at the instance of a Committee of the Radio Research Board, which is investigating the errors of wireless direction-finders. Observations made at some twelve stations throughout the country indicate that while considerable accuracy is attainable with damped waves, undamped waves give results which are vitiated by the radiation of energy from the oscillator. The problem of screening the latter has, I think, been satisfactorily solved by the author. As regards the calculation at the end of his Paper, the value of 300 assumed for the permeability of the iron seems rather optimistic, considering the low value of the magnetic fields involved.

Dr. A. Russell: The author's formulæ for the skin effect apply to the case where the magnetic field is at right angles to the screen. Has he considered the case where it is parallel to the screen?

The Author: I must thank Mr. Morton for his kind reference to the work described in the Paper. I am interested in Dr. Vincent's reference to the work of Lord Rayleigh on the escape of sound through cracks. One is reminded of the experience on closing a window ; it is the last fraction of an inch that makes all the difference in excluding sound. The ordinary oscillator gives out about a million times as much energy as is required, and the trouble is that no substantial decrease can be obtained without stopping the oscillations altogether. Possibly the use of a toroidal inductance or something of that sort would reduce the external magnetic field to within reasonable limits. In reply to Mr. Smith—I had tried to discover an established value for μ at low values of H ; but the results given by different experimenters differ widely, and the figure given in the Paper claims to be nothing more than a rough approximation to a mean value. In reply to Dr. Russell—I take it that the skin-effect formula applies to the magnetic component parallel to the screen, while the perpendicular component is mainly responsible for the large effect of a crack. Assuming that this latter component is normally screened by the circular eddy-currents which it sets up, we may suppose that a crack, by distorting the current flow, becomes subject to an oscillating potential difference between its ends, and so acts like a small Hertzian oscillator.

XX. *Viscosity Determination by Means of Orifices and Short Tubes.*

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RECEIVED JANUARY 10, 1922.

ABSTRACT.

The Paper consists of an investigation of the corrections applicable to the determination of the viscosity of liquids by flow through tubes, due to the abnormal flow at the ends.

General *expressions for the end-corrections* are obtained by the method of dimensions. These are employed in plotting the results obtained from experiments on the flow of mixtures of glycerine and water through pairs of tubes of the same diameter, but of different lengths.

The conditions that the flow at the ends may be purely viscous, and the value of the end-correction in this case, are obtained from the present experiments, and also from experiments on an orifice described in a previous Paper.

Equations are given enabling viscosity to be measured by experiments on the flow through orifices or short tubes.

Introduction.

THIS Paper consists of a full investigation of the allowance that must be made for the end-effects when measuring viscosity of liquids by flow through a capillary tube. The results enable short tubes or even cylindrical holes in plates to be used for measurements of viscosity, without previous calibration by means of a liquid of known viscosity.

In order to make the end-effects definite and independent of the surface tension, the liquid was not allowed to emerge into the air on leaving the capillary tube, but passed into a second vessel of liquid. The tube used was thick-walled, and the two ends were ground off at right angles.

The abnormal conditions at the ends cause the pressure-difference between the liquid in the two vessels to be greater than the value deduced theoretically without allowing for these end effects. The end correction usually applied is proportional to the square of the rate of flow, and is due to the kinetic energy of the liquid. The value of this correction has been for a considerable period in doubt; and even recently complete unanimity of opinion did not exist.* Hence it was thought desirable to make determinations in the present experiments.

For any particular liquid and tube, for rates of flow less than a certain critical value, the flow at the ends will be purely viscous; and the end-correction under these conditions will be proportional to the rate of flow. It has been mentioned by various writers that such a viscous end-correction must exist, but no numerical data are given. This correction is investigated theoretically and experimentally in the present Paper, one estimate of its value being obtained from the author's experimental results on viscous flow through an orifice.†

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* Edser, "General Physics for Students," 1913, p. 498.

† Proc. Phys. Soc., Vol. XXXIII., Part IV., pp. 225-230 (1921).

General Theory of the End-Corrections.

Consider two tubes of lengths l_1 and l_2, both having an internal radius r. When the rate of flow is in each case Q c.c. per second, let p_1 and p_2 be the total differences of pressure tending to cause flow in the two cases.

($p_1=h_1 g \rho$, where
h_1=difference in level between the exposed liquid surface in the two vessels
g=acceleration due to gravity
ρ=density of liquid.)

The ends may be supposed not to affect appreciably the flow at parts of the tube more than a certain distance from them. Then it follows that

$$\left.\begin{aligned} p_1&=l_1 \,.\, dp/dl+p_e \\ p_2&=l_2 \,.\, dp/dl+p_e \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where dp/dl denotes the pressure gradient causing flow at parts of the tube distant from either end; and p_e denotes the excess of pressure difference causing flow over that required to cause an equal flow if there were no end-effects. It will be noticed that p_e refers to the combined effects of the two ends.

From equation (1) it follows that

$$\left.\begin{aligned} dp/dl&=(p_1-p_2)/(l_1-l_2) \\ p_e&=(l_1 p_2-l_2 p_1)/(l_1-l_2) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

which enables dp/dl and p_e to be deduced from the pressure differences required to cause an equal flow through two tubes of the same internal diameter but different lengths.

General equations relating the end-correction p_e with the other variables concerned may be obtained by the method of dimensions. Let μ and ρ denote the viscosity and density of the liquid, and Q the volume passing per second. All the variables concerned may be related by the equation $\varphi_1(Q, r, p_e, \mu, \rho)=0$, where φ_1 is a non-dimensional function. Since there are only two independent non-dimensional products of the five variables, we may write without loss of generality

$$\varphi_2(Q\mu/p_e r^3 .\quad Q\rho/r\mu)=0,$$

and hence

$$p_e=Q\mu/r^3 . \varphi_3(Q\rho/r\mu)=Q^2\rho/r^4 . \varphi_4(Q\rho/r\mu) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

In a similar way, for parts of the tube distant from either end,

$$dp/dl=Q\mu/r^4 .\ f_3(Q\rho/r\mu)=Q^2\rho/r^5 . f_4(Q\rho/r\mu) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

When the pressure differences are independent of the viscosity, f_4 and φ_4 are constant. Under these conditions the flow is generally described as "turbulent." Similarly for purely viscous flow f_3 and φ_3 are constant, the pressure difference being independent of the density.

If $1/\sqrt{\varphi_4}=Q/r^2 . \sqrt{\rho/p_e}$ be plotted against $\sqrt{Q\rho/r\mu}$, it is easy to show that the curve will be linear both in the region where the flow is purely viscous, and where it is purely "turbulent." This method of plotting is employed in this Paper.

Description of Experiments.

To determine values of p_e, a pair of uniform thick-walled glass tubes of the same internal diameter, but of different lengths, was used. The difference in level between the surface of the liquid in the two vessels was determined for the same rate of flow in each tube. The method of experiment was almost identical with that used previously in experiments on an orifice.* Experiments were carried out with different pairs of tubes, the details of which are given below. The different sets of experiments are indicated by letters, which are also inserted by the corresponding sets of determinations in Fig. 1.

Experiment.	r	l_1	l_2	ρ	μ
(a)	0·135	0·626	2·605	1·250	6·3
(b)	0·1001	0·853	2·685	1·249	4·8
(c)	0·1001	0·853	2·685	1·227	1·067
(d)	0·1001	0·853	2·685	1·183	0·211
(e)	0·1001	0·853	2·685	1·158	0·106
(f)	0·1001	2·685	13·66	1·11	0·049
(g)	0·1001	2·685	13·66	1·05	0·022
(h)	0·1001	2·685	13·66	1·00	0·0101
(i)	0·1001	0·853	2·685	1·00	0·0094

Mixtures of glycerine and water were used for all these experiments. As some experimenters seemed to doubt whether the viscosity of very viscous liquids is independent of the rate of shear,† preliminary experiments were carried out using glycerine. The flow took place through a capillary tube, and the result of the experiments was that no variation in μ could be detected at 0°C. between rates of shear at the walls of 4·1 and 0·6 radians per second; nor at 14°C. between rates of shear of 15·7 and 0·6 radians per second.

The experimental determinations of p_e are exhibited in Fig. 1, where the non-dimensional functions derived in the section on general theory are plotted. This figure will be considered in detail in the following section of the Paper.

Values of the Viscous and Turbulent End-Corrections.

The portion (*a*), (*b*), (*c*) of the curve shown in Fig. 1 corresponds to the flow, being purely viscous not only in the tube but at each end. Hence from equation (3) we have $p_e=(Q\mu/r^3)\,\varphi_3$, where φ_3 is now a constant. The total end-correction may be considered as increasing the effective length of the tube by an amount equal to k times the internal diameter of the tube. Hence

$$p_e=(16k/\pi)(Q\mu/r^3) \quad \text{or} \quad k=\varphi_3\,.\,\pi/16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus, it will be seen that k is an absolute constant when there is purely viscous flow at the ends, provided the ends are made of definite form as described previously.

The value of k or φ_3 for these conditions should be capable of derivation from the general hydrodynamical equations when only terms involving viscosity are of importance.

A fairly accurate estimate of k is obtained from the slope of the portion (*a*), (*b*) (*c*) of the curve in Fig. 1. This gives a value of $k=0{\cdot}566\pm0{\cdot}020$. (The comparatively large probable error is chiefly due to the fact that the quantity measured is only a correction factor.)

A second estimate may be deduced from the results obtained by the author when experimenting on a small orifice.* The thickness of the orifice l was equal to 0·051 times its diameter $2r$; and it was found that the orifice could be replaced (for purely viscous flow) by an equivalent length, $2k'r$, of tube of radius r, where $k'=0{\cdot}631$. Hence, allowing for the effect of the thickness of the orifice plate, we have

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* Loc. cit.

† For references *see* the Author's paper on orifice flow, previously mentioned.

$$k=k'-0{\cdot}051=0{\cdot}580\pm0{\cdot}01.$$

This is in good agreement with the other determinations given above.

Combining the two experimental determinations and using equation (5), we have, for purely viscous flow,

$$k=0{\cdot}573\pm0{\cdot}009 \text{ and } \varphi_3=2{\cdot}92\pm0{\cdot}05 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence, by equation (3)

$$p_e=(\mu Q/r^3)\ .\ (2{\cdot}92\pm0{\cdot}05) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The portion (*d*), (*e*), (*f*) of Fig. 1 corresponds to the flow in the tube being viscous, but the end correction being due to kinetic energy. Thus the flow on emergence into the second reservoir is probably turbulent. The ordinate marked *A* corresponds to the usually accepted value of the end correction under these conditions. This will be seen to be in close agreement with the present experiments. The ordinate *B* corresponds to the value given by Edser.*

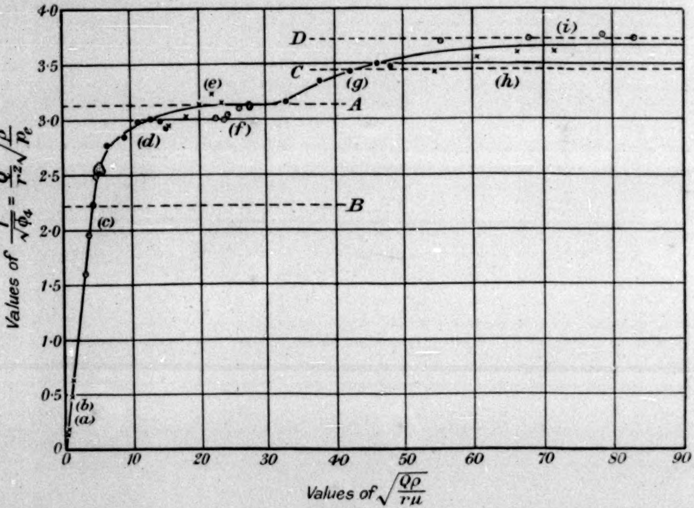

FIG. 1.

The portion (*g*) corresponds to the stage when the flow in the tube is becoming turbulent, and the part (*h*) (*i*) to the flow being turbulent both in the tube and at emergence. The ordinates *C* and *D* give the range of the values usually quoted as applying to the end effect for large pipes. Since the flow is then turbulent, the agreement between these and the present experiments acts as a further confirmation of the present determinations.

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* Loc. cit.

The transition (*g*) may have occurred at lower rates of flow than usual in the present experiments, as all the tubes were short, and the edge at the inflow end was in no case rounded.

Application to Viscosity Determinations.

When a liquid is arranged to flow through a tube as described in the Introduction the value of the end correction p_3 can be obtained from Fig. 1 if the other variables be known. If μ be not known, an approximate value can be assumed and p_e deduced. This will then yield a closer approximation to μ, and so on.

It can be seen on inspecting Fig. 1 and the figures in the Paper on flow through orifices,* that the flow will be purely viscous at all points, provided

$$Q\rho/r\mu < 16 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Under these conditions the viscosity is given by

$$\mu = \pi r^4 p / 8Q(l+2kr) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

where $k = 0{\cdot}573 \pm 0{\cdot}009$.
l=length of tube
r=radius of tube.
p=total difference in pressure causing flow.
Q=volume of liquid passing per second.

This equation enables viscosity to be determined by purely viscous flow through quite short tubes or even orifices made by drilling a hole in a plate, l being in this case the thickness of the plate.

When $Q\rho/r\mu$ has values above 250, but the flow in the tube remains purely viscous, the end correction will be within ± 3 per cent. of the value of $p_e = \rho Q^2/\pi^2 r^4$ usually assumed.

For values of $Q\rho/r\mu$ intermediate between the two above cases, the value of p_e may be obtained from Fig. 1, as described above.

When the statement (8) is not true, it will in practice be necessary to use moderately long tubes in order to determine μ with any appreciable accuracy, for otherwise the kinetic end-effect may become larger than the main term being measured and, also, the flow in the tube may become turbulent.

In conclusion, the author would like to thank Prof. W. G. Duffield for the facilities that have enabled these experiments to be carried out; and Mr. J. S. Burgess, the laboratory steward, for his kind help in all experimental matters.

DISCUSSION.

Prof. A. O. **Rankine**: The author's investigation aims at showing that it is possible to measure viscosity by means of a short tube. This reverses the usual procedure of using a long tube in order to reduce error due to doubtful end-corrections. Is there any practical reason why a long tube should not be used?

Dr. J. S. G. **Thomas** inquired as to whether the magnitude of the end correction could even now be regarded as definitely settled. The matter had been discussed in a very recent Paper (Phil. Mag., Feb., 1922), in which it was also pointed out that when employing a variable head in viscosity determinations, it was not, in general, accurate to employ the mean value of the head for the purpose of calculating the absolute value of the viscosity. The author had employed a variable head in his experiments, and it would be interesting to know whether this possible source of error necessitated any correction. In the apparatus shown, the efflux of liquid took place into a body of the liquid under experiment, whereas in commercial types of viscometers, such as the Redwood, employing a short tube, the efflux occurred into air. A comparison of the results afforded by the Redwood and Engler types of instruments with those afforded by a method employing a long capillary has been made by Higgins (Collected Researches, National Physical Laboratory, Vol. XI., pp. 1-16), and it would be interesting to know whether the results of the author's experiments are in agreement with these results. The usual end-correction applied in the case of liquids was the entrance end-correction. In the case of the efflux of gases through orifices in plates, the phenomena at the exit end were of at least equal, if not more, importance. In the case of the discharge of air through an orifice into the surrounding atmosphere, the relation between the pressure of discharge and the volume of air induced or set in motion per unit volume of the discharge was quite different over a certain region of pressure according as the orifice was one in a thick or thin plate. He inquired as to how far similar considerations might hold in the case of the efflux of liquids from long and short tubes.

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* Loc. cit.

The AUTHOR (partly communicated) : In reply to Dr. Rankine : The use of a short tube has advantages in the case of a very viscous liquid, since the apparatus can be very much smaller than would be necessary if the end-corrections were to be made negligible. It is thus much easier to keep the apparatus at a uniform constant temperature. In reply to Dr. Thomas's remarks : The author considers that the magnitude of the end-correction, under definite conditions mentioned in the Paper, is known to a fair degree of accuracy. On the other hand, a complete *theoretical* treatment may still be lacking. The Paper by Lidstone (Phil. Mag., Feb. 1922), which the author has now examined, gives the usually assumed value of the end-correction when a constant head is used. The deviation of this value, however, assumes that the viscous loss of energy and the kinetic energy removed from the system at emergence act independently. This leads to the conclusion, pointed out by Edser, that the pressure difference between the upper reservoir and the emergent liquid varies with the distance of the stream line from the axis of the tube (which is almost certainly erroneous).

Errors due to the variable head do not affect the present experiments. The author was aware of this matter and used a logarithmic method of plotting in the experiments described above, undertaken to test whether μ depended on the rate of shear. The author has not compared his results with any in which the liquid flows through an orifice or short tube *into air*. The results of such experiments might depend on whether the liquid wet the outer surface or flowed in a compact cylindrical stream. For purely viscous flow the end-correction p_e is half caused at entrance, and half at emergence. When the end correction is due to kinetic energy, an orifice does not satisfy the conditon that there shall be no overlap of the two end-regions. . However, the figures in the present Paper and that on orifice flows referred to, may be compared to find the effect of the length of the tube on the end effect. The present Paper gives no direct information as to the amount of liquid that is set in motion in the lower reservoir. Moreover, experiments in which gases are used cannot be compared with those on viscous liquids unless the variation in density in the case of the gas is negligible.

XXI. *The Position of Best Focus in the Presence of Spherical Aberration.* By

T. Smith, *F.Inst.P.* (*Optics Department, National Physical Laboratory*).

Received December 9, 1921.

ABSTRACT.

Focal variations of phase in the presence of spherical aberration are calculated directly from the axial intersection points of rays of known inclination, and the graphical method employed shows that the focus for which phase variations have a minimum value may be found and suitably interpreted when the finiteness of the wave-length is disregarded. The diagrams here employed for the representation of spherical aberration have advantages over the more usual diagrams in several important particulars, and it is suggested that the older forms should be abandoned.

The increased interest which has been taken in recent years in the basic principles underlying technical optical work appears to have led to wider acceptance of the view that geometrical reasoning leads in a number of important practical problems to conclusions at variance with those which would be deduced directly from a wave theory of light propagation. That this view is erroneous will be at once evident when it is remembered how immediately the geometrical theory follows from the postulates of the wave theory, provided it is always remembered that diffraction effects exist and are modified in the presence of large amounts of aberration. In cases where the defects are small, the diffraction system differs very little from the ideal pattern usually considered in mathematical discussions, and an optical instrument may attain its theoretical resolving power while very noticeable aberration is present. The spreading of the light patch due to diffraction and to aberration may therefore be regarded as independent effects, and when the latter is sufficiently large not to be obscured by the former the appearance of the image is in accordance with that predicted by geometrical reasoning.

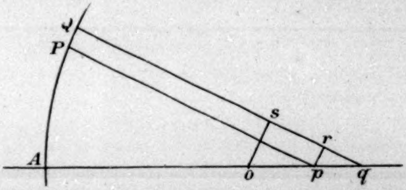


Fig. 1.—Determination of Phase Differences from Ray Positions.

A simple particular case of the supposed inconsistency is afforded by the best position of focus for the image of a point source formed by a symmetrical beam of monochromatic light when the wave front is not strictly spherical. The detailed discussion of this particular problem may, therefore, be of interest.

Suppose that APQ represents a section of the emergent wave front in a plane of symmetry, and that the normals to the surface at P and Q, two near points in this plane, meet the axis at p and q. If 0 is a fixed point on the axis through which the paraxial portion of the wave front passes at time $t=0$, the disturbance from the paraxial part of the wave front will reach p and q at times $\frac{0p}{v}$ and $\frac{0q}{v}$, where v is the velocity of light. The rays from P and Q will, in general, pass through p and q respectively sooner or later than the paraxial disturbance. Let the times be $\frac{0p}{v}+\tau$, $\frac{0q}{v}+\tau'$. Since P and Q are near points on the wave front, the rays Pp and Qq make only a slight angle with one another, and the disturbance from PQ, therefore, reaches p and r simultaneously, where r is the foot of the perpendicular from p to Qq. The lag in time at q compared with p thus depends only on the time taken by the disturbance to travel from r to q, or

$$\frac{0q}{v}+\tau'=\frac{0p}{v}+\tau+\frac{pq\cos\psi}{v}$$

where ψ is the inclination of Qq to the axis. Putting $\tau'-\tau=d\tau$, and denoting the longitudinal spherical aberration by ξ, so that $pq=-d\xi$, this equation may be integrated in the form

$$v\tau=\int_0^\xi(1-\cos\psi)d\xi$$

The integral is to be evaluated from the known variation of the spherical aberration with the inclination of the emergent ray. It is to be noted that τ is simply the difference in the times of arrival at the common point of the corresponding rays of disturbances arising from an outer zone and from the paraxial portion of the wave front, and necessarily vanishes at the lower limit of integration. Assuming simple harmonic trains of waves of wave-length λ, the disturbances which arrive simultaneously will differ in phase by

$$\frac{2\pi}{\lambda}\int_0^\xi(1-\cos\psi)d\xi$$

It is not sufficient to determine phase differences only for the geometrical intersection of ray paths, but it must be possible to determine them for an arbitrary axial point in the neighbourhood of the intersections. Let 0 be such a point. The disturbance from Q will reach 0 and s simultaneously, where s is the foot of the perpendicular from 0 to Qq. The paraxial disturbance reaches 0 at time $t=0$, and the oblique disturbance at time $t=T$. The time of arrival of the oblique disturbance at q is therefore

$$T+\frac{0q}{v}\cos\psi$$

and comparing this with the expression found before

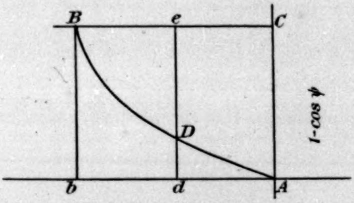
$$vT+0q\cos\psi=0q+\int_0^\xi(1-\cos\psi)\mathrm{d}\xi$$

or

$$vT=0q(1-\cos\psi)+\int_0^\xi(1-\cos\psi)d\xi$$

and from this the corresponding expression for differences of phase relative to any axial point at once follows.

The determination of phase differences from the calculated aberration is most simply effected by plotting the aberration against $1-\cos\psi$. The resulting curve is normally very smooth with slight curvature, and in a large number of cases differs imperceptibly from a circular arc. If AB represents such a curve where A is the paraxial focus and b the marginal focus, the difference of path at the paraxial focus is given in the same units as those used for the measurement of the aberration by the area ABb. At the marginal focus the difference will be represented by the area ABC, and the sign of the phase difference differs in the two cases since the areas lie on opposite sides of AB. At an intermediate axial point such as d where light comes to a focus from the zone ψ_0 of the wavefront where $1-\cos\psi_0=dD$, the difference of phase between this zone and the paraxial rays is given by the area AdD, and that between this zone and the marginal zone by the area BDe. There is evidently a point d between A and b which makes the two areas AdD and BDe equal in magnitude and opposite in sign. For this focussing position the paraxial and marginal zones give disturbances which are in the same phase, and when the total phase differences are small this is, with a smooth aberration curve, almost indistinguishable from the position which results in the greatest axial amplitude of vibration, and in such cases is accepted as the position where the best images are formed.



Longitudinal Aberration.

FIG. 2.—ABERRATION DIAGRAM FOR AN UNDERCORRECTED LENS.

When the aberration is not continuously increasing over the whole aperture the best focus will on this theory be found by an obvious modification, the total area enclosed between the curve and the ordinate on the one side being equal to the corresponding area on the other, as illustrated in Figs. 3 and 4.

Disregarding now differences of phase, the best mean focus which would be chosen in the presence of aberration on, say, a corpuscular theory would be the axial point at the centre of gravity of the ray-axis intersections. The proper weight is given to the zones for the determination of this point by plotting the longitudinal aberration against the cosine of the inclination of the ray to the axis. The curves obtained correspond exactly to those of figures 2 to 4, and the corresponding construction gives the same focussing point. The conclusion to be drawn is, that exactly the same position of focus follows from the geometrical considerations by which designers have been guided in the past as would be selected by taking into account the influence of phase discrepancies on the variation in light intensity as the tentative focus is moved along the axis.

A modified theory leads to curves in which the aberration is plotted against the square of the sine of the inclination. The curves found are similar to those described above with the single exception that they are slightly distorted in the direction of the ordinates. If the same scale is used in the two cases for the paraxial region, so that the ordinates for the new construction are $\frac{1}{2}\sin^2\psi$, the ratio of the ordinates for the angle ψ in the two diagrams is $2 : 1+\cos\psi$. The effect of this distortion on the position of the ordinate which secures equal areas on the two sides is exceedingly small, and in typical cases that have been considered the difference in the best focus position by the two methods is at least as small as one thousandth of the longitudinal aberration present. Measurements of aberrations to such an accuracy or such precise experimental settings as this difference implies are out of the question and indeed are physically without meaning. This close correspondence justifies the conclusion that any expressions for the aberrations which give reasonably accurate positions for marginal rays will be adequate for every other purpose.

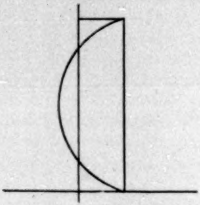


FIG. 3.—ABERRATION DIAGRAM FOR A CORRECTED LENS.

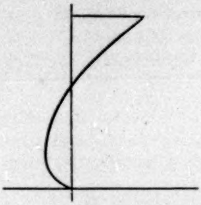


FIG. 4.—ABERRATION DIAGRAM FOR AN OVERCORRECTED LENS.

The construction of diagrams in which the longitudinal aberration is plotted against $1-\cos\psi$ or $\sin^2\psi$ have many advantages over the customary diagram in which the ordinate is proportional to $\sin\psi$ or to $\tan\psi$ in addition to the ease with which the position of best focus may be found. In the usual diagram aberrations of higher order tend to produce a curve in which changes of direction take place very suddenly, and it is difficult to draw a curve with accuracy connecting the calculated or observed points. A typical example is given in Fig. 5. It is difficult to find from such a curve with reasonable accuracy either the zone which sends a ray through the paraxial focus or the maximum amount of aberration present in intervening zones. The portion of the curve which occupies the greater part of the diagram

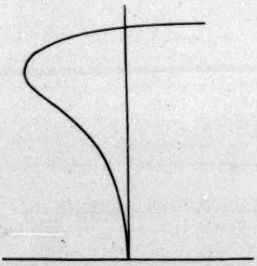


FIG. 5.—USUAL ABERRATION DIAGRAM SHOWING SLIGHT OVERCORRECTION.

corresponding to rays near the axis is not in a form which affords any useful information. With the diagrams here advocated these objections do not hold. The curve is easily drawn with accuracy, the tangent to the curve at the paraxial point A gives the first order aberration coefficient, and the law of decrement of the aberrations of different orders is easily determined. In addition changes of shape of the lenses produce effects on the curve which may be utilised effectively in shortening calculations. Alterations of a very simple kind, for example, may be represented by rotating the curve without appreciable change of shape round the point A. These and other advantages are of sufficient importance to justify the complete abandonment of the older graphical methods of exhibiting aberrations.

The aberration known as coma may be simply discussed on similar lines and with similar diagrams to those here described for spherical aberration.

DISCUSSION.

Mr. J. **GUILD**: The position of best focus is usually defined in geometrical optics as that of the circle of least confusion, but the geometrical definition adopted by the author makes it coincide with the region of maximum density of the rays. This view is interesting in connection with modern theories of vision, such as that of Prof. Joly, of Dublin, according to which vision is due, not to a two-dimensional stimulation of the surface of the retina, but to a photo-electric effect which operates in depth throughout the volume of the visual purple. On this view the Author would be right in identifying the focus with the region of maximum concentration of energy. It is of interest to know what would be the difference in practical effect between the two definitions ; it seems as though it might be appreciable in some cases.

Mr. F. **TWYMAN**: The search for a definition of the position of best focus arises from the desire to reduce to rigorous quantitative terms one's empirical estimate of the effectiveness of a lens. Unfortunately, those who have to use lenses will not be satisfied with a theoretically optimum design, but prefer that one of a number of lenses which seems to give the best results on trial. I am much struck by the last speaker's ingenious suggestion as to the volume-stimulation of the retina, but I doubt if there would be any appreciable difference in practical effect between the various definitions of focal length.

Lt.-Col. J. W. **GIFFORD** (communicated remarks) : From the point of view of trigonometrical trace, I have long felt that the question of the best height from the vertex for the incident ray at the first surface on which successful calculations can be based much needs clearing up. Thanks are due to Mr. T. Smith for his able treatment of the subject from quite a new point of view. In reference to Mr. Smith's Figs. 3 and 4, and diagrams from the geometrical aspect of the same kind in some of the text books, I would like to ask him whether the patterns there shown apply when more than two optical media are used in the construction of the lens combination. That they represent the case for the first surface only, and in a modified form when more than one curved surface and more than one refracting medium exist in the combination, is doubtless the case. But my own experience with apochromatic triples and their trace has led me to think that the curves Mr. Smith has drawn would cross and re-cross the ordinate more than twice—indeed, in that case several times—so that there the problem becomes decidedly more complex.

A simple practical formula for approximately equal areas which I have found useful is

$$H=\sqrt{\frac{(R+t/4)^2}{2}}\Big/\frac{R}{h}$$

where h is the total height to the edge, t the thickness at the axis for that height, and H the height required. R is the radius of the first surface.

This formula gives a somewhat smaller value than Kerber's $H=\frac{h}{2}\sqrt{3}$, which is consonant with the smaller amount of light per unit area transmitted as we approach the edge.

From the **AUTHOR** (communicated) : The remarks of all the contributors illustrate the absence of precise knowledge about the best focus from which we at present suffer. Theories may be constructed in large numbers which give answers for which much can be said, but every one involves assumptions for which the experimental justification is yet lacking. When the necessary precise experimental observations are available it should be possible to say to what extent symmetrical optical instruments with surfaces of specified types can have their apertures and fields of view enlarged.

In reply to Col. Gifford, the author would generally prefer $H=\frac{1}{2}h\sqrt{2}$ to $H=\frac{1}{2}h\sqrt{3}$ to find the maximum spherical aberration when the full aperture is corrected, but no precision in the position is necessary. The methods described in the paper are applicable however many times the aberration curve crosses a given ordinate. In triple apochromatic objectives the author's experience is very limited, and those he has investigated in detail suffered from no aberration which would be likely to be measurable on physical grounds. Col. Gifford's experience with oscillating spherical aberration appears to accord with the author's.

XXII. *The Determination of Poisson's Ratio and of the Absolute Stress-variation of Refractive Index.*

By F. TWYMAN, *F.Inst.P., and* J. W. PERRY.

RECEIVED FEBRUARY 18, 1922.

ABSTRACT.

The Paper describes a ready method of determining the stress-optical coefficients, the Hilger interferometer being employed. Young's modulus of elasticity and Poisson's ratio are determinable simultaneously.

AFTER the discovery by Brewster[*] of the phenomenon of double refraction in strained media, the first to make quantitative determinations of the effect was Neumann,[†] who also developed a theory of the phenomenon. Further experiments were carried out by Kerr,[‡] establishing the laws of the propagation of light in strained media. Absolute measurements were made later by Pockels,[§] whose investigations resulted in the production of a glass which, when subjected to any strain within the elastic limits, remained optically almost isotropic. Filon[||][¶] later investigated by various methods the birefringence and its dispersion in a bent beam, and advantageously proposed the use of two constants in specifying the stress-optical effect in such a beam when bent, called by him the stress optical coefficients, and defined as the "additional retardation per unit thickness per unit tension" for ordinary and extraordinary rays respectively. Filon specifies these constants in terms of "brewsters," a unit proposed[¶] also by him (1 brewster $=1{\cdot}10^{-10}$ cm.^{2}/gm.).

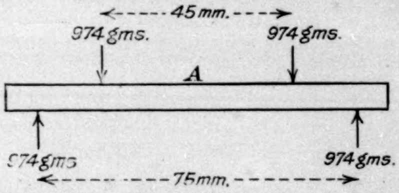


FIG. 1.

In the course of an investigation concerning the optical properties of a certain melting of glass, it became necessary to determine approximately the stress-optical coefficients. It is the purpose of the following to indicate the method employed, which differed from any known to have been published, and is applicable in connection with determinations of other small refractive index variations.[**] No great accuracy is claimed for the measurements here recorded, but the method is susceptible of refinement.

A strip of the glass was prepared, of dimensions $80\times15{\cdot}1\times2{\cdot}48$ mm., of which the two $80\times2{\cdot}48$ mm. faces were accurately plane parallel, and one $80\times15{\cdot}1$ mm.

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[*] Phil. Trans. (1816).

[†] Abh. d. Berliner Akad. (1841). II.

[‡] Phil. Mag., 5, XXVI. (1888).

[§] Ann. d. Phys., 4, 7, 745.

[||] Phil. Trans., A. CCVII. (1907) ; *see also* Proc. Camb. Phil. Soc., XI., 478, XII., 55, 314.

[¶] Proc. Roy. Soc., A., LXXXIII., 572.

[**] As, for instance, the variations of thickness and of mean refractive index from point to point of nearly plane parallel glass plates.

face polished accurately plane. It was supported on two pieces of soft wire (electric fuse wire) 75 mm. apart, so that a load (of 1948 gms.) could be applied by resting a weight symmetrically on two similar pieces of wire 45 mm. apart, as is shown in Fig. 1.

Compressive stress at *A* : 94·66 kg./cm.^{2}

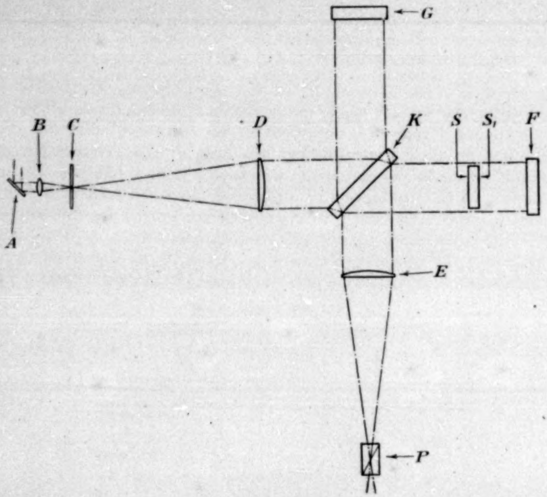

Fig. 2.

Arrangement (*a*).

The whole was placed in a Hilger prism interferometer* (Fig. 2), in the beam usually occupied by the prism, and the interferometer illuminated by the green *Hg* radiation $\lambda 5461 \mathring{A}$. *U*. The interferometer was so adjusted that uniform coloration was produced, with the strip unloaded, over the whole area presented by the strip.

On loading the strip interference bands as shown in Fig. 3 were produced. These are due to the composite effect of alteration in the length of glass path occasioned by the stress and variation of refractive index also occasioned by the stress. Since, moreover, the stress in general occasions double refraction, the appearance is a complex one, but it is readily simplified by admitting to the observer's eye light polarised either in a horizontal or in a vertical plane.

Arrangement (*b*).

Interference bands can also be obtained by covering the interferometer mirrors *F* and *G* and observing by means of the light reflected from the surfaces *s* and *s'* of the strip.

* Phil. Mag., VI., 35, 1.

If now n be the normal refractive index of the glass, $2t$ the length of glass path in the direction traversed by the light, Δn and Δt the variations in n and t due to loading the strip in the manner indicated, and m the number of complete bands observed from A to B (Fig. 3), to the approximation aimed at in the present determination we can write for the case of arrangement (*a*) :

$$\tfrac{1}{2}m_a\lambda=\Delta(n-1)t=(n-1)\Delta t+t\Delta n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

while for the case of arrangement (*b*) :

$$\tfrac{1}{2}m_b\lambda=\Delta nt=n\Delta t+t\Delta n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

from which two equations Δn and Δt are easily determinable.

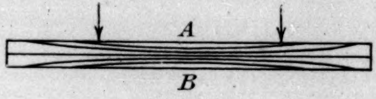

FIG. 3.

Particulars of the glass : Melt 683.* Derby Crown Glass Co.

$n_D=1{\cdot}5759 \qquad \nu=41{\cdot}3$

The following results were obtained :—

Arrangement.	Ordinary ray.			Extraordinary ray.		
	m_0	Δn_0	Δt	m	Δn_e	Δt
(*a*)	8	0·000110	1·65λ	4·3	0·000044	1·60λ
(*b*)	11·3			7·5		
Compressive stress at *A*						94·66 kg./cm.².

Thus, for any known load, Δn for both ordinary and extraordinary rays is determinable immediately from the observations. It will also be noticed that Δt is twice determined. The two values found should, of course, be identical ; that they are not so is due to the fact that m was measured by counting the bands simply as seen by the unaided eye at P, the accuracy so obtained sufficing for the immediate purpose of the experiment. More accurate means of counting the bands will readily suggest themselves to one desiring to adapt the method to more precise determinations.

It would, of course, be necessary then to correct for several sources of error, which in the more refined application of the method would no longer be negligible, arising from the prismatic form of the strip when bent. It may be shown that to

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* We have to thank the courtesy of the Derby Crown Glass Co., Ltd., for letting us have, and permitting us to publish, the theoretical composition of their melt 683 as determined from the batch used. No analysis of the actual glass was made.

Theoretical Formula :

$100\,SiO_2.\ \ 17\,PbO.\ \ 11\,Na_2O.\ \ 5\,K_2O.$

Theoretical Percentage Composition :

SiO_2	54·83
PbO	34·64
Na_2O	6·23
K_2O	4·30

the order of approximation aimed at in the present measurement such sources of error may be ignored (the greatest correction—*i.e.*, that to the value of $m_{ob}\lambda$, necessitated is approximately $0{\cdot}08\lambda$, and is thus smaller than the probable observational error).

The stress-optical coefficients as thus determined for the glass as above specified are as follows :—

$$C_0 = -5{\cdot}81 \qquad C_e = -2{\cdot}32$$

The above measurements enable us, since Δt is determined, to calculate Poisson's ratio if Young's modulus E is known for the glass. Failing a more precise knowledge of E it is possible to determine it on the same apparatus by applying the method originated by Cornu,* elaborated by Straubel,† and quite recently employed by Jessop ‡ in the determination of Poisson's ratio for specimens of plate glass. The procedure is as follows :—

A right-angled prism is so mounted under the loaded strip that light proceeding from the half-silvered mirror K (Fig. 2) is reflected normally from the under-surface of the strip, and passes back to the mirror K to be reflected to the observer's eye at P. The beam which is reflected from the curved under-surface of the strip interferes with the beam reflected from the upper surface of the prism; the curvature of the strip can, therefore, be determined by counting the interference fringes so formed and observed by the eye at P. Because of the large number of fringes with the original load, the total load was reduced to 1·006 kg., thereby rendering possible a more accurate counting. The accuracy in E thus secured is estimated at about ±3 per cent.

The following observations were made, y being measured from the centre of the strip along its major axis :—

y	7·5	10·0 mm.
$m\lambda$	6·75	11·75 λ

The radius of elastic curvature R is then given by

$$R = \frac{y_2^2 - y_1^2}{(m_2 - m_1)\lambda}$$

and Young's modulus E is calculated from

$$E = \frac{12\,RM_2}{td^3}$$

where M is the bending moment, the suffix being to distinguish this from the earlier value. E was thus found to be 6,320 kg./mm.^{2}.

Returning now to the observations firstly made, Poisson's ratio σ is to be derived

from $$\sigma = \frac{E}{12} \cdot \frac{d^2}{M_1}\,\Delta t\,;$$

thus is obtained the value $\sigma = 0{\cdot}196$.

DISCUSSION.

Mr. J. Guild inquired what fraction of a band could be counted in the determination of E.

The Author replied that the bands were counted to the nearest one-tenth.

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* C.R., LXIX., 333 (1869).

† Wied. Ann., LXVIII., 369 (1899).

‡ Phil. Mag., VI., 42 (1921).

XXIII. *An Experimental Comparison of the Viscous Properties of (a) Carbon Dioxide and Nitrous Oxide; (b) Nitrogen and Carbon Monoxide.*

By C. J. SMITH, *B.Sc., A.R.C.S., D.I.C., Research Student, Imperial College of Science and Technology.*

RECEIVED FEBRUARY 24, 1922.

ABSTRACT.

The present Paper is the result of an experimental investigation to determine the viscous properties of (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. Direct comparisons have been made in each case by observing the time required by a mercury pellet to force an equal volume of gas through a capillary tube, at atmospheric and steam temperatures. For the above groups of gases the times of fall for each gas are equal at 15·0°C. and 100·0°C., and hence, independently of any knowledge of the absolute viscosity, it has been proved that the viscous properties are identical over the above temperature range. The absolute viscosity has been obtained by comparison with air, and the mean area of collision deduced by using Chapman's formula.

THE Lewis-Langmuir theory of molecular structure has received much confirmation from the close degree of equality which exists between the physical properties of (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. In his Paper,* among other things, Langmuir showed, with the data then available, that the viscosities of nitrogen and carbon monoxide were approximately equal at 0°C., and proceeds to say: " This evidence alone should be sufficient to prove that the structure of the shell of the carbon monoxide and nitrogen molecules must be nearly the same." Similar considerations apply to the other pair carbon dioxide and nitrous oxide.

The mere fact that the viscosities of any pair of gases are equal at a particular temperature does not of itself support the view that the molecules in question have the same arrangement of electrons in the outer shell; for there are many cases of such equality in which there is no suggestion of molecular similarity: for example, the gases ethylene and cyanogen at 100°C., and ethane and hydrogen below 0·0°C. In order to infer from viscosity data identity of external molecular constitution, it is necessary to be able to show that the viscosities are equal at *all* temperatures, or, in other words, that the viscosity at any particular temperature and the rate of change of viscosity with temperature should be the same for each gas.

The object of the present research was to determine how nearly these conditions, are fulfilled by the two pairs of gases under consideration, *i.e.*, (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. Direct comparisons of the viscosities at both atmospheric and steam temperatures have been made in each case. The results support Langmuir's view to a much closer degree of approximation than can be derived from the figures which he himself quotes. It is found that nitrogen and carbon monoxide have the same viscosities both at 15·0°C. and at 100·0°C., to within two or three parts in a thousand.

APPARATUS AND METHOD OF OBSERVATION.

The apparatus used has already been described elsewhere.† Only a brief description is necessary here. The viscometer is shown in Fig. 1, and it consists of a capillary tube and a piece of ordinary quill tubing which form a complete circuit.

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* I. Langmuir, Journ. Amer. Chem. Soc., p. 904 (1919).

† A. O. Rankine, Proc. Roy. Soc., Vol. LXXXIV. (1910), and A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 601 (1921).

This quill tube contains a pellet of mercury which forces the gas in the apparatus through the capillary tube. The apparatus was specially made for the purpose from tubes carefully chosen for uniformity and circularity of cross-section. The capillary tube had a mean diameter 0·19 mm., which did not vary by more than 1 per cent. in a length of 80 cm. The fall tube was about 0·3 cm. in diameter. Atmospheric temperatures were observed by means of a calibrated mercury thermometer inserted in the jacket, and steam temperatures were obtained from observations of the barometer. The steam was admitted at both ends of the jacket and escaped freely.

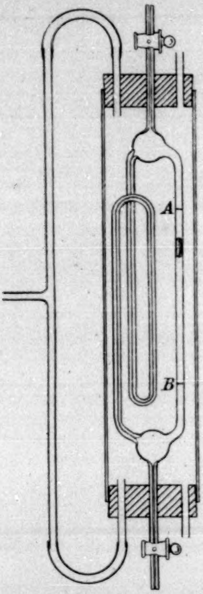

FIG. 1.

In some recent experiments Professor Rankine and the author* found it necessary to modify the original method of observation in one important particular. It had always been fully recognised that the full hydrostatic pressure, due to the weight of the mercury pellet, was never operative on account of the capillary effects due to the difference in curvature of the ends of the mercury pellet. Evidence was then obtained that the capillary effect changed with the gas in contact with the mercury pellet, and with the temperature. It also changed slowly with time. These effects have been confirmed in the present experiments.

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* A. O. Rankine, Proc. Roy. Soc., Vol. LXXXIV. (1910), and A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 601 (1921).

The capillary effect can be eliminated almost, if not entirely, by making double sets of time observations; first with the mercury pellet intact, and then with it broken into two segments, each being approximately half the whole pellet. Experiment has shown that the capillary effect is doubled when the pellet is broken into two segments, and trebled when the pellet is divided into three parts. Let the capillary effect be denoted by x, *i.e.*, a certain fraction of the full hydrostatic pressure which would be operative if the capillary effect were absent; and let t_1 and t_2 be the observed times of fall between specified marks on the viscometer. Then an examination of the formula connecting the viscosity with the pressure difference and the time of fall shows that if the temperature is constant, then

$$(1-x)t_1=(1-2x)t_2$$

or

$$x=\frac{t_2-t_1}{2t_2-t_1}$$

If t is the time of fall which would be obtained if the full hydrostatic pressure were operative, then

$$t=t_1(1-x)$$

Except for a small correction for slip,* which can be applied independently, t is proportional to the viscosity of the gas in the apparatus. Direct comparisons of viscosities thus amounted simply to the computation of the ratio of such corrected times.

Since the capillary effect may change slowly with time, the following procedure was adopted throughout in order to allow for its effective elimination. Eight observations of the time of fall were made with the pellet intact, and eight with the pellet in two segments. The order of observation was such that the second group lay between the first four and the last four of group one. The subjoined table gives a typical case and also serves to indicate the consistency of the observations. In this way any slow change in the capillary effect would certainly be eliminated.

TABLE I.

Temp. (deg.C.)	Time of fall in seconds.			
	Whole pellet.		Two segments.	
	A to B.	B to A.	A to B.	B to A.
15·9	160·3	161·0	...	...
15·9	160·5	161·0	...	...
15·95	...	...	167·1	168·4
15·93	...	...	167·6	168·5
15·9	...	...	167·6	168·2
15·9	...	...	167·8	168·5
15·9	160·5	161·3	...	...
15·93	160·5	161·7	...	...

Therefore $t_1=160{\cdot}85$ secs., $t_2=167{\cdot}96$ secs., giving for the capillary correction

$$x=0{\cdot}0406$$

whence the corrected time of fall is

$$t=154{\cdot}32 \text{ secs. at } 15{\cdot}9^\circ\text{C.}$$

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* A. O. Rankine, Proc. Roy. Soc., A. Vol. LXXXIII., p. 517 (1910).

The apparatus was cleaned with distilled water and absolute alcohol, and finally dried by being exhausted and connected to a bulb surrounded by liquid air. In order to obtain a smooth motion of the mercury pellet in the fall tube, it is necessary that the gases which are introduced into the viscometer shall be quite dry, and special precautions were taken to desiccate all the gases used. It is worthy of note that although the apparatus remained in use for more than five weeks, the motion of the pellet was just as good at the end of the period as at the beginning.

PREPARATION AND PURIFICATION OF THE GASES.

(*a*) *Air.*—The air which was used as a standard, in order to be able to calculate the absolute viscosity of the gases, was dried by being passed over phosphorus pentoxide, and then through a spiral maintained at —80°C. by means of a mixture of solid carbon dioxide and alcohol, before being introduced into the viscometer. This procedure removes the water vapour very effectively.

(*b*) *Carbon Dioxide.*—The sample of carbon dioxide which was used was prepared by the action of dilute sulphuric acid on pure anhydrous sodium carbonate. Possible traces of sulphur dioxide and water were respectively removed by passing the gas through a dilute solution of potassium permanganate, and then through concentrated sulphuric acid. The gas was then solidified in a U-tube surrounded by liquid air, and samples were finally collected over mercury. Before introduction into the viscometer the gas was again solidified by means of liquid air, and any permanent gases—the only likely impurities which may have remained in any quantity—were pumped off by means of a Toepler pump. When the liquid air was removed the carbon dioxide was collected in the pump, and then introduced directly into the evacuated viscometer. All the CO_2 had not evaporated before the sample was introduced into the apparatus ; thus any impurity of higher boiling point would be retained in the condenser.

(*c*) *Nitrous Oxide.*—The specimen of nitrous oxide was obtained from a cylinder of the gas of commercial quality. The gas was allowed to bubble slowly through a solution of caustic potash and another of ferrous sulphate, to remove traces of carbon dioxide and chlorine, and nitric oxide respectively, and then partially dried by being passed through a U-tube surrounded by a freezing mixture, and finally solidified in a U-tube maintained at liquid air temperature. The gas was collected over mercury and introduced into the viscometer with the same precautions as were observed in the case of carbon dioxide.

(*d*) *Nitrogen.*—Recently Waran * has described a method of making pure nitrogen in small quantities, which when examined spectroscopically is free from hydrogen. Attempts to use the same apparatus to obtain larger quantities of nitrogen resulted in the production of ammonia, as an impurity, in such great quantity that the process of purification was very laborious. The following modification of Waran's apparatus was therefore used. The portion of the apparatus *A B C D* was completely filled with dilute ammonia, and dilute bromine solution was placed in the thistle funnel above *B*. The delivery tube *E* dipped under an inverted tube *F*, and both were filled with recently boiled distilled water. The inverted tube *F* was closed by means of a tap K, and was connected to an exhausted glass spiral immersed in liquid air. When *K* was opened the nitrogen passed into the spiral, and most of the water

* Waran, Phil. Mag., Aug. (1921).

and possible traces of ammonia which might still be present would be retained in the spiral tube. The sample of gas was again purified by being passed slowly through a spiral tube immersed in liquid air, before being introduced into the viscometer.

(*e*) *Carbon Monoxide.*—This gas was also generated in the above apparatus, concentrated sulphuric acid replacing the ammonia and formic acid the bromine solution. It was observed that sulphuric acid attacked the tap grease liberating carbon, so that traces of sulphur dioxide and carbon dioxide may at first be present in the carbon monoxide. These gases would certainly be removed by means of the liquid air. The gas was collected and purified as in the case of the nitrogen.

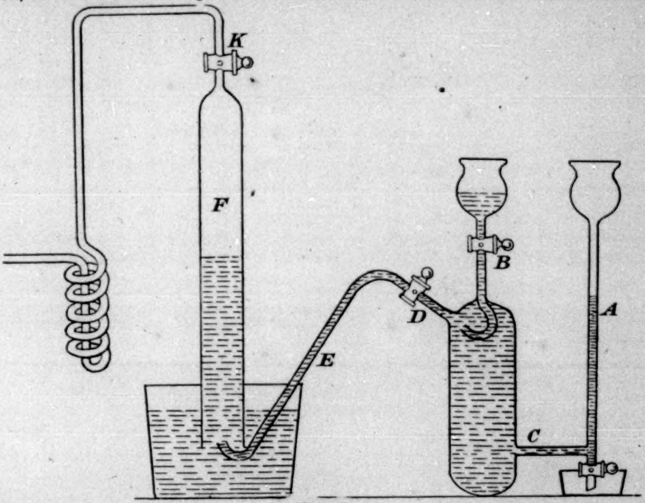

FIG. 2.

RESULTS.

In Tables II.—V. the letters in column 1 indicate the order in which the observations were made.

TABLE II.—*Carbon Dioxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C
(*a*) 16·2	133·99	139·75	0·0396	128·68	128·20	...
(*d*) 18·2	134·56	140·16	0·0385	129·38	128·10	...
				Mean	128·15	...
(*b*) 100·2	168·58	172·99	0·0249	164·38	...	164·30
(*c*) 100·0	167·90	171·76	0·0219	164·22	...	164·20
				Mean	...	164·25

TABLE III.—*Nitrous Oxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 16·3	132·83	137·23	0·0311	128·70	128·18	...
(*d*) 17·9	132·74	136·61	0·0275	129·09	127·93	...
				Mean	128·06	...
(*b*) 99·6	167·29	171·14	0·0220	163·61	...	163·77
(*c*) 99·6	167·06	170·56	0·0201	163·70	...	163·86
				Mean	...	163·82

TABLE IV.—*Nitrogen.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 15·9	160·85	167·96	0·0406	154·32	153·96	...
(*c*) 15·4	160·41	167·41	0·0401	153·97	153·79	...
(*d*) 15·5	160·74	167·96	0·0412	154·12	153·92	...
(*e*) 16·1	160·73	167·75	0·0401	154·28	153·84	...
				Mean	153·88	...
(*b*) 100·0	193·68	200·21	0·0316	187·56	...	187·56
(*f*) 99·9	193·45	199·75	0·0306	187·53	...	187·59
				Mean	...	187·58

TABLE V.—*Carbon Monoxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 18·2	161·90	169·28	0·0418	155·13	153·85	...
(*b*) 18·5	162·28	167·73	0·0420	155·46	154·06	...
(*d*) 17·4	161·39	168·59	0·0407	154·82	153·86	...
				Mean	153·92	...
(*c*) 99·7	193·39	199·49	0·0297	187·65	...	187·77
(*e*) 99·6	193·14	199·23	0·0296	187·42	...	187·58
				Mean	...	187·68

Throughout the foregoing tables the figures in the last two columns have been obtained by applying the small corrections necessary to obtain values at the two standard temperatures indicated, for purposes of comparison. The basis of the corrections is the observed temperature rate of change of the time of fall.

EXAMINATION OF RESULTS.

Taking first of all the gases CO_2 and N_2O, the mean values of the corrected times of fall are shown in Table VI.

TABLE VI.—*Carbon Dioxide and Nitrous Oxide.*

Temp. (deg. C.)	Gas.	Time of fall (seconds).	Ratio.
15·0	CO_2 N_2O	128·15 128·06	1·001
100·0	CO_2 N_2O	164·25 163·82	1·003

Here the time of fall is strictly proportional to the viscosity of the gas.* A careful examination of the detailed results shows that the experimental error is not likely to be more than 0·2 per cent. At 15·0°C. the ratio of the viscosities is closer to unity than this; and even at 100·0°C. the excess of 0·003 is not sufficient to make it safe to claim that a difference of viscosity has been detected. In any case the remarkable result emerges that both at 15·0°C. and 100·0°C. the viscosities of carbon dioxide and nitrous oxide are equal to within 0·3 per cent.

In Table VII. the data for carbon monoxide and nitrogen are given.

TABLE VII.—*Carbon Monoxide and Nitrogen.*

Temp. (deg. C.)	Gas.	Time of fall (seconds).	Ratio.
15·0	CO	153·92	1·000
	N_2	153·88	
100·0	CO	187·68	1·001
	N_2	187·58	

In this case both ratios are nearer to unity than 0·2 per cent.; so that we can say that the measurements indicate an even more precise equality of viscous behaviour than in the previous case.

Thus, these experiments prove, as closely as the method allows, that the two pairs of gases in question have, respectively, identical viscous properties at both atmospheric and steam temperatures.

In these circumstances the proper course is to disregard the small differences in the figures and to take the mean value of the times of fall as being a measure of the common viscosity. This has been done for the purpose of calculating Sutherland's constant, and the absolute viscosity by comparison with air.

SUTHERLAND'S CONSTANTS.

These have been calculated directly from the ratios of the viscosities at the two temperatures, on the assumption that Sutherland's law is valid.

TABLE VIII.

Gas.	$\frac{t_{100}}{t_{15}}=\frac{\eta_{100}}{\eta_{15}}$	Sutherland's Constant. (C).
CO_2 and N_2O	1·280	274
CO and N_2	1·219	118

Here it should be noted that the accuracy of Sutherland's constant is not great, being of the order of 5 per cent. For so small a temperature range the constant is very sensitive to variations of the viscosity ratio, in which there is a possible error of 0·3 per cent.

ABSOLUTE VALUES OF THE VISCOSITIES.

These have been obtained simply by comparing the times of fall in the gases with those of the same mercury pellet in air. For this purpose many observations were made using air in the viscometer. Small corrections have to be applied on account of the different slipping coefficients, in the manner already indicated.*

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* In general this would not be true. But for gases, having as these have, the same density and the same approximate viscosity, their slipping coefficients are equal, and therefore do not affect the above proportions.

The results are as follows :—

For CO_2 and N_2O,

$$\frac{\eta_{15}}{\eta_{15(\text{air})}}=0{\cdot}8021$$

which when corrected for slip becomes

$$0{\cdot}8011$$

For CO and N_2,

$$\frac{\eta_{15}}{\eta_{15(\text{air})}}=0{\cdot}9654$$

Here correction for slip is negligible.

Taking $1{\cdot}799\times10^{-4}$ C.G.S. units as the absolute viscosity of air at 15·0°C., the following table shows the absolute viscosities for the gases under investigation. The values at 0·0°C. have been obtained by extrapolation.

TABLE IX.—*Summary of Results.*

Gas.	Viscosity in C.G.S. units$\times10^{-4}$ at			C.
	15·0°C.	100·0°C.	0·0°C.	
CO_2 and N_2O	1·441	1·845	1·366	274
CO and N_2	1·737	2·118	1·665	118

DISCUSSION OF RESULTS.

The value of the viscosity at 0·0°C. for CO_2 and N_2O obtained in this research is $1{\cdot}366\times10^{-4}$ C.G.S. units. Probably the most reliable previous measurements for these gases are those due to Vogel.† His results are : for CO_2, $\eta_0=1{\cdot}380\times10^{-4}$ C.G.S. units, and for N_2O, $\eta_0=1{\cdot}362\times10^{-4}$ C.G.S. units.

The absolute viscosity of nitrogen has recently been determined by Prof. Kia-Lok Yen‡ in Millikan's laboratory, and he found $\eta_{23}=1{\cdot}765\times10^{-4}$ C.G.S. units, whereas the present result obtained by interpolation is $\eta_{23}=1{\cdot}773\times10^{-4}$ C.G.S. units. Vogel gives $\eta_0=1{\cdot}678\times10^{-4}$ C.G.S. units, and for carbon monoxide $\eta_0=1{\cdot}672\times10^{-4}$ C.G.S. units. The common value shown in the above table is $1{\cdot}665\times10^{-4}$ C.G.S. units.

Previous work on the variation of viscosity with temperature does not appear to have been sufficiently systematic to render of value comparisons with the present results.

CALCULATION OF MOLECULAR DIMENSIONS.

We can now proceed to calculate the mean collision area ($\bar{A}$) for the molecules, CO_2 and N_2O, and CO and N_2, by applying Chapman's formula in the manner previously indicated,§ and we find

$$\bar{A}=0{\cdot}834\times10^{-15}\text{ sq. cm. for } CO_2 \text{ and } N_2O,$$

and

$$\bar{A}=0{\cdot}767\times10^{-15}\text{ sq. cm. for CO and } N_2.$$

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* A. O. Rankine, Proc. Roy. Soc., A. Vol. LXXXIII., p. 517 (1910).

† H. Vogel, Berlin Diss. (1914).

‡ Kia-Lok Yen, Phil. Mag., Vol. XXXVIII., p. 593 (1919).

§ A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 612 (1921).

CONCLUSION.

The fact that CO_2 and N_2O (or CO and N_2) have molecular dimensions which are identical within the accuracy of measurement is consistent with Langmuir's view, to which reference has been made ; namely, that the outer electron arrangements of these molecules are the same. The present experiments indicate that this is the case with considerably greater precision than could have been deduced from previous data. It should be remembered, however, that independently of any particular theory of molecular structure, it is unlikely that CO_2 or N_2O (or CO and N_2) would differ much in those physical properties—viscous behaviour being one—which depend on the total mass of the molecule and the number of electrons external to the nuclei. For CO_2 and N_2O have equal molecular weights, viz., 44, and equal numbers of extra-nuclear electrons, viz., 16 ; and CO and N_2 both have a molecular weight of 28 and extra-nuclear electrons 10 in number. The equalities of apparent molecular size which have been indicated do not of themselves prove any special electronic arrangement ; but merely suggest that, whatever the arrangements are, they are nearly identical.

To detect appreciable differences we should almost certainly have to examine properties dependent on the distribution of atomic nuclei in the molecules. For example we may expect differences in the molecular moments of inertia. The moment of inertia is almost entirely attributable to the masses of the nuclei of the constituent atoms, the electrons making an insignificant contribution to the mass. Even if the electrons in CO_2 and N_2O (or CO and N_2) are distributed in an identical manner, the molecular moments of inertia will be unequal on account of the differing masses of the atoms in the molecule. It seems probable that the spectroscopic method, applied by Imes* and others to the gaseous hydrogen halides, would lead to the discovery of such differences of rotational inertia. This would involve the measurement of the far infra-red absorption spectra for the gases in question, and the treatment of the results in terms of the quantum theory.

In conclusion, I wish to thank Professor Rankine for his unfailing help and advice while these experiments have been made, and also the Government Grant Committee of the Royal Society for a grant which enabled the research to be undertaken.

DISCUSSION.

Prof. A. O. **RANKINE**: The author, who has worked with praiseworthy perseverance, has succeeded in establishing beyond dispute a close identity in viscous properties between the two pairs of gases investigated. The *ratios* of the viscosities have been measured with an extremely high accuracy, since they are the ratios of times which are practically equal. For the *absolute* viscosities, however, less accuracy must be claimed, as their measurement depends on the calibration of a stop-watch. It is remarkable that gases which resemble one another so closely in their physical properties should differ so widely in other—*e.g.*, their physiological properties. It seems clear that chemical and physiological properties are determined by the distribution of the nuclei, and are independent of the outer electron system. A further curious fact has been brought to light—viz., that while the surface-tension effects of mercury are the same when in contact with various gases which differ widely in other respects, the behaviour of a mercury pellet in presence of CO_2 is very different from that in presence of N_2O. Dr. Percy Phillips, in an unpublished investigation, attempted to obtain the variation of the viscosity of N_2O in the neighbourhood of the critical state, but failed because the angle of contact of the mercury with the glass became 180 deg., so that the gas slipped past the pellet, whereas no such difficulty had arisen in the case of CO_2 near the critical state.

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* Imes, Astrophysic. Journ., Vol. L., p. 251 (1919).

Dr. D. OWEN congratulated the author on his work, the interest of which arose from the connection which the viscosity of a gas bore with the size and shape of the molecule. The author remarks that all theories of atomic structure which identify the electron system of the molecule of N_2 with that of CO can equally claim the support of his results. But since the Lewis-Langmuir theory does lead to a close equality in structure in the case of the gases in question, it seemed fair to place the agreement found in the present Paper to the credit of that theory. In regarding theories of the atom and molecule one wondered whether there were certain properties only explicable on a statical theory, others requiring a dynamical theory; one was reminded of the apparent antinomy in the province of radiation between the undulatory and the quantum theories. The accuracy obtained by the device of splitting the mercury pellet—viz., ·2 per cent.—is remarkably good. Even in very carefully cleaned tubes it appears that the correction in the case of a single pellet may amount to 4 per cent., and it is subject of congratulation that by the device of splitting the pellet this has been reduced to $\frac{1}{5}$ per cent. He would like, in conclusion, to ask the author whether he could give a simple physical interpretation of Sutherland's Constant—which has the dimensions of a temperature, and which appears so constantly in relation to subjects like that of the present Paper.

Dr. F. Ll. HOPWOOD remarked that the suggestion hardly commended itself that physiological properties depended on the nuclei, but rather on the *configuration* of the constituent atoms or *shape* of the molecule (as indicated by the experiments of Prof. Hamburger). The author's results gave a measure of the mean collision *area* of the molecule, and did not necessarily give any information concerning its *configuration*.

Mr. R. S. WHIPPLE suggested that a photographic method might be of advantage in measuring the time of fall of the pellets.

Dr. H. P. WARAN: I would suggest that the difficulty which the author experienced in obtaining nitrogen free from ammonia was due to his using too strong a solution of the latter. My original apparatus can be modified by providing a side tube near the tube *C*, Fig. 2, and maintaining a steady flow of the reacting liquids into and out of the apparatus. Very dilute ammonia can then be used, and the trace of this substance in the resulting nitrogen will be so small that it can be absorbed along with the water vapour by the acid phosphorus pentoxide in the bulb between *K* and *D*, the parts *E*, *F* being dispensed with. Nitrogen thus obtained is spectroscopically pure, and any quantity can be generated.

Dr. J. S. G. THOMAS: The values of the viscosities of gases are of such importance to gas technologists that results over a wider range of temperature would be extremely valuable. The work of Stanton and Pannell on flow of gases, and the formula deduced from their results by Lees, constitute a very valuable contribution from pure science to technology, and the value of this contribution would be still greater were values of the viscosities of gases, and more especially of mixtures of gases, available at temperatures up to, say, 600°C. Little seems to have been done in the matter of determining the relation of the viscosity of a gaseous mixture to the viscosities of its constituents. The best that can be done for purposes of technology is to calculate the viscosity of the mixture from its composition, the viscosities of the several constituents at the appropriate temperature being calculated from the values available, by the application of Sutherland's formula. The procedure is a very doubtful one, and it is not surprising to find that the pressure drop actually found in a pipe system through which, say, coal gas at 300°C. is flowing, is very different from that determined by calculation. The accurate determination of viscosities of pure gases and mixtures at higher temperatures would be of value not only from the point of view of the determination of atomic structure, but also from the point of view of the practical application of the results.

The AUTHOR, replying to Dr. Owen: The results which have been obtained for CO_2 and N_2O (or CO and N_2) show that the electronic configurations of these molecules, as nearly as the precision of the method enables us to say, give rise to *mean* collision areas which are identical. Langmuir places the extra-nuclear electrons in the same positions for each pair of these gases, and hence the results support his theory, but they also support any theory of molecular structure which places these electrons in identical positions. He also says, however, that the molecules of CO_2 and N_2O would have the same shape as three contiguous neon atoms in a line. Professor Rankine has shown that this is the case. It is in this latter respect that the results of the present research lend definite support to the Lewis-Langmuir theory.

In regard to Sutherland's constant C, which appears in the formula

$$\eta=k\,\frac{T^{\frac{1}{2}}}{1+C/T}$$

(k=const. T=absol. temp.)

C, as it stands in the formula, is clearly a temperature, and it is proportional to the work done in separating two molecules originally in contact to an infinite distance apart.

It may be of interest to quote a remark by Dr. Oxley which will be found in Nature, July 21, 1921, p. 652 : "On the cubical atom theory as developed by Lewis and Langmuir it appears that the molecules of CO_2 and N_2O have almost identical electron configurations. A. O. Rankine has shown from viscosity data that each electron system is equivalent to that of three adjoining neon atoms in line. The writer is not aware that attention has been directed to the fact that the specific susceptibilities of gaseous CO_2 and N_2O are $-0{\cdot}423\times10^{-6}$ and $-0{\cdot}429\times10^{-6}$ (T. Soné, Science Reps. Tohoku, Vol. VIII., p. 162, 1919, and Proc. Phys. and Math. Soc., Japan, Vol II., p. 84, 1920), and their molecular susceptibilities $-18{\cdot}6\times10^{-6}$ and $-18{\cdot}8\times10^{-6}$ respectively.

"The electron systems of the two molecules are apparently identical, but the net positive charges on the nuclei are 8—6—8 for CO_2 and 7—8—7 for N_2O. If, therefore, atomic nuclei possess rotations and are a source of magnetic moment, it must be assumed that the re-distribution of the positive charges in the manner indicated involves no change of angular momentum."

DEMONSTRATION of the Optical Sonometer. *By* F. TWYMAN, *F.Inst.P.* (*A. Hilger, Ltd.*).

THE apparatus is designed to record the pressure variation caused by sound waves. The sound wave is received by the horn A attached to the diaphragm box B (*see* Fig. 1). In the box is a diaphragm C, consisting of a celluloid film a fraction of a wave-length in thickness, a small portion of the inner face of which is platinized, silvered or gilt.* Light is directed from the light source D by a condenser E through the slit F, and brought to a focus on the diaphragm C. Lenses G and H form an image of the slit on the photographic paper (or film) on the drum J. The lens H being cylindrical, with its axis parallel to the drum, the beam of light is brought to an intense point image, so that when the drum rotates a record of the deflection of the diaphragm is made. The apparatus is arranged to take a 30 or 100 candle-power Pointolite lamp.

Liberal adjustment is provided, so that the spot of light may be focussed on the the drum at any distance from 4 to 20 inches, according to the amplitude of vibration to be dealt with.

The recording drum is arranged to take celluloid or paper film 1⅜ inches wide. For general use, the paper film will be found sufficiently sensitive, but in cases when the amplitude is large and waves of high frequency, standard cinematograph film will be found advantageous on account of its superior speed and orthochromatic quality.† On account of its orthochromatic quality, cinema film is more suitable for use with the gilt diaphragm.

The drum is enclosed in a light tight box, and provided with an automatic shutter, arranged to expose from one-sixth to one complete revolution of the drum, as desired; thus, from 1 to 6 records can be made on one film according to the length it is desired to expose. The drum takes 37 inches of film.

The drum is driven by a spring motor provided with a speed indicator. The speeds thus shown are, however, only to be taken as a rough guide, and when accurate timing is required a time marker should be used.

Time Marker.

This consists of an electrically-driven tuning-fork (placed at some distance from the apparatus in order to guard against interference with the sound to be recorded), such that a small mirror on the fork reflects light taken from the light source of the recording apparatus, back to the drum, where it is focussed near the edge of the film; both sets of variations are photographed together, and thus an accurate time reference is secured.

The apparatus can also be supplied with the axis of the diaphragm in a horizontal plane.

Examples of Records.

Three records showing the sound waves of a mallet, human voice and whistling are reproduced (Figs. 2, 3, 4).

One must not, of course, accept these curves as representing the time variation of air pressure without investigation.

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* The silver gives better reflection while untarnished, but the gilt is more durable.

† Kodak Cine ortho film is about 300 H & D. Paper film about 75 H & D.

The free period of vibration of the membranes of the size here shown in use appears to be approximately 200 per second. On singing up the scale one finds, either with or without the use of the horn a big increase of response at about that

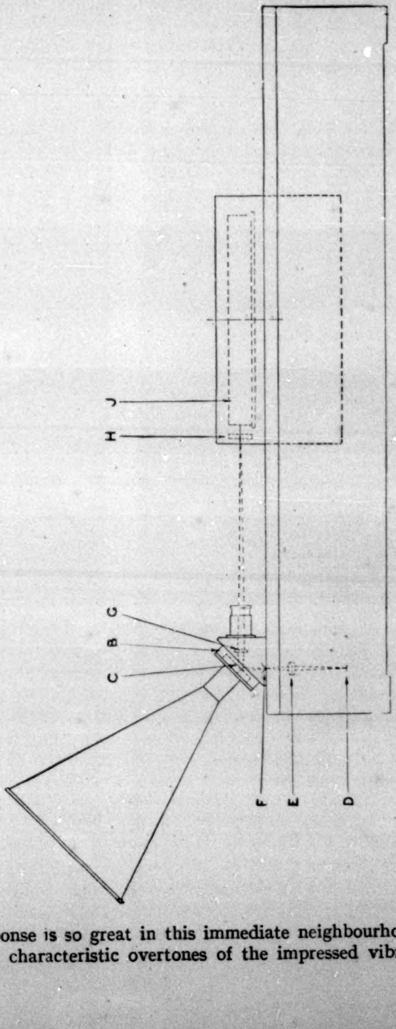

FIG. 1.—OPTICAL SONOMETER.

pitch; the response is so great in this immediate neighbourhood as to obliterate by contrast the characteristic overtones of the impressed vibration. But if one keeps away from the immediate neighbourhood of this fundamental note, one feels reasonably sure that a good qualitative record is given of the pressure variations, and one arrives at conviction on this point from the records of a number of different sounds.

If it should be desired to know variations of pressure quantitatively, the effect of the constants of the film and their reactions must be calculated. It may be of interest to quote ways in which the constants of a film may be in practice determined.

1. *Mass of the Film.*

If on an interferometer the curvature of the film be determined when it is horizontal by observation from above and from below, and if, in addition, an observation be made with the film vertical, the mass of the film may be determined as a function of its diameter and tension.

Fig. 2.—Leather Covered Mallet on Wood. Frequency about 250 per sec.

Fig. 3.—Singing. Frequency about 200 per sec.

Fig. 4.—Whistling. Frequency about 1,300 per sec.

2. *Tension of the Film.*

This can be determined in two ways: By application of air pressure, or by observing the additional deflection obtained with a horizontal film on placing a small ring of known weight upon it in a concentric position, the observations being made, as in the former case, on an interferometer.

One would hope that analysis would reveal that variation of the tension with deflection of the membrane in the region of operative pressures could be neglected.

3. *Energy Dissipation of the Film in Vibrating.*

The information obtained from (1) and (2) together, with the physical properties of the film, are sufficient to enable the energy dissipation to be calculated.

This must be known if the pressure variation in the region of the sympathetic vibrations are required, since it is the dissipation of energy which determines the amplitude.

XXIV. *A New Apparatus for the Measurement of the Polarisation-Capacities of Platinum Plates in Sulphuric Acid. By* A. GRIFFITHS, *D.Sc., and* W. T. HEYS, *B.Sc., Birkbeck College, London.*

RECEIVED FEBRUARY 23, 1922.

ABSTRACT.

A new and simple apparatus for the measurement of the capacities of electrolytic cells has been developed, which allows corrections to be made for the leakage and self-depolarisation of the cell. It gives results consistent to one or two per cent., and of the same order of magnitude as those obtained by other observers.

(1). INTRODUCTION.

THE authors are engaged in a study* of the variation of the polarisation-capacity of an electrolytic cell, consisting of platinum plates in sulphuric acid, when subjected to an intermittent current, and they have developed an apparatus containing some original features which are described below. In order to correct for the rapid leakage or self-depolarisation which apparently constitutes nearly all the difficulty in measurements of this kind, they employ a falling-plate method for making connections of short, variable, and calculable periods.

(2). METHOD.

In Fig. 1 E denotes a 2, 4, or 6-volt battery, x the electrolytic cell, C the charging condenser (1 microfarad capacity), K the auxiliary condenser (4 microfarads capacity) and G the ballistic galvanometer.

The dots numbered 1 to 6 represent brushes which are connected to the component parts of the apparatus, as suggested in Fig. 1. Fixed on a heavy insulating carrier are two brass plates P and Q (Fig. 2), whose shapes and relative positions are so chosen that as the plates fall between vertical guides past the brushes. The following momentary contacts are made :—

The battery E is connected to the condenser C, charging it. The condenser C is connected in parallel with the electrolytic cell x and the auxiliary condenser K, and thus shares its charge with them. The condenser K is discharged through the galvanometer G.

Immediately after the breaking of the contacts between C, x and K, both C and K have acquired voltages equal to the polarisation back E.M.F. of the electrolytic cell x. As the condenser C is affected by the residual charge, not it but the condenser K is discharged through the galvanometer G to determine the value of the back E.M.F. of the electrolytic cell.

Regarding the cell as a pure capacity of 20 microfarad in series with an electrolytic resistance of 1^{ω}, and taking the resistances of the connecting wires and brushes in series with the condensers C and K as $1/100^{\omega}$ each, it can be shown that the cell will receive all of its final charge except one-tenth per cent. in less than 10^{-5} secs. The cell x is, of course, connected to the condensers C and K for a much longer time than this, so that the effect of the self-depolarisation of the cell may not be inappreciable. Examination of Fig. 2 will show that the brush 4 attached to the electrolytic cell x may by a lateral displacement cross the plate P by any of the paths α, β or γ, and thus that the time during which the cell is in contact with the condensers C

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* Phil. Mag., March, 1895, p. 232 ; Proc. Phys. Soc., Vol. XXIX., Part IV., June 15th (1917).

and *K* may easily be varied by intervals calculable in terms of the (constant) height of fall of the carrier and the length of the path of the brush. By plotting the values of the back E.M.F. of the cell against the time during which it has been free to depolarise, we may extrapolate and thus obtain the initial value of the back E.M.F. of the cell.

(3). RESULTS.

(i.) *Effect of Leakage and Calculation of Results.*—The back E.M.F. of the electrolytic cell was measured at intervals of 0·002, 0·004, and 0·006 seconds after the connection of the condenser *C* with the cell *x* and the condenser *K*. Some typical results with two bright platinum electrodes 1 cm. × 1·5 cm. immersed in dilute sulphuric acid (1 part conc. acid to 12 parts distilled water) are given below :—

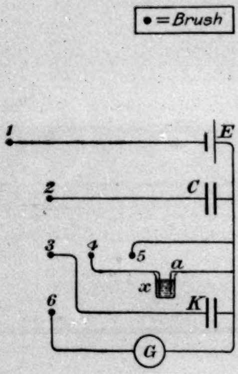

FIG. 1.

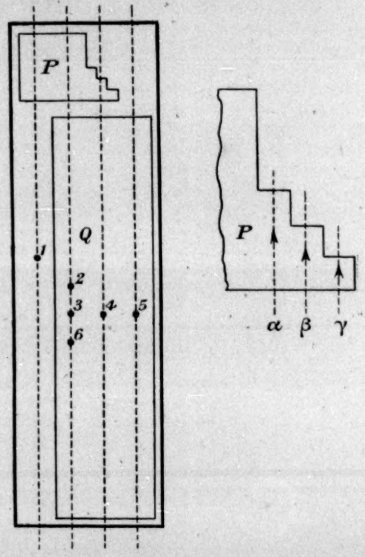

FIG. 2.

Time of self-depolarisation in seconds (approx.).	Back electromotive-force in volts.	
	October 14.	November 17.
0·002	0·1073	0·1095
0·004	0·1057	0·1063
0·006	0·1036	0·1047

Extrapolation by means of the graph (Fig. 3) gives the initial back E.M.F.'s as 0·1095 and 0·112 volts, so that the capacities of the cell on the two dates were 13·3 and 12·9 microfarad respectively.

(ii.) ***Variation of the Capacity of Electrolytic Cells with Time and with Different Specimens of Electrodes.***—In every case the electrodes were rectangular—1 cm. by 1·5 cm.

Date.	Capacity of cell in microfarads.
August 15th, 1921	46·3
September 29th	17·2
October 14th	13·3
November 17th	12·9

Electrodes.	Capacity in microfarads.
A and *B*	12·9
C and *D*	8·7
P and *Q*	5·0

(iii.) *Variation of the Back E.M.F. of an Electrolytic Cell with the Charge to which it is Subjected.*—The back E.M.F. of an electrolytic cell produced by the passage through the cell of a definite quantity of electricity is not proportional to the charge passed, but increases more slowly than the latter. This indicates an increase in the capacity of the cell.

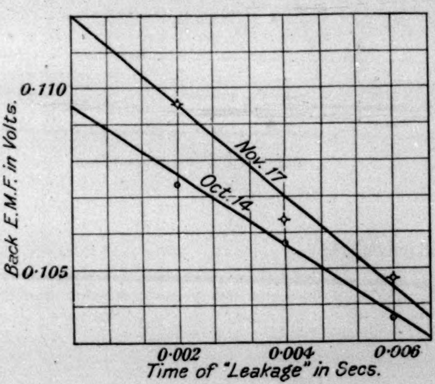

FIG. 3.

Charge passed through the cell in micro-coulombs.	Back electromotive-force of the cell in volts.
1·56	0·091
3·17	0·167
6·74	0·232
12·34	0·332
21·98	0·404
29·82	0·437

It may be added that the back E.M.F. developed depends to a slight extent upon the direction in which the cell is charged.

(iv.) *Measurement of the Capacity of the Cell for Small Charges.*—On repeating any measurement slight discrepancies are often found, seldom amounting to 2 per cent. It was found that if the battery of accumulators be removed from the circuit and the plate dropped as usual, a small deflection was still obtained. The following is a series of typical readings (in cms.): +0·25, +0·65, +0·25, —0·25, +0·25, —0·3, —0·15, +0·4, +0·35, +0·35. To make some allowance for the results of these "dummy" experiments, a dummy experiment was performed before and after each ordinary experiment, and the mean of the deflections in the dummy experiments subtracted from the deflection obtained in the ordinary experiment. As an example the following series of readings is appended :—

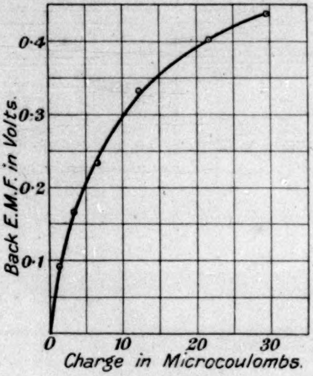

FIG. 4.

Order of experiment.	Deflection in cm.	Mean of deflections in 1, 3 and 5.	Corrected deflections.
1. (Dummy)	+0·15	...	...
2.	+7·0	...	+6·83
3. (Dummy)	+0·2	+0·17	...
4.	+7·0	...	+6·83
5. (Dummy)	+0·15	...	...

The following table gives the results of a series of experiments in which the above correction was applied :—

Order of experiment.	Anode of cell.	Voltage of charging condenser.	Galvanometer deflection in cm. Time of leakage in secs.			Initial value of back E.M.F. in volts.	Charges passed through cell in micro-coulombs.
			0·002	0·004	0·006		
1	C	2	6·83	6·78	6·56	0·140	0·960
9	C	2	6·95	6·85	6·82	0·140	0·960
3	C	4	12·80*	12·95	12·88	...	...
7	C	4	12·88	12·81	12·80	0·259	1·977
5	C	6	18·27	18·19	18·16	0·367	3·023
2	D	2	7·09	6·98	6·90	0·144	0·950
10	D	2	7·02	7·02	6·93	0·142	0·954
4	D	4	13·84	13·78	13·73	0·278	1·966
8	D	4	13·75	13·69	13·65	0·276	1·930
6	D	6	19·98	19·93	19·90	0·403	2·926

Capacity of charging condenser $C=\frac{2}{3}$ microfarad.

Capacity of auxiliary condenser $K=2$ microfarads.

Electrodes C and D of bright platinum, 1×1 sq. cm., in dilute sulphuric acid.

The graph (Fig. 5) shows that there is an approximately linear relationship between the charge passed and the back E.M.F. developed. The capacity of the cell for small charges=6·85 microfarads, *i.e.*, 0·068 microfarads per sq. mm. for each electrode.

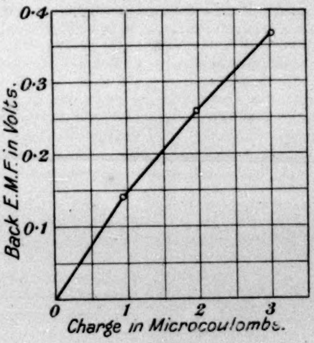

FIG. 5.

In conclusion the authors desire to express their indebtedness to Messrs. H. G. Bell and G. Temple for help and suggestions.

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* It will be noted that out of all the results only one has to be rejected.

DISCUSSION.

Dr. D. OWEN: I should like to ask the authors whether they have worked with platinum-black electrodes, and if so with what results. The authors, like others who have worked at this subject, use the term "Polarisation Capacity." The value of this, for plates of area of 1 cm.^{2}, is calculated to be of the order of 10 microfarads. The cell is thus regarded as electrically equivalent to a resistance (of the order of 1 ohm in the present experiments) in series with a leaky condenser. Judging from the curves of Fig. 3, the resistance of the leak is several thousands of ohms. Now, if such a system be subjected to an alternating voltage it should show marked signs of the presence of this electrostatic capacity. It should have a considerable reactance, far greater than the ohmic resistance of the acid of the cell, which should reduce the value of the current and at the same time cause this to lead the voltage considerably in regard to phase. Accurate measurement of electrolytic resistance by alternating currents should thus be rendered difficult or impossible—which is quite contrary to what is known to be the case. It must be inferred that the interpretation of the phenomena of electrolytic polarisation at electrodes in terms of an assumed electrostatic capacity is not in general justifiable; though doubtless with electrodes of certain metals, such as aluminium, in appropriate electrolytes, such capacity may be present.

Dr. A. RUSSELL: The Society is much indebted to the authors for bringing this subject before it, as the conception of "polarisation capacity" is not a generally familiar one. Experience on A. C. circuits with plates immersed in soda, for instance, shows that such plates act in some ways like a condenser of very variable capacity.

Dr. GRIFFITHS: I am not sure what Dr. Owen means by "leakage-resistance." I think that changes take place on the surface of the platinum electrode, and also in the liquid in the neighbourhood of the electrode. The surface tends to come back to its original condition; and through diffusion the liquid tends to revert to its original uniformity. Until more is known of the subject it does not seem unwise to follow the example of others in using the term "polarisation-capacity." As a matter of fact, appreciable phase differences have been obtained by some workers. The term "leakage-resistance" does not occur in the Paper, and I have avoided it in presenting the latter. I have tried to determine the capacity of platinum-black, but have not succeeded with my present apparatus. The rate of depolarisation is so great that extrapolation is not permissible with the experimental results obtained. Success might be obtained if the time of connection were reduced considerably, or, as the rate of depolarisation will diminish with diminished electromotive forces, if feebler charges were given to the electrolytic cell, and if my present galvanometer were replaced by one much more sensitive. Success might also be attained by using an auxiliary condenser of great capacity and with smaller charging E.M.F.s.

DEMONSTRATION of Some Electrical Properties of Neon-filled Lamps. By *Messrs.* S. O. PEARSON, *B.Sc., and* H. ST. G. ANSON.

THE Demonstration exhibited the behaviour of "Osglim" lamps (made by the General Electric Company) when shunted by a condenser and connected in series with a high resistance to a D.C. voltage supply. In these circumstances an intermittent current passes through the lamp, as indicated by the intermittency of the orange glow on the cathode (formed in this case as a letter of the alphabet of conconsiderable area, the anode being much smaller); this fact was also shown by use of a loud-speaking telephone. The frequency of the flashes can be varied by altering the capacity and resistance of the circuit. In a particular case a resistance of 1 megohm and a capacity of 1 mf. on a 200-volt supply gave a frequency of about 1·2 flashes per second.

The phenomenon may be explained as follows: When cold the lamp fails to pass any current until subjected to an E.M.F. of about 171 volts. If, however, the voltage be reduced when the lamp is glowing, current continues to pass until about 140 volts is reached, when the current falls suddenly to zero. When, therefore, an E.M.F. of 200 volts is applied, some time elapses while the condenser is charging up to the necessary 171 volts. At the end of this time the lamp begins to take current, the current increases and the voltage across the lamp falls to the limit of 140 volts, when the lamp goes out. This cycle of operations then repeats indefinitely.

It was suggested that the arrangement might be used at low frequency for flashing signs for advertising, &c., at audio-frequency for telephonic measurements, and at high frequency (up to about 15,000 ~) for radio-signalling.

The phenomenon was discovered by accident by Mr. Anson.

DISCUSSION.

DR. E. H. RAYNER: It will be most valuable to have an account of this interesting phenomenon in the records of the Society. I should imagine that the highest frequency as yet obtained with the apparatus—viz., 15,000 ~—is by no means an upper limit, as I have seen a neon lamp taking 60,000 ~. A tube 8 inches long was rotated about a perpendicular axis through its middle point at a distance of 1 foot from a C.W. aerial. The apparent illumination was thus drawn out into a disk with radial striæ, the spacing of which indicated the frequency I have mentioned.

DR. J. H. VINCENT: Can the author tell us whether the lamp exhibits any degree of unilateral conductivity? If so, a good many possible applications suggest themselves. It would also be interesting to know how far the critical voltages depend on the construction of the lamp. The usefulness of the arrangement would be greatly extended if the working voltage could be brought down to, say, 12 volts by suitable design. Has the author made any experiments with suitably arranged inductances instead of condensers?

MR. J. W. RYDE: Some results which I have obtained may help to answer the last speaker's questions. It appears that the electrical quantities concerned are related as follows:

$$I=(V-e)/\{(MA)^{-1}+R\}$$

where

I = steady current through lamp,
V = voltage across lamp,
e = constant (viz., the extrapolated intercept on the V-axis of the straight part of the I/V curve when produced),
M = a constant,
A = area of cathode,
R = resistance fitted in series with the lamp.

It will be seen that if the electrodes be of unequal area, the conductivity will be unilateral. The constant M is of the order 10^{-4}, and depends on the shape and material of the electrodes, the nature and purity of the gas in the lamp, and the pressure. It is particularly affected by occluded impurities. If the gas be pure, there is no change of current with time; but the current drops to an asymptote with time if impurities be admitted—as they are in practice in order to reduce the critical voltage and to prevent sputtering and consequent blackening of the glass.

Prof. C. L. **Fortescue**: I must congratulate the authors on their interesting exhibit. I should like to know whether the frequency is constant, say, to one part in a thousand or ten thousand. As regards the use of the apparatus for wireless transmission, the question of efficiency arises, and my impression is that there would be considerable waste of energy. As to priority of discovery, the late William Duddell used a somewhat similar arrangement in a demonstration at the Royal Institution about 11 years ago; but, instead of a lamp such as those before us, he employed a bulb which had been improvised for the purpose.

Mr. F. E. **Smith**: We have been told that Mr. Anson discovered this phenomenon "by accident," but such accidents only happen to observant persons. I should say the arrangement would be useful for low rather than high frequencies. For the latter purpose the thermionic valve is perfectly satisfactory, while low frequencies can only be obtained with it by using cumbrous inductances and condensers. If the frequency obtained with the neon lamp turns out to be constant, the apparatus might be used for measuring frequency.

Dr. G. B. **Bryan** inquired whether the pressure in the bulb remained constant during the life of the lamp.

Mr. W. C. S. **Phillips** (communicated): In the case of an "Osglim" lamp used as a night-light, I have noticed some curious facts which may be of interest. When current is switched on, as much as 10 seconds may pass before the lamp lights up. The supply is A.C., but when the frequency of the illumination is tested by moving the hand rapidly, one gets the impression that this latter frequency is somewhat lower than that of the supply. Again, when current is off, so that one electrode is alive and the other insulated, a glow can be noticed on the glass of the bulb in any region to which the hand is approached, as if a capacity-current were flowing through the body of the observer.

The **President**, remarking that Mr. Anson had begun making discoveries very early in life, invited the authors to undertake a written account for the Society's Proceedings.

XXV. *An Experiment on Molecular Gyroscopic Action.* By J. W. FISHER, B.Sc., *King's College, London.*

RECEIVED MARCH 13, 1922.

(COMMUNICATED BY PROF. O. W. RICHARDSON.)

ABSTRACT.

Starting from the inference, pointed out by O. W. Richardson on the basis of electron theory, that the magnetization of a substance implies a moment of momentum about the axis of magnetization, the consequences of applying a rotating magnetic field to a specimen of magnetic material are analysed.

It is shown that magnetization in a direction transverse to the plane of the rotating field should result, in sense related to the direction of rotation of the field by the left-hand-screw rule; and proportional in intensity to the first power of the angular velocity of the field and the susceptibility of the material.

Experimental tests were made, using a magnetometer, but without success, disturbing effects having not as yet been eliminated.

ACCORDING to the electron theory the magnetization of a substance implies a moment of momentum about the axis of magnetization due to a resultant rotation of the electrons in one sense. This consequence was first pointed out, in connection with the electron theory, by O. W. Richardson* who found for the moment of momentum U per unit volume, taking account of both positive and negative electrons, the value

$$U_z = 2\frac{m}{e} I_z \left(\frac{1 - \dfrac{eMA}{Ema}}{1 - \dfrac{A}{a}} \right)$$

where M and m are the masses of the positive and negative electrons respectively, E and e their charges, A and a the average values of the projections of the areas of the orbits perpendicular to the axis of magnetization divided by the times in which they are described. I_z here denotes the intensity of magnetization. Let us assume that the magnetization arises entirely from the orbital motion of negative electrons, so that we can write

$$U_z = 2\frac{m}{e} I_z$$

Suppose now that a magnetizable substance is placed in a rotating magnetic field of strength H, which in the diagram we suppose revolving in the plane yx with uniform angular velocity Ω. If at any instant it coincides in direction with oy, then by the above relation there will be a moment of momentum about that axis equal to $2\frac{m}{e} I_y$. On account of the rotation of H, however, the rate of change of this moment of momentum about the perpendicular axis ox will be $2\frac{m}{e} I_y \Omega$, so that there should, by the conservation of angular momentum, be a mechanical couple of this amount acting on the material and tending to turn it about the axis ox. The material, however, being fixed, and the field revolving, the elementary magnets must, in part at any rate, revolve with the field, and the more so the stronger the magnetizing field (as is shown by the falling off of the hysteresis in strong rotating fields). We should expect, then, the magnetic axes of the elementary magnets to be deflected, under the directive force of the above-mentioned gyrostatic couple, in a direction perpendicular to the plane of rotation of the field, thus giving rise to a transverse intensity of magnetization. For the case of revolving negative electrons this transverse field will be directed in a left-handed screw sense with the direction of rotation of the external field.

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* Richardson, Phys. Rev., Vol. XXVI., p. 248 (1908).

To calculate the magnitude of this transverse intensity we introduce a co-ordinate system x' y' z' revolving with the field, and we suppose the field H to be always directed along the oy' axis. M being the moment of momentum of an elementary magnet of magnetic moment m we have

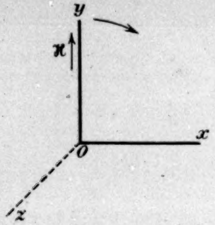

FIG. 1.

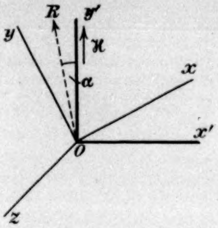

FIG. 2.

$$M=2\frac{m}{e}\,\mathrm{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We have, in a fixed co-ordinate system x, y, z, such that oz and oz' always coincide, $dM/dt=C$, where C is the moment of the external forces. Denoting differentiation with respect to the time by d'/dt, in the rotating system we can write

$$\frac{dM}{dt}=\frac{d'M}{dt}+[\Omega, M] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Ω being the vector angular velocity. For C, however, we write

$$C=[H, \mathrm{m}] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and therefore from (2) and (3)

$$\frac{d'M}{dt}=[H, \mathrm{m}]-[\Omega, M]=[H, \mathrm{m}]-[2\frac{m}{e}\Omega, \mathrm{m}]=[H-2\frac{m}{e}\Omega, \mathrm{m}] \quad . \quad . \quad (4)$$

We are now dealing with the rotating system, and have no longer any relative rotation of H to consider. The expression (4) gives the couple acting on an elementary magnet in the rotating system, and we see that the equivalent field is given by $H-2\frac{m}{e}\overline{\Omega}$; $\overline{\Omega}$ is directed along the axis of rotation oz and is therefore perpendicular to $\overline{H}$. The vector $\overline{H}-2\frac{m}{e}\overline{\Omega}$ has a fixed direction in the plane $y'oz$, and makes an angle α with H such that $\alpha=\tan^{-1}\left(2\frac{m}{e}\cdot\frac{\Omega}{H}\right)$. In the rotating system the direction of this vector will determine the axis of magnetization, and so there will be a transverse component parallel to the axis of rotation. The magnitude of the equivalent field does not differ appreciably from H; therefore, if I is the resultant intensity, the transverse component will be $I_z=I \,.\, \sin\alpha=I \,.\, \alpha=I \,.\, 2\frac{m}{e} \,.\, \frac{\Omega}{H}$, since α is small.

I/H is, however, the susceptibility k of the material, and so we have finally for the transverse magnetization

$$I_z=2\frac{m}{e}\,\Omega k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

To be sure the force at a point in the interior of a substance in a magnetizing field H is not H, but $H+\frac{4\pi}{3}I+X$, where I is the intensity of magnetization and X is the force due to the magnets in the immediate vicinity of the point under consideration. The latter vanishes in the case of fortuitous or regular cubic distributions of the elementary magnets,* and we may in any case, as a first approximation, write it proportional to I, so that the total force becomes $H+\left(\frac{4\pi}{3}+\beta\right)I$.

The justification for considering only the part H in the expression for the couple depends on the fact that the second component, being determined by the polarization I, has the same direction as I and consequently does not oppose a general rotation of the magnetic axis in the same sense. H is the only force independent of the orientation of the axes of the molecular magnets, and in the case under consideration H and I are no longer coincident but are inclined to one another at the angle α.

The effect being proportional to k it is clearly best to experiment with a material having a maximum possible value of k, *i.e.*, iron, and to work with fields for which k attains its greatest value. Precisely the same formula gives the intensity in Barnett's experiments on magnetization by rotation, but in this case the susceptibility k corresponds to small field strengths of the order $2\frac{m}{e}\,\Omega$, and is, therefore, according to the well-known curves for iron, considerably smaller than that corresponding to an external magnetizing field of the order of 1 Gauss or thereabouts, such as can be applied in the method described. From the results given by Barnett in one of his Papers† it appears that the susceptibility (including end effects) is about 5 for these small fields. We can, on the other hand, with field strengths of the order of 1 Gauss, easily reach values of k of 100–150, or, say 20 times as large, and herein lies the chief advantage of the above method. It possesses on the other hand a considerable disadvantage in that the large primary field can very easily give rise to strong fields very much greater than that due to the effect in question, and in the trials made I have not succeeded in eliminating these spurious effects with the apparatus which was available.

Experimental Tests.—In order to eliminate the stray fields as much as possible the rotating field was produced by two alternating currents differing in phase by 90° and exciting two suitably disposed electro-magnets, the specimen itself being a rectangular bar of soft iron passing between the pole-pieces of the magnets. In this case, theoretically at any rate, the mean value of the stray fields should be zero. It was sought to detect with a magnetometer a variation in the longitudinal magnetization of the rod, due to the effect in question, on changing the direction of rotation of the field, in which case the change in intensity of magnetization should be twice that given by the above formula.

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* H. A. Lorentz, Theory of Electrons, p. 306.

† Barnett, Phys. Rev., VI., 1915, p. 270.

It was, however, impossible to make any observations, as the irregular disturbances were at best about twenty times that of the effect looked for, and appeared to be due in almost equal measure to varying external fields and to variations in the alternating currents exciting the magnets. A couple of ancient single-phase alternators (frequency 50 ∿), coupled together with 90° phase difference, had to be used as generators, and the irregularities of the alternating currents followed, no doubt, from fluctuations of the exciting currents, which were derived from the mains, and to commutating action at the slip-rings due to bad contact and unevenness. No appreciable advantage was found to follow from fitting several brushes to each slip-ring. As the rectification of these deficiencies involved setting up the apparatus anew in a more favourable position and with a more suitable generator, the experiments had, at any rate for the time being, to be abandoned.

In conclusion, I wish to express my best thanks to Professor Richardson for his interest in the problem, and for the opportunity afforded to carry out the experiments in the Wheatstone laboratory.

DISCUSSION.

Dr. E. H. Rayner alluded to the experimental difficulties occasioned by the distortion of the magnetic flux, owing partly to eddy currents in the iron bar and partly also to the non-uniform distribution of magnetic flux along the bar, only the middle section of which would be exposed to the magnetising field. The time-variation of the field-strength could be minimised by using a single alternator with polyphase windings and stationary armature, slip-rings being absent in this case from the magnetising circuits. An excellent machine of this kind belonging to the National Physical Laboratory might perhaps be placed at the Author's disposal.

Dr. L. Simons referred to some research he had seen last July being carried on in the Physics laboratory at Bristol. In this case an iron wire was hung vertically by means of a fibre fixed to its end. When a vertical magnetic field was put on, the wire experienced a momentary rotating couple. Elaborate precautions were necessary to eliminate disturbing factors such as lack of symmetry and electric trams which otherwise masked the effect under investigation. It seemed as though Mr. Fisher were looking for an effect of the same order of magnitude, and that consequently similar difficulties in the elimination of disturbing factors would arise.

Dr. D. Owen inquired as to the magnitude of the effect to be observed. If it were large enough the errors could be suppressed by using a substance of lower permeability than iron, and preferably a non-conductor of electricity which would be free from eddy currents. It might then be of advantage to employ high-frequency fields, since the effect to be measured is proportional to the frequency.

Prof. O. W. Richardson, replying on behalf of the author, thanked Dr. Rayner for his kind offer of a suitable alternator, of which advantage would be taken when time permitted. He thought that the effect of eddy currents had been fully considered by the author, although the latter had not mentioned the fact in his Paper. The experiments of Prof. Chattock and Mr. Bates, referred to by Dr. Simons, aimed to measure the rotatory mechanical impulse caused by magnetisation which Prof. Richardson attempted to detect experimentally, and made calculations about, in 1908. The effect which Mr. Fisher had sought to detect was of a somewhat more complex nature, but it was clearly a development of the same order of ideas. The use of higher frequencies, suggested by Dr. Owen, would accentuate the difficulties introduced by eddy currents, so that, in using conducting substances at any rate, there would be a certain optimum frequency at which to work. The magnitude of the effect would not be excessively small, though its measurement would be a delicate matter. Under the conditions referred to in the Paper it would be some 20 or 30 times as great as the effect detected by Barnett, which is the converse of the rotatory impulse effect due to magnetisation.

XXVI. *On the Viscous Properties and Molecular Dimensions of Silicane.*

By A. O. RANKINE, *D.Sc., Professor of Physics, and* C. J. SMITH, *B.Sc., A.R.C.S:., D.I.C., Research Student, Imperial College of Science and Technology, London.*

RECEIVED APRIL 14, 1922.

ABSTRACT.

The absolute values of the viscosity of the gas silicane have been determined for the temperatures 15°C. and 100°C. The data have been used to calculate the mean collision area presented by the silicane molecule in the gaseous state. Use is then made of the knowledge of this dimension in an attempt to elucidate the structure of the silicane molecule in relation to those of other gaseous hydrides whose molecules have equal numbers of extra-nuclear electrons.

SILICANE is a gaseous compound having the molecular composition SiH_4, thus resembling the well-known gas methane (CH_4). It is the last known member of a regular series commencing with argon and forming the sequence, argon, hydrogen chloride, sulphuretted hydrogen, phosphine and silicane—A, HCl, H_2S, PH_3, SiH_4. For each of these five molecules the " molecular number," *i.e.*, the sum of the atomic numbers of the various constituent atoms, is the same, namely, 18 ; and there are substantial grounds for believing that the central regions of all of them have practically the same dimensions, the differences in the mean collision areas of the complete molecules arising from external protruberances corresponding to the hydrogen atoms. The evidence for this is based mainly upon the results of the calculations of molecular size derived from the viscous properties of the first four gases of the sequence.* In order to extend the investigation as far as possible it was necessary to know corresponding dimensions for the silicane molecule, and we have therefore measured for this gas the necessary viscosity data.

Silicane is a gas not frequently made, and its preparation is attended with some difficulty. We have, however, succeeded, after some failures, in making a sufficient sample in a pure state by a method nearly the same as that described by Moissan.† Once prepared, silicane is quite stable, and there is no difficulty in measuring its viscosity with the same degree of accuracy as for the other gases previously mentioned.

Method of Observation.

The method which we have used to determine the viscosity of silicane at atmospheric and steam temperatures has already been described in detail.‡ It amounts to filling the viscometer with air, and then with the gas under investigation, and comparing the times taken for the same mercury pellet to drive equal volumes of gas through the capillary tube. This ratio, with appropriate corrections, gives the relative viscosities of air and the gas ; and from it, assuming the absolute viscosity of air, that of the gas is deduced. For the variation with temperature, a knowledge of which is also necessary for estimating molecular dimensions, we compare the times of transpiration for the gas at atmospheric and steam temperatures.

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* A. O. Rankine, Trans. Far. Soc., Vol. XVII., Part 3 (1922).

† Moissan, C. R., 134, p. 571 (1902).

‡ A. O. Rankine, and C. J. Smith, Phil. Mag., Vol. XLII., p. 601 (1921), and C. J. Smith, Proc. Phys. Soc., Vol. XXXIV., p. 155 (1922).

Preparation of the Silicane.

The sample of silicane was obtained by the action of hydrochloric acid on magnesium silicide—Mg_2Si. The purity of the magnesium silicide was tested spectroscopically by examining the spark spectrum in the ultra-violet region, when only magnesium and silicon were found to be present. The apparatus is shown diagrammatically in Fig. 1. The flask, *C*, contained 5 gm. of magnesium silicide. At first no liquids were present in the apparatus except strong sulphuric acid in the wash bottle, *A*. Commercial hydrogen, desiccated by bubbling through concentrated sulphuric acid, and then freed from possible traces of carbon dioxide and sulphur dioxide by circulation through a U-tube *B* in liquid air, was used to displace the air from the generating apparatus. When the air had been entirely displaced the flask *C* was surrounded by ice. This caused a sufficient reduction of pressure in the apparatus to enable 200 c.c. of water to be dropped into *C*, also some water into the scrubber *D*, and finally about 50 cc. of strong hydrochloric acid into *C*. The gas which is generated has been shown by Moissan (*loc. cit.*) to consist of two hydrides of silicon—SiH_4 and Si_2H_6—and a large excess of hydrogen. (This mixture is spontaneously inflammable when brought in contact with air—a fact which necessitates the exclusion of air from the generating apparatus in the manner described.) The mixed gases were freed from traces of hydrogen chloride by being bubbled through the water in *D*, and dried by being passed through tubes—200 cm. long—containing phosphorus pentoxide. Ultimately the silicon hydrides were solidified in a U-tube *H* surrounded by liquid air. The stream of hydrogen was kept bubbling through *A* throughout the process; when not required in *C* it was diverted by means of the three-way tap *T*. The pressure gauge *E* was inserted to detect whether any blocking up took place in the apparatus.

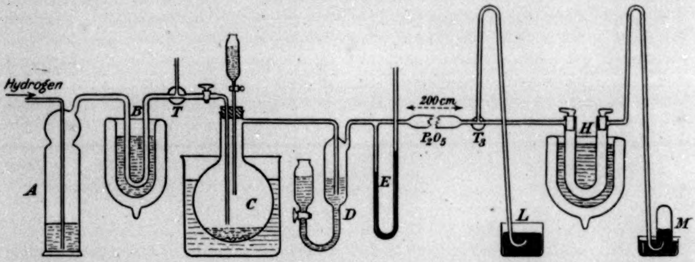

FIG. 1.

When sufficient solid had collected in *H*, the three-way tap T_3 served to cut off *H* from the rest of the apparatus, and also provided an alternative escape through *L* for the gases still being generated. The liquid air around *H* was then replaced by alcohol cooled to —80°C. by means of solid CO_2. This temperature is well above the boiling point of silicane, which thus evaporated. The escaping gas was collected over mercury at *M*, and consisted of a mixture of hydrogen and silicane, rich in the latter constituent.

Before introduction to the viscometer the silicane was purified by fractional distillation. It was condensed at liquid air temperature, and the permanent gases, mainly hydrogen, were pumped off. The liquid air was then removed and alcohol and CO_2 at —80°C. substituted. This retained possible traces of water and silico-ethane (Si_2H_6) (which boils at a comparatively high temperature well above 0°C.), and allowed the purified silicane to evaporate and be collected. This same process was repeated twice before measurements with the gas were commenced.

After the observations had been made the gas in the viscometer was found to be wholly condensible at liquid air temperature. The purity of the silicane was thereby confirmed.

RESULTS.

Dry Air.

The air which was used for comparison was dried by being passed over phosphorus pentoxide, and then through a tube maintained at —80°C. by means of solid CO_2 and alcohol, before being introduced into the viscometer. Three sets of observations in the neighbourhood of 15·0°C. were made, the results being consistent to within 0·1 per cent. The mean time of fall corrected to 15·0°C. was 104·70 secs.

Silicane.

TABLE I.

	Temp. (°C.)	Time of fall (secs.)		Capillary correction (x)	Corrected time (t)	Time of fall.	
		Whole pellet.	Two segments			15·0°C.	100·0°C.
(*a*)	15·3	69·06	72·18	0·0413	66·21	66·15	...
(*c*)	15·9	69·54	73·17	0·0473	66·25	66·06	...
					Mean	66·11	...
(*b*)	99·5	85·60	87·70	0·0234	83·60	...	83·70

The order in which the observations were made is shown by the letters in the first column. Each figure recorded in Column 3 is the mean of eight observations, mutually consistent to within 0·6 per cent., and those in column 4 the mean of six, mutually consistent to within 0·4 per cent.

In the above table small corrections have been made to reduce the observations to the standard temperatures 15·0°C. and 100·0°C.

The ratio of the times of fall for silicane and air at 15·0°C. is thus

$$\frac{t_{\mathrm{SiH_4}}}{t_{\mathrm{Air}}}=\frac{66{\cdot}06}{104{\cdot}70}=0{\cdot}6314$$

Correcting for slip in the manner previously indicated,* this ratio becomes 0·6304, whence, on the basis that the viscosity of air at 15·0°C. is $1{\cdot}799\times10^{-4}$ C.G.S. units, we have for silicane

$$\eta_{15}=0{\cdot}6304\times1{\cdot}799\times10^{-4}=1{\cdot}134\times10^{-4}\text{ C.G.S. units.}$$

Also

$$\frac{\eta_{100}}{\eta_{15}}=\frac{t_{100}}{t_{15}}=\frac{83{\cdot}70}{66{\cdot}11}=1{\cdot}266$$

whence for silicane

$$\eta_{100}=1{\cdot}436\times10^{-4}\text{ C.G.S. units.}$$

If we assume that Sutherland's law holds for the gas silicane over the range of temperature used in these experiments, we obtain for silicane $C=229$, where C is Sutherland's constant. Extrapolating to 0·0°C. we get

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* A. O. Rankine, Proc. Roy. Soc., A. Vol. LXXXIII., p. 517.

$$\eta_0=1{\cdot}078\times10^{-4}\text{ C.G.S. units.}$$

Calculation of Molecular Dimensions.

The mean area presented for collision by molecules of silicane in thermal agitation may be calculated by means of Chapman's formula, interpreted in the way already frequently described.[*] The result obtained is

$$\bar{A}=0{\cdot}989\times10^{-15}\text{ cm.}^2.$$

Comparing it with the corresponding area for the argon molecule, namely, $0{\cdot}648\times10^{-15}$ cm.^{2}, we find

$$\frac{\bar{A}_{\text{silicane}}}{\bar{A}_{\text{argon}}}=\frac{0{\cdot}989}{0{\cdot}648}=1{\cdot}53.$$

Summary of Results for Silicane.

Viscosity in C.G.S. units $\times 10^{-4}$			Sutherland's Constant C.	Mean collision area of molecule $\bar{A}$.
15°C.	100°C.	0°C.		
1·134	1·436	1·078	229	$0{\cdot}989\times10^{-14}$cm.^{2}

DISCUSSION OF RESULTS.

Attention has already been called[†] to the probable relation between the structures of the five gaseous molecules which form the sequence—

SiH_4 PH_3 H_2S HCl A

Any member of the sequence can be regarded as a system in which the stable configuration of extra-nuclear electrons of the argon atom has been attained by the addition of a sufficient number of the single electrons of hydrogen atoms, the nuclei of the latter remaining external to the system. In SiH_4, for example, four additional electrons are required for this purpose, the atomic number of silicon being four less than the argon number; thus four hydrogen atoms are needed to complete the stable molecule. Reasons based on crystal measurements have already been adduced[†] for believing that there are no appreciable differences of effective size of the completed electron configurations, and for attributing any variation of apparent magnitude in the above molecules to the effect of protuberances corresponding to the nuclei of the attached hydrogen atoms. Certainly the mean collision areas of the molecules continue to increase as we pass from argon to phosphine, and the new data available for SiH_4 show that this molecule is no exception to the rule. The numerical values for the mean collision areas, shown as ratios to that of argon, are

SiH_4	PH_3	H_2S	HCl	A
1·53	1·41	1·19	1·04	1·00

and these results are also shown graphically in Fig. 2, where the absolute mean collision areas are plotted against the number of hydrogen atoms in the molecule. The dotted line is not intended to denote continuity, for the number of hydrogen atoms must obviously be integral, but merely to emphasise the relative magnitudes of the successive increments.

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[*] A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 612, Nov., 1921; and A. O. Rankine, Proc. Phys. Soc., Vol. XXXIII., p. 362 (1921).

[†] A. O. Rankine, Trans. Faraday Soc., Vol. XVII., Part 3 (1922).

It will be noticed that although the increments increase more and more rapidly in passing from argon to phosphine, the newly determined increment from PH_3 to SiH_4 is of smaller magnitude than the two previous steps. It is suggested that the explanation of this result is to be found in a combination of two effects which operate in opposite senses.

The fact that at first the successive increments increase with each addition of a hydrogen atom is consistent with the view already put forward that the protuberances for which the hydrogen nuclei are responsible grow with increasing number. This growth may very plausibly be attributed to the retreat from one another of the hydrogen nuclei which occupy the outskirts of the molecule, on account of their increasing mutual repulsion. Thus, if we suppose that the contributions of each hydrogen protuberance to the mean collision area are independent of one another, we should expect the graph in Fig. 2 to display an upward curvature throughout. But if the prominences are large enough and numerous enough it is improbable that the contributions would be thus independent.

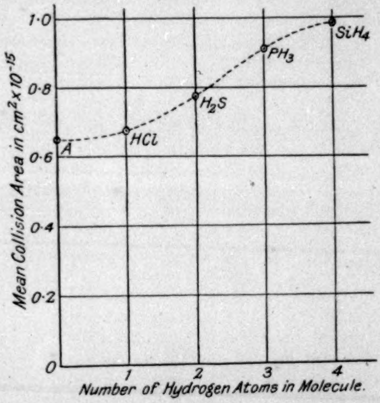

FIG. 2.

Let us picture the molecule for purposes of collision as a hard central sphere of the argon size, having a certain number of prominences corresponding to the hydrogen atoms. The area which the molecule will present as a target for any given orientation will depend upon the degree of mutual overlapping of the projections of the central sphere and the prominences. In the case of HCl, where there is only one prominence, there can be no question of mutual eclipses of prominences themselves, as distinct from eclipses of prominences by the central sphere. The comparative mean collision areas of HCl and A are 1·04 and 1·00, and we may therefore regard the excess 0·04 as the contribution of the single hydrogen prominence.

In the next molecule, H_2S, we have two hydrogen prominences which, for reasons of symmetry, we may expect to be on opposite sides of the central sphere. In these positions they also will be unable to eclipse one another except when already overlapped by the central sphere. Thus each will make an independent contribution to the mean collision area, the amount of each being $\frac{1}{2} \times 0{\cdot}19 = 0{\cdot}095$ times the mean collision area of argon. The excess of 0·095 over 0·04 may be taken as indicating precisely that each hydrogen protuberance in H_2S is greater than the single one in HCl.

When we come to PH_3, however, we can no longer assume that the effect of overlapping prominences is zero. There are now three prominences distributed round the central sphere, and if they are large enough there will obviously be certain orientations of the molecule in which mutual eclipsing of prominences occurs in which the central sphere plays no part. The effect of this will be to reduce the value of the mean collision area, and thereby to diminish the effect of any enlargement of the protuberances which may have occurred in passing from H_2S to PH_3. Actually, in spite of this, the increment between H_2S and PH_3 is larger than that between HCl and H_2S (0·22 as compared with 0·15). The effect of multiple overlapping is, apparently, in this case insufficient to obscure completely the enlargement of the protuberances.

In SiH_4 the prominences are four in number, and the operation of the kind of overlapping referred to in the last paragraph may be expected to be much more marked on this account. A little consideration shows that in the limit, when the number of prominences becomes very large, a single additional one of the same magnitude, which does not give rise to any general enlargement of those already there, may be inserted without increasing appreciably the mean collision area of the molecule. The fact, therefore, that there is so considerable an increase (0·12) between PH_3 and SiH_4 is not inconsistent with the view that the hydrogen prominences in the latter molecule are still greater in size, and that their growth has been masked to a considerable extent in the manner indicated.

Since we have not at present sufficient information to enable us to assign a probable shape to the prominences, the matter cannot be treated quantitatively; but qualitatively the new data for the silicane molecule support the conclusions already deduced from previously known data, namely (*a*) that these hydrogen compounds have central regions of configuration nearly identical with that of an argon atom, and only differ in dimensions by reason of the outlying hydrogen nuclei; and (*b*) that the distances of the hydrogen nuclei from the centre of the molecule increase more and more rapidly as the nuclei multiply.

DISCUSSION.

Prof. O. W. **Richardson**: Am I right in understanding that the authors' results conflict with those obtained by X-ray measurement of crystal lattices?

The **Authors**: The conflict to which Prof. Richardson has referred is apparent rather than real. The only point of contact between the two methods is afforded by the inert gases, the crystallographic diameters of whose atoms can be extrapolated from those of other elements in the same series. Thus Cl and O are found by Sir W. H. Bragg to have nearly the same diameters, and it is assumed from continuity that the diameter in argon agrees with these. The diameters found by the present method are somewhat larger, but the dimensions to be measured are not the same in the two cases. It must be assumed not that the atoms have hard shells which impinge on one another, but rather that they approach one another up to the distance at which the force between them changes sign and becomes a repulsion. This distance, being larger than the radius of the outer electron system, gives a different figure from that obtained by crystal measurements.

XXVII. *The Pressure-Gradient in Liquids Flowing Through Cones.*

By W. N. BOND, *B.Sc.* (*Eng.*), *M.Sc.* (*Lond.*), *Lecturer in Physics, University College, Reading.*

RECEIVED MAY 1, 1922.

ABSTRACT.

The pressure gradient in a liquid that flows through a conical tube is considered as depending on three effects: the variation in kinetic energy with distance along the axis; the loss due to purely viscous flow; and the loss caused by eddying. Expressions are developed giving approximately the gradient due to the three effects independently. The independence of the effects due to varying velocity and to "losses" is assumed as a working hypothesis. It is hence shown that these two effects should be capable of determination by two sets of experiments on any chosen cone, the direction of flow being reversed in the second experiments.

The effect of the entrance conditions is discussed.

The results of experiments on three ambroid cones of semi-angles 15½', 1°51' and 4°2' respectively are given. The conicality and losses effects are deduced, and these are plotted, the values for all the cones being reduced to a common basis. The results are compared with the theoretical values, experiments on a parallel ambroid tube and Stanton's experiments on parallel tubes of drawn brass. Two interesting conditions are mentioned, and a peculiarity is found in the case of divergent flow at moderate speeds through the cones of larger semi-angle.

Introduction.

THE present Paper consists of an attempt to extend the knowledge relating to the flow of liquids through conical tubes of circular cross-section by investigating how the pressure gradient along the walls of the tube is related to the rate of flow of the liquid and to the other variables concerned. For previous work on the subject, reference may be made to Chapters III. and IV. of "Gibson's Hydraulics," 1919.

Conical pipes were chosen, in order to make the shape easy to specify and also to facilitate as complete a theoretical treatment as possible. The restriction of the work to liquids enables variation in density to be neglected, but viscosity and density must be considered in these and any similar experiments.

General Theory of Pressure Gradient.

General equations may be obtained from the theory of dimensions before proceeding to more detailed theories.

Consider a right circular cone of semi-angle α. Let dp/dl denote the variation in pressure at the boundary with distance measured in the direction of flow, parallel to the axis; the radius of the circular section at this point being Υ.

(All pressure differences due to actual difference of level are for convenience throughout supposed to have been deducted.)

Let Q=volume of liquid passing in unit time (cm^3/sec).
ρ=density of liquid.
μ=viscosity of liquid.

Then we have

$$dp/dl=Q^2\rho/\Upsilon^5 \,.\, f_1(Q\rho/\Upsilon\mu,\ \alpha)=Q\mu/\Upsilon^4 \,.\, f_2(Q\rho/\Upsilon\mu,\ \alpha). \quad . \quad . \quad . \quad \text{(I)}$$

Detailed Theory of Pressure Gradients.

Three main factors which affect the pressure gradient have to be considered. Firstly, differences of pressure due to the variation of the velocity and kinetic energy of the liquid with the distance along the cone. Secondly, losses of pressure due to purely viscous flow. And thirdly, losses due to eddying and hence indirectly to viscosity.

The pressure gradient due to variation in kinetic energy depends on how the velocity varies across any transverse section, and may be obtained on the supposition that the velocity has a value v which is constant over any sphere of radius R described with the apex of the cone as centre. For a diverging cone the pressure gradient due to conicality alone is given by

$$\frac{dp}{dR} = -\frac{\rho}{2} \cdot \frac{d(v^2)}{dR}$$

or

$$\frac{dp}{dl} = \frac{\sin^4\alpha}{2\pi^2(1-\cos\alpha)^2} \cdot \tan\alpha \cdot \frac{\rho Q^2}{\Upsilon^5}$$

$$\longrightarrow 2\alpha\rho Q^2/\pi^2\Upsilon^5 \text{ for small values of } \alpha, \text{ with an error of under 1 per cent. for } \alpha=10^\circ \quad . \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

If the velocity across any transverse section be assumed to vary in the way it is found to do for turbulent flow in straight pipes, a similar theory gives a value about 12 per cent. larger than Equation (II.). This value has been verified experimentally. (For references, *see* Lea's "Hydraulics," 1916, p. 147.)

The calculation of the pressure gradient for the case of purely viscous flow, the author finds has been given by W. J. Harrison,* so it need only be given briefly, with application to the present problem. Since the lines of flow are straight lines passing through the apex of the cone, we have

$$v = \varphi(\theta)/R^2$$

where R, θ are the polar co-ordinates of the point.

Then, using the general hydrodynamical equations in which the terms depending on kinetic energy are neglected, we obtain

$$\varphi''' + \varphi'' \cot\theta + \varphi' (6 - \operatorname{cosec}^2\theta) = 0.$$

Hence

$$\varphi = C(\cos 2\theta - \cos 2\alpha)$$

and

$$\left(\frac{dp}{dR}\right)_{\theta=\alpha} = -\frac{Q\mu}{R^4} \cdot \frac{3}{\pi} \cdot \frac{3\cos^2\alpha - 1}{2\cos^3\alpha - 3\cos^2\alpha + 1}$$

or

$$\frac{dp}{dl} = -\frac{3}{\pi} \cdot \frac{\sin^4\alpha(3\cos^2\alpha - 1)}{\cos\alpha(2\cos^3\alpha - 3\cos^2\alpha + 1)} \cdot \frac{Q\mu}{\Upsilon^4}$$

$$\longrightarrow -8Q\mu/\pi\Upsilon^4 \text{ for small values of } \alpha \text{ with an error of under 4 per cent. for } \alpha=10^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

Thus, in this case, the pressure gradient at any point is to a close approximation the same as if the tube were of the same diameter but parallel. For cones of small angle an expression was previously derived by Gibson,† which gives the same result for such angles as the limiting value given in Equation (III.) above.

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* Proc. Camb. Phil. Soc., Vol. XIX., pp. 311-12.

† Phil Mag., 18, 1909, p. 38.

When the flow is almost purely viscous, the velocity variation across any transverse section is approximately parabolic, and hence the small pressure gradient due to differences of kinetic energy (which may be supposed superposed on the gradient due to viscous forces) may be deduced. Thus, for a diverging cone, we have approximately

$$Q\frac{dp}{dR}=-\int_0^{\Upsilon}\tfrac{1}{2}\rho(2\pi\Upsilon\,.\,d\Upsilon)v\frac{d(v^2)}{dR}$$

or

$$dp/dl=4\alpha\,\rho Q^2/\pi^2\Upsilon^5 \text{ (approximately)} \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

Thus the gradient due to conicality has under these conditions about double the value it has when the velocity is approximately constant over any transverse section.

The loss of pressure due to turbulence is supposed to depend on the nature of the boundary ; and there is considerable evidence that it depends on the condition under which the liquid enters the tube.[*]

In experiments on parallel pipes it is usual to let the liquid pass through a considerable length of similar pipe before it reaches the section experimented on. It is not possible to extend a diverging cone in this manner, since it is limited in the upstream directions by its apex. Hence it is necessary to make the liquid enter a diverging cone from a parallel or converging section, the transition being made as gradual as possible.

The pressure gradient due to turbulence may depend on the angle of the cone besides on the other variables. Since no data are available in regard to this effect, we may assume that, as in the case of a parallel pipe, the pressure gradient due to eddying may be represented by

$$dp/dl=A\,\rho Q^2/\Upsilon^5+B\,\rho^{1-n}\mu^n Q^{2-n}/\Upsilon^{5-n} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(V.)}$$

There A, B and n are positive constants, depending on the nature of the surface roughness.[†]

It is interesting to notice that Stanton[‡] found experimentally that the pressure losses for different diameters of parallel pipe of the same material were in the same ratio as would have been expected had the linear dimensions of the roughness been made proportional to the diameter of the pipe. Since such geometrical similarity of roughness probably did not exist, it follows that the absolute magnitude of the roughness is of little importance, provided the roughness be small. Hence the roughness can only influence the eddy losses in virtue of the nature of its contour.

Reversible and Non-reversible Effects.

Throughout this Paper the pressure gradient is termed positive when the pressure increases on passing down the tube in the direction of motion. Adopting this convention, it will be seen that the gradient due to " losses " is always negative, but that due to varying diameter or conicality is positive for diverging and negative for converging cones. If these effects act approximately as if independent, experiments on any cone with flow first in one direction and then in the reverse, should give values of the two effects separately. This is assumed as a working hypothesis, and may be considered in conjunction with the experimental results given below.

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[*] L. Schiller, Zeits. Math. u. Mechanik, Vol. I., No. 6, pp. 436-444.

[†] Lees, Proc. R. Soc., A., Vol. XCI.

[‡] Stanton and Pannell, Phil. Trans., A., Vol. CCXIV., p. 299

Since for purely turbulent flow the eddy loss is roughly proportional to the square of the rate of flow, it may be seen that it is probably not feasible to separate these two effects by means of experiments on cones of the same angle but of different diameters, since the two effects vary almost in proportion.

For divergent cones two effects in particular are of interest. Firstly, for any angle of cone, there is a critical condition when the gradient due to viscous loss is just neutralised by that due to divergence, and there is thus zero pressure gradient at the point. Secondly, for very high speeds there is some angle depending on the roughness of the walls, for which the eddy loss is approximately neutralised by the divergent gain. The semi-angle would be of the order of four minutes.

Deduction of the Pressure Gradient from Observed Pressure Differences.

Since in these experiments the difference in pressure between two points is observed, account must be taken of the fact that the radius Υ of the circular section varies continuously between the two pressure holes.

Consider the extreme cases in which Equation (I.) may be written

$$dp/dl = a\,\rho Q^2/\Upsilon^5 \text{ and } b\mu Q/\Upsilon^4,$$

where a and b are constant. The observed pressure difference δp between points separated by a distance δl along the axis is given by

$$\delta p/\delta l = a\,\rho Q^2[1/\Upsilon^4]/4[\Upsilon] \text{ and } b\mu Q[1/\Upsilon^3]/3[\Upsilon]$$

in the two cases. Hence, the pressure gradient is given in the two cases by

$$\frac{dp}{dl} = \frac{4[\Upsilon]}{\Upsilon_m{}^5[1/\Upsilon^4]} \cdot \frac{\delta p}{\delta l} \text{ and } \frac{3[\Upsilon]}{\Upsilon_m{}^4[1/\Upsilon^3]} \cdot \frac{\delta p}{\delta l} \cdot\cdot \quad . \quad . \quad . \quad . \quad . \qquad \text{(VI.)}$$

at a point where the radius is Υ_m. This may for convenience be chosen as the mean of the radii at the pressure holes.

The results obtained by these two extreme suppositions do not differ much in the present experiments.

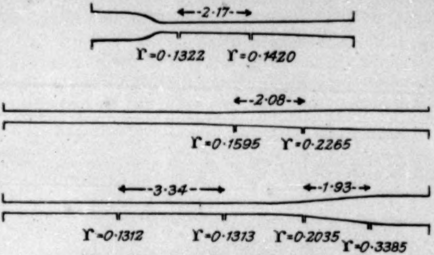
The first part of Equation (VI.) is therefore used, since it is nearly correct for large rates of flow, and only causes in the case of one cone at the very lowest speeds an error of about 10 per cent. In all other cases the error is quite negligible.

Experiments.

Small ambroid tubes were used for the experiments. These are shown in section in Fig. 1. The inside was cut to a conical form of the desired angle by means of suitable rimers, care being taken to make the deviation from the conical boundary at the narrow end a gradual curve. Small holes were drilled in the walls at chosen positions and a pair of holes was connected by tubes to a manometer. A simple water manometer, water over mercury, paraffin over water " U " tubes, and inclined tube water manometers were used. The liquid passed through the tube was water, except in a few experiments where mixtures of glycerine and water were used. An upper limit to the rate of flow practicable was set by the gases dissolved in the water being evolved at points of low pressure.

To minimise this trouble the cone was inclined so that the side holes were below, and the liquid emerged at the upper end. Thus any bubbles were quickly removed by the moving liquid and did not get into the manometer tubes.

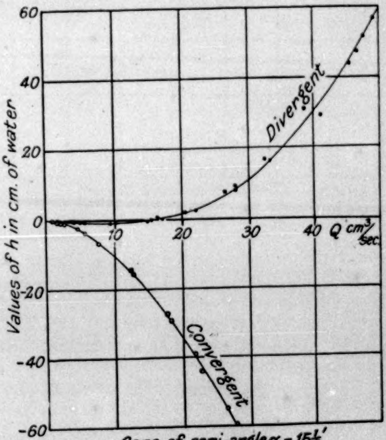
The tubes were measured with travelling microscopes both in their complete form (the tubes being transparent) and also after being cut transversely at the pressure holes.



Mean diameter of pressure holes, 0·08 cm
The length measurements are in centimetres.

FIG. 1.

In plotting the results of experiments on each tube it has been found most convenient to draw a graph relating h/Q and Q, where h is the excess of the pressure at the down-stream pressure hole over that at the hole further up stream, measured in centimetres of water.



Cone of semi-angle α = 15½′

FIG. 2

Fig. 2 shows the (h, Q) curve for the cone with semi-angle $\alpha = 15\frac{1}{2}'$, and it will be seen that for the divergent case there is a point at which the viscous loss and the kinetic gain combine to give zero pressure difference.

Fig. 3 shows the $(h/Q, Q)$ curve for the same experiments. It will be seen that for small rates of flow the curves approximate to a constant value, due to viscous loss. For large rates of flow the curves approximate to straight lines through the origin, and hence to the condition $h \propto Q^2$.

The dotted curve with its ordinates the mean of those in the two original curves should, according to our supposition, represent the pressure gradient due to losses. The difference between the ordinate of this curve and that of either of the original curves should represent the gradient due to varying kinetic energy (*i.e.*, due to conicality).

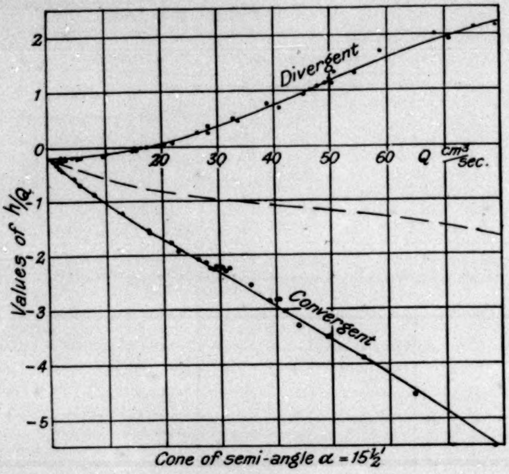


FIG. 3.

Figs. 4 and 5 show similar curves for the cases when $a=1°51'$, and $a=4°2'$. In Fig. 5 the mean curve is almost along the axis, and cannot claim to the least accuracy. It will be seen that the ratio of the effects due to eddy losses and to conicality decreases considerably as a is increased, varying in the case of the present experiments from about 50 to 1/50. The curves in Figs. 3 to 5 all reach a small negative value when $Q \longrightarrow O$, due to the existence of purely viscous losses.

In Figs. 4 and 5 the curves for the experiments on divergent cones show a marked peculiarity after h/Q attains a positive value and before Q becomes very large. This irregularity is peculiar to the divergent cones of not very small angle, and hence the " mean " curve in Fig. 4 has been drawn as in Fig. 3, the peculiarity of the curve for diverging flow being ignored. This is equivalent to the supposition that the peculiarity is due to some effect of conicality and is confined to the diverging case.

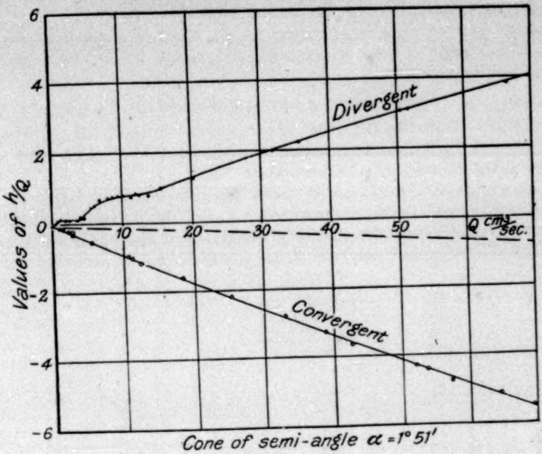

6
4
2
0
-2
-4
-6
Values of h/Q
10
20
30
40
50
Q cm³/sec.
Divergent
Convergent
Cone of semi-angle α = 1° 51′

FIG. 4.

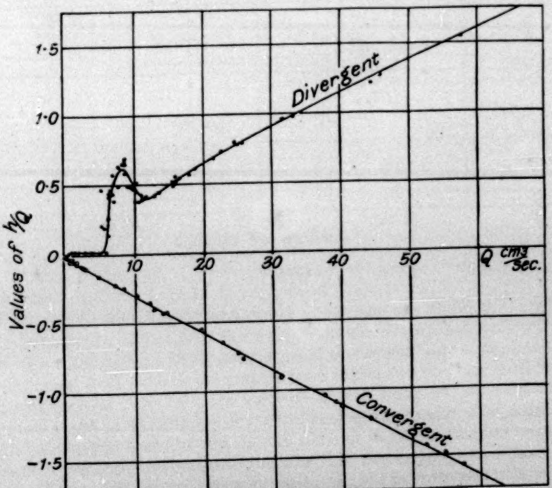

1·5
1·0
0·5
0
-0·5
-1·0
-1·5
Values of h/Q
10
20
30
40
50
Q cm³/sec.
Divergent
Convergent
Cone of semi-angle α = 4° 2′

FIG. 5.

In order to reduce the experiments to a common basis, we may apply an equation of the form of Equation (I.), first, to the effect of conicality treated alone. If squares of the angle of the cone be neglected, which we see by Equations (II.) and (IV.) is legitimate, we have

$$(\Upsilon^5/aQ^2\rho)dp/dl \text{ a function of } Q\rho/\Upsilon\mu \text{ only.} \quad . \quad . \quad . \quad . \quad \text{(VII.)}$$

which, according to Equation (I.), tends approximately to the limit $\pm 2/\pi^2$ or $\pm 0{\cdot}203$ for large values of $Q\rho/\Upsilon\mu$, and according to equation (IV.) should tend to double this limiting value for very small values of $Q\rho/\Upsilon\mu$.

This expression (VII.) evaluated from the observations with the aid of Equation (VI.) is plotted against values of $Q\rho/\Upsilon\mu$ in Fig. 6. For the larger angles of cone two curves are drawn in the neighbourhood of the peculiarity, one being obtained from the convergent and one from the divergent experiment. For large values of $Q\rho/\Upsilon\mu$ the curves agree with the limit mentioned above, $(2/\pi^2)(1{\cdot}12) = 0{\cdot}227$. For low values the ordinate does not attain the value $4/\pi^2$, but it is possible that if the experiments had been extended to lower values of Q this limit might have been reached. A Paper read to this Society by Prof. Hemmy* should yield information on this point, but he does not state whether his experiments were on a diverging or a converging cone. Since the peculiarity of the divergent case is not proportional to a, it is probably due to the initial condition of the liquid entering. The liquid may, over this range, perhaps flow for some distance in a narrow diverging stream near the centre of the tube, leaving the surrounding liquid comparatively at rest.

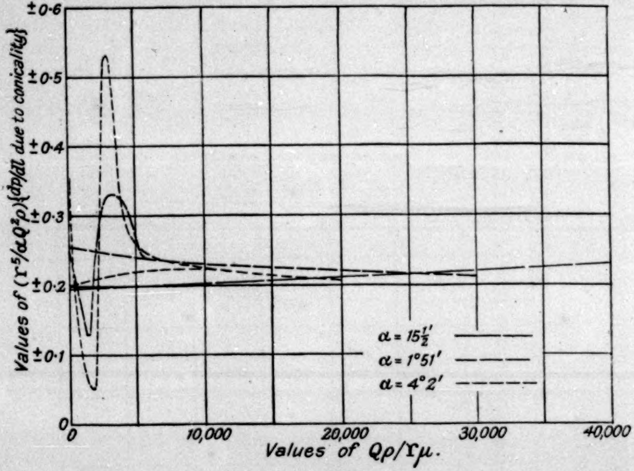


FIG. 6.

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* Prof. A. S. Hemmy, Proc. Phys. Soc., Vol. XXXIV, Dec., 1921, p. 22.

Again applying an equation of the form of Equation (I.) to the gradient due to viscous and turbulent losses treated apart from the conicality effect, we have

$$(\Upsilon^4/Q\mu)dp/dl \text{ a function of } Q\rho/\Upsilon\mu \text{ alone} \quad . \quad . \quad . \quad . \quad . \quad \text{(VIII.)}$$

with the limiting value for small values of the latter of $-8/\pi = 2{\cdot}55$ (by Equation III.).

This expression (VIII.) may be plotted with the aid of Equation (VI.), and is given in Fig. 7.

Curves are also plotted in this figure, obtained from experiments on the parallel pipe shown in Fig. 1. The pipe was tested with flow in each direction, and the difference in the results could not be accounted for by difference in diameter at the two pressure holes. It is thus probably due to the difference in initial conditions of the liquid depending on the shape of the entrance. The importance of this effect on the eddy losses, and the impossibility of a long up-stream to a diverging cone, must be remembered. A curve is also plotted in Fig. 7 obtained from Stanton and Pannell's experiments (*loc. cit.*) on drawn brass tubes.

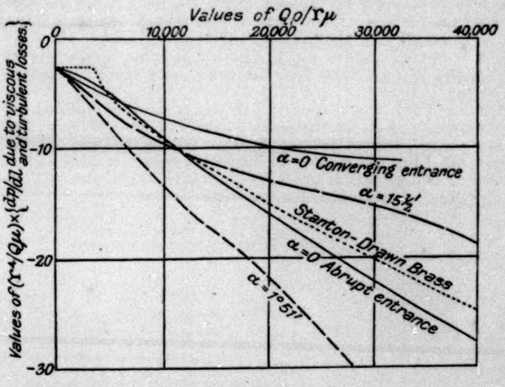

FIG. 7.

All the curves reach values very close to $-8/\pi$ for small rates of flow. The present experiments do not, however, give a stable viscous flow over so big a range as is found for long parallel tubes. This is probably due to the impossibility of the same up-stream conditions. The eddy losses, which depend on the roughness, are not constant.

Since the hypothesis of reversability leads to values of $(\Upsilon^5/aQ^2\rho)dp/dl$ that agree with previous independent estimates, it must be concluded that the mean curve in Figs. 3 and 4 represents the effect of the losses with a fair amount of accuracy. Hence the losses are approximately the same whether the cone be a converging or a diverging one. From these considerations and a comparison of the values of the losses effect in all the present experiments, it may be concluded that the losses term is not affected to any large extent by the angle of the cone.

The difference between the various curves in Fig. 7 can, however, easily be accounted for by differences of entrance conditions (the effect of which was considered above) and difference in roughness of the different tubes. The large effect of tubes having different surface roughness may be seen by reference to Stanton and Pannell's paper (*loc. cit.*).

Hence it is seen that for cones of small and moderate semi-angle, the hypothesis of the approximate independence of the conicality and losses effects is confirmed. For this hypothesis, together with that relating to the reversibility of the effects, leads to estimates of the values of the two effects which are self-consistent and in agreement with other theoretical and experimental estimates.

In conclusion, the author would like to thank Prof. W. G. Duffield for the facilities that have enabled this investigation to be carried out, and Mr. J. S. Burgess for his kind help throughout the course of the experiments.

DISCUSSION.

Dr. H. Chatley remarked that the irregularity in the curves of Fig. 6 appeared to be greatest in the neighbourhood of 5 deg., a fact which suggests analogy with the drift angle of aerofoils. A similar phenomenon is dealt with in Gibson's Hydraulics.

Mr. R. S. Whipple thought it would be interesting to indicate the lines of flow by means of colouring matter, as was done by Prof. Hele-Shaw, and to project an image of them on to a screen.

The Author, in reply, said that whereas Dr. Gibson mentions a critical angle of 5 deg., the irregularity referred to by Dr. Chatley seems to take place more or less at all angles, at much the same rate of flow. It must be noted, however, that all the tubes had the same diameter at the narrow end. It would certainly be interesting to use colouring matter, as this would indicate whether, at the speeds at which the irregularity occurs, there is a stream down the middle of the tube which breaks away from the remainder of the liquid.

List of Elements and Their Isotopes. By F. W. ASTON, *M.A., D.Sc., F.R.S.*

GIVEN in course of a lecture delivered before the Society on May 26, 1922.

Table of Elements and Isotopes.

Element.	Atomic Number.	Atomic Weight.	Minimum number of Isotopes.	Masses of Isotopes in order of intensity.
H	1	1·008	1	1·008
He	2	3·99	1	4
Li	3	6·94	2	7, 6
Be	4	9·1	1	9
B	5	10·9	2	11, 10
C	6	12·00	1	12
N	7	14·01	1	14
O	8	16·00	1	16
F	9	19·00	1	19
Ne	10	20·20	2	20, 22, (21)
Na	11	23·00	1	23
Mg	12	24·32	3	24, 25, 26
Si	14	28·3	2	28, 29, (30)
P	15	31·04	1	31
S	16	32·06	1	32
Cl	17	35·46	2	35, 37, (39)
A	18	39·88	2	40, 36
K	19	39·10	2	39, 41
Ca	20	40·07	(2)	(40, 44)
Ni	28	58·68	2	58, 60
Zn	30	65·37	(4)	(64, 66, 68, 70)
As	33	74·96	1	75
Br	35	79·92	2	79, 81
Kr	36	82·92	6	84, 86, 82, 83, 80, 78
Rb	37	85·45	2	85, 87
I	53	126·92	1	127
X	54	130·2	5 (7)	129, 132, 131, 134, 136, (128), (130 ?)
Cs	55	132·81	1	133
Hg	80	200·6	(6)	(197-200), 202, 204

(Numbers in brackets are provisional only).

XXVIII. *On Whittaker's Quantum Mechanism.*

By H. STANLEY ALLEN, *M.A., D.Sc., Reader in Physics in the University of Edinburgh.*

RECEIVED JUNE 5, 1922.

ABSTRACT.

A brief description is given of the mechanism of Prof. E. T. Whittaker which compels all exchanges between the kinetic energy of electrons and radiant energy to conform to the quantum condition. A modified form of the mechanism is suggested, consisting of two magnetons, or two ring electrons, placed near together with their planes parallel. It is pointed out that the quantum mechanism gives the law of force postulated by Langmuir in his model of a static atom, which leads to results identical with those obtained from the circular orbits of Bohr's theory.

§1. IN a suggestive Paper read before the Royal Society of Edinburgh on 8th May, 1922, Prof. E. T. Whittaker has described a mechanism within the atom so constituted as to compel all exchanges between the kinetic energy of electrons and radiant energy to take place in amounts $h\nu$, where h is Planck's constant and ν is the frequency of the radiation. The object of the present note is to direct the attention of physicists to this communication, and to point out that the same mechanism may be employed in forming a model of static atom, in which electrons, instead of revolving in orbits of a more or less complicated description, may be regarded as occupying, or oscillating about, positions of equilibrium.

§2. Prof. Whittaker infers that when an atom is caused to emit radiation by collision with an electron, the mechanism within the atom must be such that the approaching electron induces in it what he terms a "magnetic current"—*i.e.*, the magnetic analogue of an electric current: or at any rate something which behaves like a magnetic current. To illustrate this a model is described which is essentially similar to one of those which have been proposed by Sir Alfred Ewing* for the purpose of explaining induced magnetisation. The magnetic structure consists of a number of magnetic poles revolving in a circle of radius a, their corresponding poles of contrary sign being at rest at the centre of the circle. When this magnetic wheel is rotating about its axis with angular velocity $\dot{\psi}$, it sets up an electric field such that an electron (charge e, mass m) situated at a point on the axis at a distance x from the centre is acted on by a force $Mea^2\dot{\psi}/(a^2+x^2)^{3/2}$ along the axis. Here M denotes the sum of the magnetic poles (of one sign) in the magnetic wheel. When the electron is moving along the axis, it creates a magnetic field which tends to set the wheel in rotation. The equations of motion of the electron and of the magnetic wheel having been written in the form

$$m\ddot{x}+\frac{Mea^2\dot{\psi}}{(a^2+x^2)^{3/2}}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$A\ddot{\psi}-\frac{Mea^2\dot{x}}{(a^2+x^2)^{3/2}}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where A denotes the moment of inertia of the wheel, two first integrals are obtained without difficulty. One of these is the equation of conservation of energy of the system, and may be written

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* Ewing, Proc. Roy. Soc., Vol. C., p. 449 (1922); Phil. Mag., Vol. XLIII., p. 493 (1922); Proc. Roy. Soc. Edin., Vol. XLII., p. 97 (1922).

$$\tfrac{1}{2}A\dot{\psi}^2+\tfrac{1}{2}m\dot{x}^2=\tfrac{1}{2}mu^2 \qquad \text{. . . . . (3)}$$

assuming that initially the wheel is at rest, and the electron is projected from $x=-\infty$ with velocity u. The second integral of the equations of motion is written by Prof. Whittaker in the form

$$A\dot{\psi}-\frac{Mex}{(a^2+x^2)^{1/2}}=Me \qquad \text{. . . . . (4)}$$

It is worthy of notice that this equation may be interpreted as representing conservation of angular momentum, provided we take into consideration angular momentum in the electromagnetic field. Adding Me to each side, the equation may be written

$$A\dot{\psi}+Me\left\{1-\frac{x}{(a^2+x^2)^{1/2}}\right\}=2Me \qquad \text{. . . . . . . (5)}$$

or

$$A\dot{\psi}+\frac{Me\Omega}{2\pi}=2Me \qquad \text{. . . . . (6)}$$

where Ω denotes the solid angle subtended by the magnetic wheel at the electron. In this equation the first term represents the angular momentum of the magnetic wheel, and it may be observed in passing that this also (or rather the corresponding energy) is regarded later in Prof. Whittaker's Paper as located in the electromagnetic field. The second term may be taken to correspond to the angular momentum in the field arising from the presence of the electron and what is, essentially, a magneton. The result can be obtained from an extension of McLaren's theorem with regard to the angular momentum of a magneton.*

It follows from these equations that when the initial volocity u is small, the electron travels to a definite point on the axis where it comes to rest and the motion is reversed. When u has the value $eM/\sqrt{(Am)}$, the electron is able to get just as far as the magnetic structure, but not beyond it ; when u is greater than this value, the reversal point is on the further side of the magnetic structure, and when u has the value $2eM/\sqrt{(Am)}$, or any greater value, the electron is able to pass completely through and out of the magnetic system, so as to be free from its influence.

In this latter case ω, the final value of $\dot{\psi}$ (namely, when $x=\infty$) is given by

$$A\omega=2eM \qquad \text{. . . . . (7)}$$

and an amount of energy $U=\tfrac{1}{2}A\omega^2=\dfrac{2e^2M^2}{A}$ is lost by the electron and gained by the magnetic structure. *Unless the initial energy of the electron is as great as U, the electron gives up no energy to the magnetic structure, but experiences an "elastic impact"; if, however, the initial energy of the electron is greater than U, it gives up exactly the amount U of energy, and retains the rest.*

§ 3. The energy absorbed by the atom from the moving electron becomes resident in the atom as the energy of a " magnetic current." Prof. Whittaker then considers the transformation of this energy into the radiant form by the mechanism within the atom. Just as a closed electric current is equivalent to a magnetic shell, so a magnetic current is equivalent to an electric shell—an electric shell being to all intents and purposes what is called a charged condenser in electrostatics. It is found that the charge on either plate of the condenser which is equivalent to the magnetic current is numerically equal to the charge on the bombarding electron. Now a Hertzian oscillator is essentially a condenser in the act of discharging, the frequency ν being given by

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* S. B. McLaren, Phil. Mag., Vol. XXVI., p. 800 (1913) ; H. S. Allen, Phil. Mag., Vol. XLI., p. 119 (1921).

$$V=\frac{1}{2\pi\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

It is assumed that there are two factors associated with the vibrator in any particular atom which play the same part as the capacity C and the inductance L in the differential equation for the oscillatory discharge of a condenser.

Now the quantity $e^2\sqrt{(L/C)}$ has the same dimensions in every possible system of units, namely—the dimensions of action. This quantity, which may be regarded as a natural constant of Action, is accordingly represented by h/π, so that

$$\sqrt{\frac{L}{C}}=\frac{h}{\pi e^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

This may be taken to indicate that the Hertzian oscillators in the atoms are similar to each other in structure and differ only in scale.

From (8) and (9) we have

$$h\nu=\frac{e^2}{2C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

The right hand side of this equation represents the energy of the charged condenser which, in turn, is equal to U, the energy of the " magnetic current." So we find

$$h\nu=U \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

which is precisely Planck's equation connecting the frequency of the emitted radiation with the amount of kinetic energy absorbed from the bombarding electron.

The original Paper, together with other Papers which formed contributions to the ensuing discussion on Quantum Theory and Atomic Structure, should be consulted for further particulars, modifications and developments of the suggested mechanism.*

§ 4. In the discussion on Prof. Whittaker's Paper I suggested that the magnetic wheel in his model might be replaced by two thin anchor rings, representing the magnetons either of McLaren or of Parson, placed near together with their planes parallel and having a common axis. To obtain the required distribution of the lines of magnetic force the rings must be placed so that the magnetic force between them is one of repulsion. Such an arrangement is equivalent to a magnetic shell forming the curved surface of a cylinder having its edges coincident with the two rings. It is of interest to consider what modifications are required in Prof. Whittaker's equations if we substitute such a magnetic shell for his magnetic wheel. If φ denote the strength of the shell and d the distance between the two rings, I find that in equations (1) to (7) above the quantity M must be replaced by $2\pi\varphi d$. If we prefer to think of two ring electrons with currents circulating round them in opposite directions, φ is proportional to the strength of the current in each ring.

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* Proc. Roy. Soc. Edin., Vol. XLII. (1922).

§ 5. We pass on to consider how Whittaker's mechanism may be employed in constructing a model of a static atom with electrons occupying fixed positions, or oscillating about positions of equilibrium. Dr. Irving Langmuir, in a Paper read before the American Physical Society,[*] and in a Discussion on the Quantum Theory at the Edinburgh meeting of the British Association, has discussed the forces within a static atom. "If in addition to the Coulomb forces between charged particles we assume the existence of another force (quantum force) equal to $Fq=1/mr^3 \,.\, (nh/2\pi)^2$ acting between an electron and a nucleus, we find that a stationary electron is in stable equilibrium when its distance r from a nucleus is the same as the radius of a circular orbit corresponding to a stationary state in Bohr's theory. The total energy of the electron is also the same as that given by Bohr's theory. The frequency of oscillation about the position of equilibrium is identical with the frequency of revolution of the electron in the Bohr atom. Thus the Rydberg constant and the Balmer series can be deduced from this law of force without assuming moving electrons." Now it appears that Whittaker's mechanism provides a force of exactly the type required in Langmuir's theory.

We assume that the angular momentum of the magnetic wheel in its steady state is determined by Nicholson's quantum relation, so that, from (7)—

$$A\omega=2eM=nh/2\pi \quad \dots\dots\dots\dots \tag{12}$$

where n is an integer.

As we have seen in § 2, the force on an electron at a point on the axis of the wheel is $Mea^2\dot{\psi}/(a^2+x^2)^{3/2}$. When $\dot{\psi}=\omega$ and x is large in comparison with a this becomes $Mea^2\omega/r^3$, or, substituting the values of Me and ω given by (12)—

$$\frac{a^2}{2Ar^3}\left(\frac{nh}{2\pi}\right)^2$$

This agrees precisely with Langmuir's "quantum force," provided $A=\frac{1}{2}ma^2$. This might be taken to indicate that the moment of inertia of the magnetic wheel is the same as would be obtained by imagining a mass equal to that of an electron distributed uniformly over a circular disc of radius a. It must, however, be remembered that we are to regard the inertia of the magnetic structure as located in the electromagnetic field.

§ 6. Taking the electrostatic attraction between electron and nucleus as e^2/r^2, and assuming the quantum force to be a repulsion of amount $1/mr^3 .\ (nh/2\pi)^2$, the distance between electron and nucleus in the equilibrium position is—

$$r_0=\frac{n^2h^2}{4\pi^2me^2}$$

which is the radius of a circular orbit in Bohr's theory. Hence the force of repulsion may be written as r_0e^2/r^3. Assuming the massive nucleus to remain fixed and the electron to be displaced from its equilibrium position by a small distance, dr, we easily find for the value of the restoring force $\frac{e^2}{r_0^3}dr$. The frequency of the resulting oscillation is $\frac{1}{2\pi}\sqrt{\frac{e^2}{r_0^3 m}}$, which reduces to $\frac{4\pi^2 m e^4}{n^3 h^3}$, the value for the frequency of revolution of the electron in the Bohr atom. These results are included amongst those stated by Langmuir, but it may be pointed out that Langmuir's theory may be applied to a nucleus of finite mass M, leading to the same correction for the mass of the nucleus as is given by Bohr's theory. Thus, in the value of the Rydberg constant we must substitute for m the quantity $\frac{Mm}{M+m}$, as in Bohr's theory. It appears, therefore, that the quantum mechanism leads to results which are identical with those of Bohr's theory, so long as we consider circular orbits, and it is not impossible that a more accurate investigation, taking into account terms of higher order in the displacement of the electron, would lead to results comparable with those obtained in Sommerfeld's theory from a consideration of elliptic orbits.

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[*] Langmuir, Phys. Rev., Vol. XVIII., p. 104 (1921).

DEMONSTRATION of a New Method of Producing Visual Effects by Means of Sound. *By* DR. E. E. FOURNIER D'ALBE.

A MEMBRANE of thin rubber is stretched horizontally across the end of a tube about 2 in. in diameter, and a drop of mercury, about 1 in. in diameter, is placed upon the membrane. Sound waves entering the tube produce oscillations in the membrane, and these are communicated to the edges of the mercury drop, when they travel to the centre and out again. The incoming and outgoing waves produce stationary ripples in the mercury surface, which remain in position so long as the note is held, and change with every variation in the pitch of the note, the change taking place in about 1/20th of a second, so that a piece of music can be made practically " visible " by means of the pattern of ripples. The wave-length varies inversely as the pitch of the note.

The patterns were thrown on a screen, so that the audience could follow a piece of music played on a gramophone.

XXIX. *The Neon Tube as a Means of Producing Intermittent Currents.* By S. O. PEARSON, *B.Sc., and* H. ST. G. ANSON.

RECEIVED JUNE 7, 1922.

(COMMUNICATED BY A. RUSSELL, M.A., D.Sc.)

ABSTRACT.

The Paper relates to an experiment in which a neon tube and a condenser in parallel are connected in series with a high resistance and source of current.

In these circumstances intermittent current is found to pass through the lamp, and the Paper discusses the conditions governing the frequency and duration of the resulting flashes.

IN order to explain clearly the phenomenon to be described here it will be necessary first to consider briefly some of the chief characteristics of a neon tube. The tube consists essentially of a glass blub enclosing two metal electrodes and filled with neon gas at a low pressure. Experiments were carried out with a number of neon lamps as supplied by the General Electric Co. under the trade name of "Osglim."

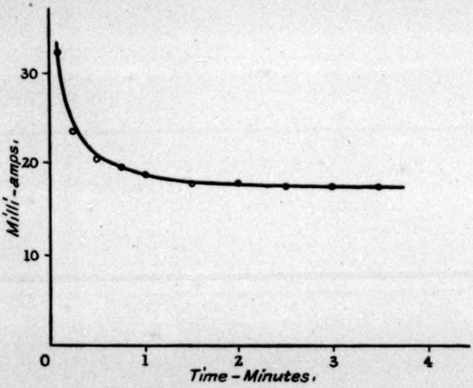

FIG. 1.—VARIATION OF CURRENT WITH TIME.

In these lamps the electrodes have various shapes according to the purpose for which they are intended, *e.g.*, letters of the alphabet, &c.

When a suitable difference of potential is applied to the electrodes a discharge takes place through the gas and an orange-coloured glow is formed over the entire surface of the negative electrode or cathode. Under normal conditions the anode does not glow at all. It will be seen later that the effective resistance between the electrodes, when the lamp is glowing, decreases with increase of current; hence a steadying or ballasting resistance is connected in series and mounted inside the cap of the commercial type of lamp. In order to carry out the experiments described the resistances were removed from the caps and connected externally, thus enabling the actual difference of potential between the electrodes themselves to be measured.

When the lamp is first switched on the temperature of the gas in the bulb begins to rise, and hence its pressure increases. This may account for the fact that the current does not reach a steady value until several minutes have elapsed after first switching on. The initial value may be more than double the final steady value of the current for a given applied voltage. The curve of Fig. 1 shows how the current varied with time after first switching on with the lamp cold; the pressure across the electrodes was kept constant at 175 volts. The curve appears to be very similar in shape to an ordinary curve of cooling or " die-away " curve. It is evidently necessary to allow an interval of at least three or four minutes to elapse before measuring the current for any given voltage.

D.C. CHARACTERISTIC.

The steady current flowing through the lamp was measured for various values of the potential difference applied to the electrodes. It was found that as the voltage was gradually increased from zero, no current flowed and the lamp did not commence to glow until a value of 171 volts was reached. At this point the current suddenly increased from zero to 28·4 milliamps, and the glow was established. This critical voltage seems to be quite definite for a given temperature, but it was found that with the lamp hot after running for 15 minutes the critical voltage had risen to 179 volts. The initial value of the current at 171 volts was over 28 milliamps, but it reached a steady value of 14·7 ma. after about five minutes. The steady values of the current were obtained for increasing values of the voltage, and from the curve of Fig. 2 it will be observed that the current follows very approximately a straight line law. On reducing the voltage in stages it was found that the current did not cease to flow until the potential difference had fallen to

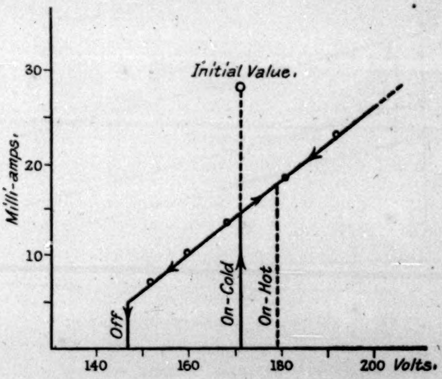

FIG. 2.—CHARACTERISTIC CURVE OF NEON TUBE.

147 volts—a figure well below the critical starting voltage. Mr. J. H. Ryde has shown that the steady current taken by the lamp is given by :—

$$I=(V-e)/\{(MA)^{-1}+R\}$$

where

V=voltage across lamp and series resistance,
e=constant (viz., the extrapolated intercept on the V-axis of the straight part of the V, I curve),
M=a constant of the order of 10^{-4}, dependent on the area and material of the electrodes and the pressure and purity of the gas,
A=area of cathode,
R=resistance fitted in series with lamp.

In order to determine the effects of temperature on the steady values of the current and on the critical values of the voltage, a lamp was immersed in water and tests made with the water at various temperatures between zero and 100°C. The curves of Fig. 3 show how the steady values of the current are dependent on the temperature—the higher the temperature the lower the current for any given voltage. Three curves are given, namely, at 200, 180 and 170 volts respectively. This seems to show that the change of current with time after first switching on is, to some considerable degree, accounted for by the rising temperature. It has been stated by some observers that this change in current with time is due to occluded impurities which are admitted in order to reduce the critical voltages and to prevent spluttering and consequent blackening of the glass ; and that if the gas be pure there is no change of current with time. This implies that temperature plays no part—a conclusion which does not seem to tally with the experimental results given.

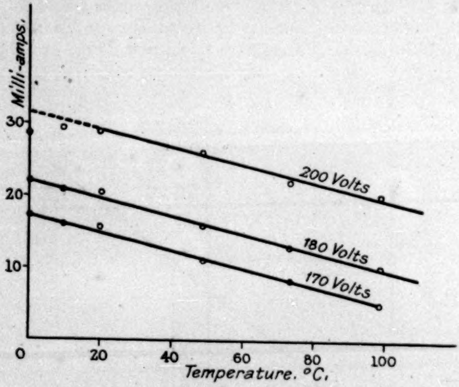

FIG. 3.—EFFECT OF TEMPERATURE ON CURRENT.

Fig. 4 shows the effect of the temperature of the surrounding medium on the upper and lower critical voltages.

FLASHING OF THE LAMP.

We have seen that no current flows until the voltage across the electrodes reaches a certain definite value ; that is, the resistance is to all intents and purposes infinitely great for all values of voltage up to the value at which the discharge through the gas commences. When a condenser of capacity K farads is connected in series with a high resistance R and a constant voltage V is suddenly applied to the ends of the circuit, the condenser will begin to acquire a charge and the voltage v across its terminals will rise according to the law $v=V\left(1-\varepsilon^{-\frac{t}{KR}}\right)$, where t is time in seconds after switching on. If the lamp is connected in parallel with the condenser as in Fig. 5, it will obviously flash on as soon as the voltage across the

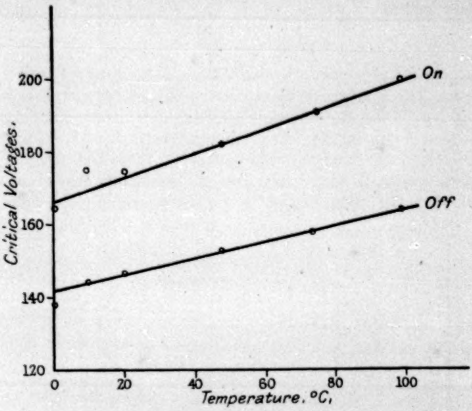

FIG. 4.—EFFECT OF TEMPERATURE ON CRITICAL VOLTAGES.

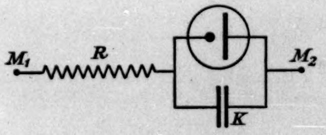

Diagram of Connections.

FIG. 5.

condenser reaches the critical value at which the discharge through the gas commences. When this occurs the condenser becomes shunted by a fairly low resistance—that of the lamp—and begins to lose its charge in consequence. The lamp will continue to glow until the pressure across it and the condenser falls to the lower critical voltage at which the current through the lamp ceases. If a and b represent the upper and lower critical values respectively of the voltage, the glow will last while the voltage falls from *a* to *b*. As soon as the glow ceases, the condenser charges up again from *b* to *a*, and so the process is repeated indefinitely.

By adjusting the values of *R* and *K* the frequency of flashing can be varied over a very wide range. The authors have obtained frequencies as low as one flash in several minutes and so high as to be above the range of audibility. The audible frequencies of flashing are best detected by means of a telephone connected in the condenser branch circuit.

CALCULATION OF PERIODIC TIME.

The dark period lasts whilst the condenser is charging up from the voltage *b* to the voltage *a*. This is quite easily calculated if we know definitely the values of *a* and *b*. There is no difficulty in measuring *a* experimentally, since the gas is quite cool when the glow commences ; but in the case of the lower critical voltage *b*, a difficulty arises—the duration of the light period is a very small fraction of a second ; in fact, in some cases it may only last for one-millionth of a second. We shall see later that the light period is very short compared to the dark period and hence there is very little heating effect on the gas when the lamp is flashing, which means that the gas is still cool when the light period comes to an end. To measure the lower critical voltage experimentally it is necessary to reduce the applied voltage gradually with the lamp glowing, which allows the gas to heat up ; and so the true value with the gas cool cannot be obtained.

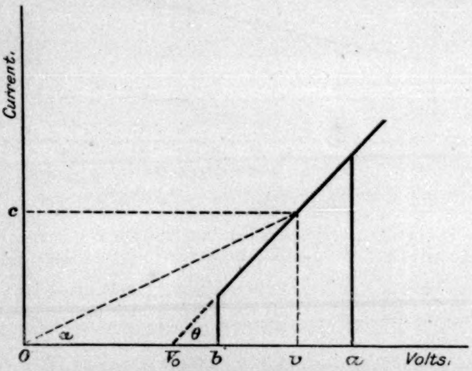

FIG. 6.—TYPICAL CHARACTERISTIC CURVE.

The time of one dark period differs from the time of one complete cycle by much less than 1 per cent. Hence, if the time of one complete cycle is measured it is possible to calculate the lower critical voltage very approximately.

Let T_1=duration of one dark period

=time for condenser to charge from *b* to *a*,

then

$$T_1 = KR \log (V-b)/(V-a) \quad . \; . \; . \; . \; . \; . \quad (1)$$

In a particular case the values were as follows :—

$T_1 = 0{\cdot}83$ sec.
$K = 1{\cdot}0$ microfarad
$R = 10^6$ ohms
$V = 200$ volts
$a = 171$ volts

from which we get :—

Lower critical voltage, $b = 139{\cdot}6$ volts.

In order to calculate the duration of one light period it is necessary to know how the resistance of the glowing lamp varies with the voltage across the electrodes. Fig. 6 shows a typical characteristic curve, and from it we can deduce an expression for the resistance in terms of voltage across the electrodes. Put

$$\begin{aligned} \cot\theta &= r' \\ &= (v - v_0)/c \end{aligned}$$

Now $c = v/r$, where $r =$ effective resistance of lamp.

Therefore $\qquad r' = r(v - v_0)/v$

and $\qquad r = v \,.\, r'/(v - v_0)$ ohms.

We are now in a position to find the duration of one light period.

Let v=voltage across the condenser at any time t seconds after commencement of flash. When $\quad t = o, \; v = a.$

Charging current from supply and flowing through R

$i_1 = (V - v)/R$, where $V =$ constant applied voltage.

Current flowing through lamp

$$\begin{aligned} i_2 &= v/r \\ &= (v - v_0)/r' \end{aligned}$$

Resultant current flowing from condenser, or rate of discharge

$$\begin{aligned} i &= i_2 - i_1 \\ &= (v - v_0)/r' - (V - v)/R \\ &= v(1/R + 1/r') - (V/R + v_0/r') \end{aligned}$$

Also rate of discharge

$$i = -\frac{dq}{dt} = -K\frac{dv}{dt}$$

Hence $$\frac{dv}{dt} + \frac{1}{K}\,(1/R + 1/r')v - \frac{1}{K}\,(V/R + v_0/r') = 0,$$

the solution of which is

$$v = \frac{Vr' + v_0R}{R + r} + A\varepsilon^{-\frac{R+r'}{KRr'}t},$$

where A is a constant which can be determined from the initial conditions, viz., when $t = 0$, $v = a$.

Hence
$$A=-\frac{Vr'+v_0R}{R+r'}$$

Therefore voltage across lamp at any time t after commencement of flash is
$$v=\frac{Vr'+v_0R}{R+r'}\left(1-\varepsilon^{-\frac{R+r'}{KRr'}t}\right)+a\varepsilon^{-\frac{R+r'}{KRr'}t}$$

Let
$$\frac{Vr'+v_0R}{R+r'}=Q$$

Then
$$v=Q+(a-Q)\varepsilon^{-\frac{R+r'}{KRr'}t}$$

When v falls to b,
$$t=T_2=\text{duration of light period.}$$

Therefore
$$b=Q+(a-Q)\varepsilon^{-\frac{R+r'}{KRr'}T_2}$$

or
$$T_2=\frac{KRr'}{R+r'}\log\frac{a-Q}{b-Q} \quad \ldots\ldots\ldots\ldots \quad (2)$$

where
$$Q=\frac{Vr'+v_0R}{R+r'}$$

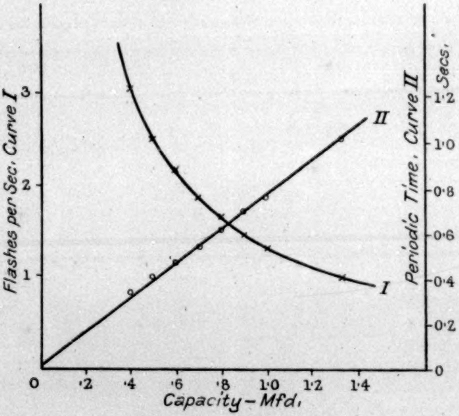

FIG. 7.—VARIATION OF FREQUENCY WITH CAPACITY.

In the case considered above the duration of the light period worked out at about 0·002 second.

The total time of one complete period is equal to the sum of the times of the dark and light periods respectively.

That is,
$$T=T_1+T_2$$
$$=KR\left\{\log\frac{V-b}{V-a}+\frac{r'}{Rr'}\log\frac{a-Q}{b-Q}\right\}$$

We see from the above that the time of one complete cycle is directly proportional to the capacity in parallel and the resistance in series with the lamp. This was proved experimentally, and in Fig. 7 curves are given showing how the periodic time and its reciprocal, the number of flashes per second, vary with change of capacity. Change of applied E.M.F. also affects the periodic time, and two curves are shown in Fig. 8. In both cases the series resistance was 1 megohm, but two different capacities, namely, 1 microfarad and 1·333 microfarads, were used. It will be noticed that the lower part of each curve is drawn dotted and made to meet the V-axis at 171 volts. This is obviously the case, for when the voltage is below 171, *i.e.*, below the critical starting voltage, no flashing can possibly occur.

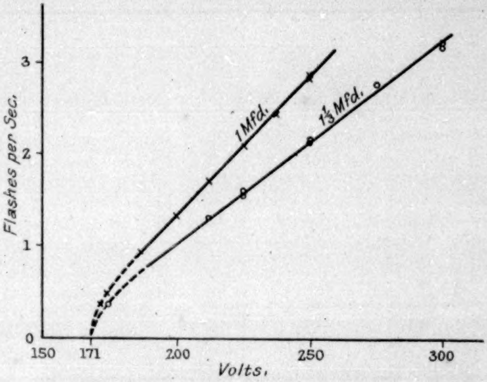

FIG. 8.—VARIATION OF FREQUENCY WITH APPLIED VOLTAGE.

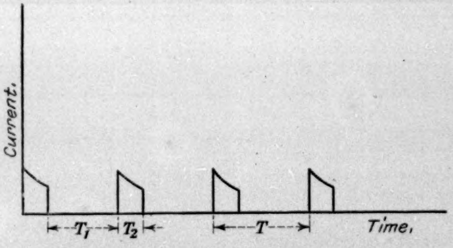

FIG. 9.—WAVE SHAPE OF INTERMITTENT CURRENT.

From a consideration of the laws of charge and discharge of a condenser, in conjunction with the characteristic curve of a neon tube, it follows that the wave-shape of the intermittent current obtained from such an arrangement will be somewhat as shown in Fig. 9. This may possibly be modified by insertion of inductance or by including a tuned or resonant circuit for any particular frequency.

The device which has been described here provides a very simple means of producing intermittent currents without the use of any moving mechanism or contacts, and should prove particularly useful over the audible range of frequencies.

There is no doubt that the three-electrode thermionic valve provides the most satisfactory means of generating very high or radio frequencies, but in order to get an audio frequency current from a thermionic valve, very costly apparatus is necessary, whereas, with the flashing neon lamp the apparatus is extremely simple and inexpensive.

Among the suggested possible uses of the arrangement are :—

(1) Audio frequency—Telephonic measurements.
Stroboscopic measurements.

(2) High frequency—Radio communication and measurements.

DEMONSTRATION of (*a*) *A Magnetic Pivot, and* (*b*) *A Self-charging Electroscope.*

By Major C. E. S. PHILLIPS, *O.B.E.*

A MAGNETIC PIVOT.

THIS device may take various forms, and is intended to provide a means of rapidly setting up a delicate pivot in the laboratory. In the first of two examples exhibited a steel knitting needle stood vertically with its lower end resting upon a plate of glass, while the upper end was supported (without touching) by a permanent magnet held in a clamp immediately above.

The magnet could be lowered till the needle was nearly lifted from the glass plate, and, since the upper end was free and the lower end only pressing upon its bearing point with an extremely small force, the friction of the system was reduced to a minimum. It was then shown that the needle was very free to rotate about its axis, and that when a short arm was attached radially to the centre of it, and the needle itself slightly inclined, a control was afforded by the fact of the arm always moving so as to place its mass centre as low as possible. Under these conditions, but with the needle held nearly vertically, a slight lateral movement of the base-plate gave rise to a large deflection of the radial arm—possibly a useful application as an optical lever.

The other exhibit was a small sewing needle standing vertically with its point resting lightly upon the surface of mercury, while its upper end was supported by a magnet as before. It was explained that a convenient form of the pivot is also obtained by placing the magnet above the concave surface of a watch glass and a needle below with its point held against the convex surface of the glass. In this way large masses of magnetic material can be supported for rotational experiments in which it is required to eliminate friction as far as possible. Many other forms of the device will suggest themselves.

A SELF-CHARGING ELECTROSCOPE.

This electroscope was originally designed and used in 1910-11 for radioactive measurements, but was then found to have a serious defect. In 1911 Prof. Zeleny described an instrument working upon something the same plan, but both this and an improved form suggested in 1915 by Prof. Horton are not free from this same defect—viz., the occasional adhesion of the gold leaf to the charged surface with which it periodically comes into contact. In the original form of this electroscope the leaf moved intermittently against a carbon point, which was kept charged to a constant potential of —200 volts, while the leaf stem was connected to an earthed ionisation-chamber, so that with, say, radium present the leaf was continually charging up, and then periodically discharging by touching the carbon point. The number of kicks of the leaf per minute was, of course, dependent upon the extent of the ionisation in the chamber. In the electroscope exhibited the occasional "adhesion" of the leaf is prevented by coating the object touched by the leaf with a layer of fine carbon (arc lamp) powder. The powdered electrode may in fact be approached so closely to the leaf (or fine carbon filament suspended from a gold leaf hinge) that an almost imperceptible movement results which continues regularly and without appreciable adhesion. It was also explained that the arrangement has been made to give a sharp sound audible in a telephone at the instant the leaf or fibre strikes against the powdered electrode, and that this result is being utilised in the design of a recording electroscope. It is expected that the device will prove useful for the measurement of X-ray intensity, &c., and enable electroscopic observations to be made at a safe distance from the source of radiation.

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